THE EJECTION RESONANCE AND THE ANGULAR MOMENTUM OF THE EARTH-MOON SYSTEM. Wm. R. Ward and R. M. Canup, Southwest Research Institute, Boulder, CO 80302

Recently, Cuk and Stewart [1] have suggested that the Earth-Moon system may have had its angular momentum modified by a solar resonance. As the early Moon’s orbit expands due to tidal interaction with the Earth (mass \( M \)) it can be captured into the ejection resonance, which occurs when the precession period of the Moon (mass \( m \)) equals the orbit period of the Earth [2]. Capture excites the Moon’s orbital eccentricity and drains angular momentum from the Earth-Moon system. If the angular momentum loss is substantial as recently advocated by Cuk and Stewart - who numerically integrated the evolution of the lunar orbit with an ersatz version of a constant \( Q \) tidal model - this could allow for a broader range of lunar forming impact scenarios than previously considered viable. Here we further examine this possibility by using complimentary semi-analytical methods in the context of a popular tidal model due to Mignard [3] that assumes a constant time lag, \( \Delta t \), of a body’s response to tidal distortion.

The equations for the tidal evolution of the lunar semi-major axis \( a \) and eccentricity \( e \) vs. \( \tau \equiv t / t_T \) due to Earth tides read
\[
\frac{\Delta a}{\Delta \tau} = \left[ \frac{s}{n} \right] f_1(e) - f_2(e) \left( R / a \right)^8 \]
\[
\frac{\Delta e^2}{\Delta \tau} = \left[ \frac{s}{n} \right] g_1(e) - g_2(e) \left( R / a \right)^8
\]
where \( s \) is the Earth’s spin rate, \( n \) is the Moon’s mean motion, \( f_1, f_2, g_1, \) and \( g_2 \) are functions of \( e \) [4], \( R \) is the Earth’s radius, and \( t_T \) is a characteristic tidal time scale. Since tides conserve angular momentum, we can set
\[
C_0 ds / d \tau = -dL_o / d \tau |_{\Delta \tau},
\]
where \( C_0 \) is the Earth’s spin moment of inertia and \( L_o \equiv m(GMa)^{2 / 3} (1 - e^2)^{1 / 2} \) is the Moon’s orbital angular momentum. The corresponding evolution expressions due to satellite tides are then given by using the lunar spin rate, \( s_M \), in place of \( s \) in the \( \dot{a} \) and \( \dot{e} \) expressions and then multiplying the RHS by
\[
A \equiv \frac{k_M}{k} \frac{\Delta t_M}{\Delta t} \left( \frac{M}{m} \right)^2 \left( \frac{R_m}{R} \right)^5 = 10 \frac{k_M}{k} \frac{\Delta t_M}{\Delta t},
\]
which is a ratio of physical parameters of the two bodies that scales the relative strength of tides on the Moon to tides on the Earth, with \( R_m, k_M \) and \( \Delta t_M \) being the Moon’s radius, tidal Love number, and lag time, respectively and \( k, \Delta t \) are the corresponding quantities for the Earth. The lunar spin rate is then found from setting \( C_M ds_M / d \tau = -dL_o / d \tau |_{\Delta \tau_M} \), where \( C_M \) is the lunar moment of inertia.

In addition to the tidal rates, we need the Lagrange equations associated with the ejection resonance, viz.,
\[
\frac{de}{d \tau} = \frac{15}{4} e (1 - e^2)^{1 / 2} \left( \frac{a}{R} \right)^{3 / 2} \left( \frac{n_\odot}{n_R} \right)^2 (n_\odot t_T) \sin 2 \varphi
\]
\[
\frac{d\varphi}{d \tau} = \left[ \frac{\Lambda^2 (s / n_R)^2}{(1 - e^2)^2 (a / R)^{7 / 2}} \right] - 1 + \frac{3}{4} (1 - e^2)^{1 / 2} \left( \frac{a}{R} \right)^{3 / 2} \left( \frac{n_\odot}{n_R} \right)^2 \left( 1 + 5 \cos 2 \varphi \right) n_\odot t_T
\]
where \( n_\odot \) is the mean motion of the Earth about the Sun and \( n_R \equiv (GM / R^3)^{1 / 2} \). The resonance angle \( \varphi \) measures the difference between the solar longitude, \( \lambda_\odot \), and the position of the moon’s perigee, \( \varphi \), as seen from the Earth. In writing \( \varphi \), we have set
\[
J_2 = J_4 (s / n_R)^2 \quad \text{with} \quad J_4 = 0.315 \quad \text{to take into account the effect of the Earth’s spin rate on its oblateness, and then defined}
\]
\[
\Lambda \equiv (3 J_4 n_R / 2 n_\odot) \left( \frac{3}{2} \right)^{1 / 2} = 54.2.
\]
To evolve the Earth-Moon-Sun system, we use a 2nd order Runge-Kutta routine.
to integrate the tidal and evection equations together. The simulations are started with the Moon in a nearly circular orbit inside the evection resonance location. Tides push the orbit outward until the resonance is encountered, at which time the eccentricity starts to increase rapidly. Eventually, \( e \) becomes so large that the tidal expansion of the orbit stalls in this tidal model. Past this point both \( a \) and \( e \) begin to decrease. The system angular momentum, \( L = C_\oplus s + C_M s_M + L_o \), also decreases. Just how much angular momentum is drained by the Sun depends on the duration of resonance occupancy.

In the left-hand figure below, with initial values \( a/R = 4, \ e \sim 0, 2\pi / s = 2.5\text{hrs}, \ \varphi = \pi/2 \) and an \( A \) of 9, the Moon escapes the evection resonance very soon after the semi-major axis stalls. Once the Moon is out, its semi-major axis resumes an outward migration (dashed line = perigee distance), while the eccentricity undergoes a slow increase. This is qualitatively similar to some earlier investigations of this mechanism [4]. The bottom panel shows the normalized system angular momentum, \( L' \equiv L / C_\oplus R \), which has a starting value of 0.64, but is only slightly changed by the time of escape and remains constant thereafter.

By contrast, the right-hand figure shows the evolution with the same initial conditions except for the starting value of the resonance angle, \( \varphi = 0 \). Here, resonance escape occurs late in the evolution, after the orbit has contracted back to \( a \sim 6R \). By then, significant angular momentum has been removed from the system, leaving \( L \) pretty close to its current value of 0.35. This is qualitatively similar to the results of Cuk and Stewart [1]. Both types of outcomes appear possible with this tidal model.

This work was supported by the NLSI and NASA’s LASER programs.