SATELLITE RECOIL FROM A CIRCUMPLANETARY DISK. Wm. R. Ward and R. M. Canup, Southwest Research Institute, Boulder, CO 80302 ward@swri.boulder.edu.

The gravitational interaction of a satellite with a Roche-interior disk orbiting a primary M_p is examined. A satellite of mass $M_s = \mu_s M_p$ launches spiral density waves in the disk of mass $M_d = \mu_d M_p$ at resonance sites, and the associated torques cause the satellite and disk to mutually recoil [1]. The most powerful zero-order resonances (inner Lindblad resonances that are independent of the satellite's eccentricity, e, and inclination, I), occur where the ratio of the disk's orbital frequency, $\Omega(r)$, to that of the satellite, Ω_s, is m/m-1; the order m (a positive integer) denotes the number of arms in the spiral wave. For a satellite that forms just exterior to the disk, there are many resonances, but eventually the mutual separation equals the distance to the most interior ($m = 2$) resonance at $r = 2^{-2/3}a = 0.630a$, where $a(t)$ is the satellite's semi-major axis. From this point the lowest order resonances remaining in the disk are the $m = 2$, first-order resonances with forcing functions that depend linearly on e and I. There are two such resonances; an inner Lindblad resonance (ILR) first order in e, and an inner vertical resonance (IVR) first order in I. These resonances lie near where $\Omega/\Omega_s = m+1/m$-1, which for $m = 2$, implies $r = 3^{-2/3}a = 0.481a$. Their exact positions are split by the apsidal and nodal precession rates of the satellite and disk material which cause the IVR to fall slightly interior to the ILR [2].

First-order torques, $\{T_L, T_V\}$ are relatively weak compared to their zero-order counterparts, and the outward migration of the satellite slows accordingly. Both the $m = 2$ ILR and IVR have pattern speeds $\Omega_{ps} = m\Omega_s/m+1$. The torques change the perpendicular component of the angular momentum, $L = M_s a^2\Omega_s\sqrt{1-e^2}\cos I$, and the energy, $E = -M_s(a\Omega_s)^2/2$, of the satellite at rates $\dot{L} = T_L + T_V$; $\dot{E} = \Omega_{ps}(T_L + T_V)$. Combining, one can show that [3]

$$\frac{d}{da}\sqrt{1-e^2}\cos I = \frac{1}{2a}(\Omega_s/\Omega_{ps} - \sqrt{1-e^2}\cos I) \quad \textbf{(1)}$$

which is easily integrated to give $\sqrt{1-e^2}\cos I = (m+\sqrt{a_1/a})/(m+1)$ assuming $e_o = I_o = 0$, where a_1 denotes the satellite's position when the last zero-order resonance leaves the disk. This expression is valid for $m = 2$ as long as the first-order resonances dominate the evolution.

Angular momentum conservation arguments can be used to estimate the final value of $\sqrt{1-e^2}\cos I$. As the moon recedes, the ILR and then the IVR leave the disk and weaken precipitously. However, if the IVR leaves the disk, the satellite's migration stalls and a quasi-equilibrium is established between the viscous spreading rate of the disk and the outward drift rate of the satellite which maintains the IVR near the disk edge. The IVR and ILR continue to draw angular momentum from the disk even as it spreads so that its mass and surface density drop. If the disk is not too large, the disk mass is eventually accreted by the primary along with a portion, *i.e.*, $L_{acc} = M_d\sqrt{GM_pR}$, of the disk's original angular momentum; the remaining angular momentum winds up in the moon, *i.e.*,

$$M_s(GM_pa_f)^{1/2}\sqrt{1-e^2}\cos I = L_T - L_{acc} \quad \textbf{(2)}$$

where L_T is the initial total angular momentum of all orbiting material and a_f is the final orbital semi-major axis when the disk is exhausted. Combining with eqn (1), one finds that

$$\sqrt{1-e^2}\cos I = 2f/(3f-1) \quad \textbf{(3)}$$

where $f \equiv (L_T - L_{acc})/L_1$, and $L_1 \equiv M_s(GM_pa_1)^{1/2}$. This can rearranged to read

$$\varepsilon \equiv \sqrt{e^2\cos^2 I + \sin^2 I} = \sqrt{(5f-1)(f-1)}/(3f-1) \quad \textbf{(4)}$$

The quantity ε can be considered a measure of the epicyclic energy of the satellite. As $f \to \infty$, $\sqrt{1-e^2}\cos I \to 2/3$, and $\varepsilon \to \sqrt{5}/3 = 0.745$ is the upper limit attainable by this process.

To relate f to $r_1 = \alpha a_1 = 2^{-2/3}a_1$ and M_d/M_s, we define $\chi(r) \equiv L_d/M_d(GM_pr)^{1/2}$ and write $L_T = L_1(1+\sqrt{\alpha}\,\chi_1 M_d/M_s)$, to infer

$$f = 1 + M_d/M_s(\chi_1\sqrt{\alpha} - \sqrt{R/a_1}) \quad \textbf{(5)}$$

where $\chi_1 \equiv \chi(r_1)$. Combining eqns (4) and (5) yields the epicyclic quantity ε as a function of non-dimensional quantities $M' \equiv \chi_1 M_d/M_s$, $a_1' \equiv \chi_1^2 a_1/R$ as shown in Figure 1.

The validity of these curves probably breaks down if the final position of the disk, $r_f = \alpha a_f$, is predicted to be past the Roche distance, $r_R \sim 3R$ (appropriate for the Earth-Moon system) because the disk may spawn other satellites instead and no longer torque the first formed satellite efficiently. The final edge distance is

$$r_f = \alpha(3f-1)^2 a_1/4 \qquad \textbf{(6)}$$

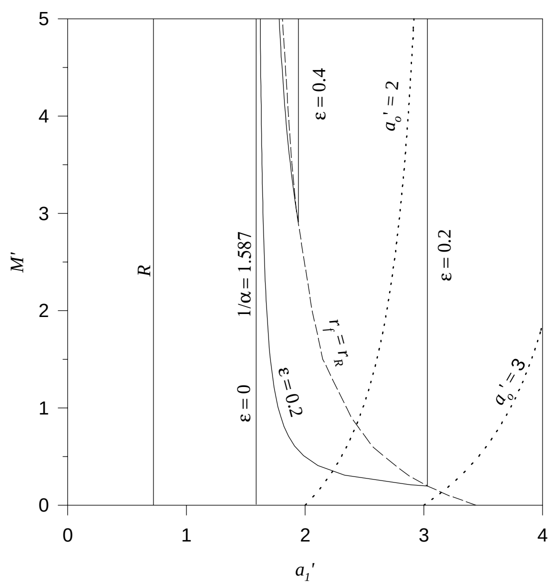

Figure 1: The epicycle strength (solid curves) generated during satellite recoil. Dashed line indicates disks that spread to the Roche distance before depletion. Dotted curves indicate colocation point of satellite and disk edge.

Setting $r_f = r_R$ results in the dashed boundary shown in Figure 1 assuming a value of 0.85 for χ; the surface of the planet, R, is also shown for this value of χ.

It is convenient to define an orbital distance by $L_T = (M_S + \chi_o M_d)\sqrt{GM_p a_o}$, $\chi_o \equiv \chi(a_o)$. This is the position the satellite would have if its evolution were extrapolated backward until it touched the disk's (also evolving) edge. In terms of M',

$$\sqrt{a_o/a_1} = (1 + \sqrt{\alpha}M')/(1 + M'\chi_o/\chi_1) \qquad \textbf{(7)}$$

If we assume that χ is a weak function and that $\chi_1/\chi_o \approx 1$, we can plot eqn (7) (dotted lines) in Figure 1. The intersections of the two families of curves indicate the disk mass required to produce a given value of ε from starting satellite-disk position, a_o.

Finally we consider the function $\chi(r)$; for a power-law disk with surface density $\sigma \propto r^{-n}$, it takes the form

$$\chi(r) = \left(\frac{2-n}{5/2-n}\right)\frac{[1-(R/r)^{5/2-n}]}{[1-(R/r)^{2-n}]} \qquad \textbf{(8)}$$

For a constant surface density, $n = 0$, χ approaches 4/5 as $r/R \to \infty$. For low n and modest $r/R \sim 3$, eqn (8) is fairly insensitive to r.

As an example, consider the Earth-Moon system. Recently, Ward and Canup [3] have suggested that interactions of the newly formed Moon with a remnant of its precursor disk could be responsible for the ~5° inclination of the Moon to the ecliptic. Backward tidal integrations imply that when the Moon resided close to the Earth its orbit was inclined by ~ 10-12° to the equator [4-5]. This implies a minimum of $\varepsilon = \sin I \sim 0.2$. For $\chi = 0.85$, a colocation distance of $a_o \sim 3R$ translates to a non-dimensional distance $a_o' \sim \chi^2 a_o/R \sim 2.2$. Figure 1 indicates that the requisite value of ε can be generated by M' of a few tenths.

So far our results have not depended specifically on a knowledge of the torques T_L and T_V. However, the ratio e/I does depend on their relative strengths. Generally, the full strength ILR is much stronger than the IVR, but two factors mitigate this: (1) Density waves from the ILR propagate toward the disk edge, while bending waves from the IVR propagate toward the primary. If the resonances are closer to the disk edge than the wave damping length, the angular momentum flux carried by the density waves may reflect at the disk boundary and return to resonance largely undamped, which greatly reduces the net torque. On the other hand, torque reduction is less for the bending waves because of the greater round trip path length. (2) Since the ILR is external to the IVR, it lies outside the disk when the IVR is at its edge. Thus, the e/I ratio will depend on the detailed wave mechanics of a particular satellite-disk pair. In general, however, the viscous damping length tends to increase with disk mass so that lower mass disks may favor eccentricity growth whereas higher mass disks may have an increasing inclination contribution to ε.

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References: [1] Goldreich and Tremaine, 1980. *Astrophys. J.* **241**, 425-441. [2] Shu, F., 1984. In *Planetary Rings* (Univ. Ariz. Press); [3] Ward W. R., and R. M. Canup, 2000. *Nature*, in press. [4] Goldreich, P., 1966. *Rev. Geophys.* **5**, 411-439. [5] Touma, J., and J. Wisdom, 1994. *Astron. J.*, **108**, 1943-1961.