# A POWER-LAW DISTRIBUTION OF SOLAR MAGNETIC FIELDS OVER MORE THAN FIVE DECADES IN FLUX

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# ABSTRACT

Solar flares, coronal mass ejections, and indeed phenomena on all scales observed on the Sun, are inextricably linked with the Sun's magnetic field. The solar surface is covered with magnetic features observed on many spatial scales, which evolve on differing timescales: the largest features, sunspots, follow an 11-year cycle; the smallest seem to follow no cycle. Here, we analyze magnetograms from *Solar and Heliospheric Observatory* (*SOHO*)/Michelson Doppler Imager (full disk and high resolution) and *Hinode*/Solar Optical Telescope to determine the fluxes of all currently observable surface magnetic features. We show that by using a "clumping" algorithm, which counts a single "flux massif" as one feature, all feature fluxes, regardless of flux strength, follow the same distribution—a power law with slope  $-1.85 \pm 0.14$ —between  $2 \times 10^{17}$  and  $10^{23}$  Mx. A power law suggests that the mechanisms creating surface magnetic features are scale-free. This implies that either all surface magnetic features are generated by the same mechanism, or that they are dominated by surface processes (such as fragmentation, coalescence, and cancellation) in a way which leads to a scale-free distribution.

Key words: Sun: magnetic fields - Sun: photosphere

## 1. INTRODUCTION

One hundred years ago, Hale (1908) discovered that sunspots are regions on the Sun's surface through which intense magnetic fields thread. It is now well known that sunspots are not the only locations through which magnetic fields penetrate the Sun's surface. Indeed, the Sun's surface is a patchwork of positive and negative magnetic features covering a vast range of sizes (Schrijver & Zwaan 2000; Solanki et al. 2006; Figure 1). The largest features are sunspots which have areas of  $2 \times 10^{19}$  cm<sup>2</sup> and fluxes of several  $10^{22}$  Mx. With current instrumentation, the smallest observable features have areas of just a few times  $10^{14}$  cm<sup>2</sup> and fluxes of order  $10^{16}$  Mx. However, the existence of much smaller features have been detected by using the Hanle effect (Trujillo Bueno et al. 2002).

Sunspots and their associated plage fields are distributed in two active-region bands lying between  $\pm 40^{\circ}$  latitude (Carrington 1858). Smaller magnetic features seem to be randomly distributed over the entire solar surface. In 1843, after a 17-year observing period, Samuel Schwabe discovered that the numbers of sunspots follow a cyclic pattern with a period of 11 years, known as the solar cycle. However, newly emerged smallscale bipoles, known as ephemeral regions (Harvey & Martin 1973), appear to be weakly anticorrelated with the solar cycle (Hagenaar et al. 2003). On the other hand, the general array of mixed-polarity small-scale features does not seem to follow any cyclic behavior.

The entire solar atmosphere is filled with a complex entanglement of magnetic loops resulting from the patchwork nature of magnetic features that carpet the Sun's surface (e.g., Schrijver et al. 1997b; Close et al. 2003, 2005b). Buffeting by overshoots of convection cells from the Sun's interior means that the magnetic carpet is highly dynamic, resulting in energy being pumped into the outer solar atmosphere where, due to the complexity of the magnetic field, it may be stored in the form of electric currents (e.g., Priest 1982). Huge amounts of magnetic energy can be stored in this way, with solar flares (e.g., Benz 2008) probably the best known example of the sudden release of some of this stored "free" magnetic energy. Indeed, it is this highly complex and dynamic behavior of the Sun's magnetic field which results in the outer solar atmosphere, the corona, being heated to some 150–200 times the temperature of the solar surface (e.g., Close et al. 2004, 2005a; Priest et al. 2002; Vlahos et al. 2002; Haynes et al. 2007; Démoulin 2007).

A number of authors (Bogdan et al. 1988; Abramenko & Longcope 2005; Canfield & Russell 2007) have determined distributions of surface fluxes or areas for sunspots and activeregion fields. These papers report that distributions relating to active regions are log normal. However, a couple of papers (Tang et al. 1984; Schrijver et al. 1997a) find that the fluxes of active-region features are distributed exponentially, while Das & Das Gupta (1982) find that the sunspot areas follow a log-normal distribution, but sunspot fluxes with flux greater than  $3 \times 10^{19}$  Mx follow a power law with slope -1.9. In the quiet Sun, however, the distribution of small-scale emerging features has been found to be fitted by either a single exponential (Schrijver et al. 1997b) or double exponential (Hagenaar et al. 2003). The fluxes of all quiet-Sun features, rather than just those that are emerging, were found to follow a Weibull distribution (Parnell 2002). Hagenaar et al. (2003) considered ephemeralregion fluxes and compared the resulting distribution to that found for sunspot fluxes observed by Harvey & Zwaan (1993); they claimed that bipolar active regions that emerge into the Sun's surface are part of a smoothly decreasing distribution that spans almost 4 orders of magnitude in flux and 8 orders of magnitude in frequency. However, their histogram of flux frequencies contained no data in the middle 1-2 decades of flux.

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**Figure 1.** Vast range of sizes of positive (white) and negative (black) magnetic features on the solar surface are shown (a) around a sunspot (2006 December 10) and (b) in the quiet Sun (2007 December 21). Both images cover the same area  $(1.21 \times 10^{20} \text{ cm}^2)$  and have a pixel area of  $1.17 \times 10^{14} \text{ cm}^2$ . In order to reveal the complexity of the fields in each region, the left image has been saturated at  $\pm 1200 \text{ Mx}$  per pixel, while the right has been saturated at  $\pm 50 \text{ Mx}$  per pixel. The images show the circular polarization (the Stokes "V" parameter) in the blue wing of the 6302 Å spectral absorption line. These images were made using the Spectro-Polarimeter on board *Hinode*/SOT, which takes line profiles over a slit of 0'.15; the slit moves to map the required area.

These varied results are somewhat confusing. Magnetic fields on the Sun are generated via dynamo action (e.g., Parker 1955, 1979; Moffatt 1978; Choudhuri 1998; Dikpati & Gilman 2006; Weiss & Thompson 2009) within the solar interior and the above results might at first be interpreted to suggest that active-region and quiet-Sun fluxes are unrelated, supporting the idea of largeand small-scale dynamos acting independently on distinct scales (Cattaneo 1999; Cattaneo & Hughes 2001). We argue that the differing flux distributions discussed above result instead from identifying and counting features in different ways, so the results of the above papers cannot be directly compared with each other.

In this paper, we study photospheric magnetic field data from different instruments in order to investigate the distribution of magnetic features over a wide range of scales. First, in Section 2, we describe the data sets used and their preparation. Then, in Section 3, we explain the method used to identify the magnetic features. The results are presented in Section 4, before the conclusions are discussed in Section 5.

## 2. DATA SETS

Three types of data sets are analyzed from: (1) *Hinode*, Solar Optical Telescope/Narrow-band Filter Imager (SOT/ NFI); (2) *Solar and Heliospheric Observatory* (*SOHO*), Michelson Doppler Imager—high resolution (MDI HR); (3) *SOHO*, MDI full disk (MDI FD). They are all prepared in a similar, although not identical, manner to reduce noise with details given below. Concerns about the use of slightly different preparation methods and the differences this might make to our results are discussed at the end of Section 4.

The first data set comes from taking the ratio of pairs of Stokes V and Stokes I images of the Sun center. These data were taken on 2007 September 19 by Hinode SOT/NFI (Kosugi et al. 2007; Tsuneta et al. 2008) using the blue wing of the Na D line. The data have a pixel area of  $0.16 \times 0.16$  arcsec<sup>2</sup> and a cadence of  $\approx 45$  s. Before taking the ratio of V/I, a pedestal of 800 counts was subtracted from I. These uncalibrated singleline-wing magnetograms are calibrated using the linear calibration constant of  $6555 \pm 600$  G, which is found for a similar SOT/NFI data set and simultaneous MDI HR data set. Details of this calibration were published in Parnell et al. (2008). These NFI magnetograms are then despiked to remove cosmic rays and deconvolved to reduce stray light using an NFI point-spread function (PSF) that was derived by C. E. DeForest et al. (2009, in preparation) using NFI observations from the 2008 July eclipse. We applied a fast Fourier transform filter to reduce the effect of P-modes and related noise. This filter eliminated any features traveling faster than the 7 km s<sup>-1</sup> sound speed. The filter was apodized in velocity  $(\omega/k)$  as well as in temporal frequency ( $\omega$ ) in order to avoid introducing artifacts in the filtered data. Finally, the images were smoothed spatially, using a 2 pixel FWHM Gaussian kernel, and temporally, using a 3 minute FWHM Gaussian weighting function, and resampled (in the same step) to a 1.5 minute regular cadence. In order to avoid the SOT oil bubble (Ichimoto et al. 2008) and the left-hand glint reported by C. E. DeForest et al. (2009, in preparation), we considered a  $853 \times 600$  region from the lower right-hand side of the original data. An example magnetogram is shown in Figure 2(a).

The SOHO MDI (Scherrer et al. 1995) HR data were taken on 2005 October 13 and lasted 17 hr with a cadence of 1 minute and a pixel area of  $0.61 \times 0.61$  arcsec<sup>2</sup>. Full details of the preparation for these data are given in Lamb et al. (2008), but for completeness we give a brief description here. The data are despiked and a radial cosine correction is performed. The data are derotated before being temporally smoothed with a 10 minute FWHM Gaussian giving a cadence of 5 minutes and then spatially smoothed with a 3 pixel FWHM Gaussian kernel. An example magnetogram is shown in Figure 2(b).

The final data type are *SOHO* MDI FD data sets. Three series of MDI FD magnetograms taken in three different years (1998 May 7, 2001 December 8–9, and 2007 December 19) are analyzed. These data series lasted 11, 13, and 5.5 hr, respectively, and each had a cadence of 1 minute. The data used are the



Figure 2. Example magnetograms from (a) SOT/NFI Stokes V/I images taken on 2007 September 19, (b) MDI high resolution taken on 2005 October 13, and (c) MDI full disk taken on 1998 May 7. The relative areas covered by the SOT/NFI and the MDI HR data are shown by the white rectangle and white square, respectively, on the MDI FD image for comparison. All frames have been saturated at  $\pm 200$  Mx cm<sup>-2</sup>.

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Instrument	Region	Duration	Cadence	Area	Pixel Area	Sigma	
		(hr)	(minutes)	(arcsec <sup>2</sup> )	(arcsec <sup>2</sup> )	$(\times 10^{16} Mx)$	
SOT/NFI 07	Quiet	5	1.5	$141 \times 162$	0.16 × 0.16	0.07	
MDI HR 05	Quiet	17	5	$246\times246$	$0.61\times 0.61$	0.54	
MDI FD 98	Active	11	5	$< 60^{\circ}$		35.0	
MDI FD 01	Active	13	5	$< 60^{\circ}$		35.0	
MDI FD 07	Quiet	5.5	5	$< 60^{\circ}$		35.0	

 Table 1

 Details of the Five Data Sets Used

**Notes.** The area of the MDI FD pixels is not given since projection effects significantly affect the true area of the region observed in each pixel: the areas vary by a factor of 2 from the center to the edge of the observed area.

original level 1.8 corrected full-disk data from MDI. To these data a radial cosine correction was also performed and the magnetograms were despiked. Each pixel value was multiplied by the true area of the Sun that it represented, thus converting the pixel values into Maxwells. The data were temporally smoothed using a boxcar average over 10 frames and all data outside  $60^{\circ}$  from Sun center were ignored to avoid pixels that had overly large area multipliers. The data were then reduced to a 5 minute cadence for analysis. An example magnetogram from 1998 May is shown in Figure 2(c).

For all these three data sets the measured photon noise was calculated by determining the FWHM of a Gaussian fitted to the near-zero pixels. Table 1 provides details of the  $1\sigma$  noise level, final area analyzed, cadence, and time interval covered by each of these data sets.

#### 3. IDENTIFYING MAGNETIC FEATURES

There are three main ways of identifying magnetic features (DeForest et al. 2007) and in each case the features identified are different.

- 1. *Clumping* (Parnell 2002) which finds flux massifs—a collection of contiguous same-signed pixels with absolute values greater than a lower cutoff, such that each feature is the flux equivalent of a mountain massif.
- 2. *Downhill* (Welsch & Longcope 2003) which finds flux peaks—individual "summits" within flux massifs. Flux massifs are divided into flux peaks along saddle lines, thus forming single-peaked collections of same-signed pixels with absolute values above a lower cutoff.
- 3. *Curvature* (Strous 1994; Hagenaar et al. 1999) which finds flux cores—collections of same-sign pixels about local maxima (minima) that form a convex surface and which have absolute values above a lower cutoff. These features are typically much smaller than the other two types of features observed since only the core flux in the feature is measured. Hagenaar et al. (1999) determined that a correction by a factor of 3 gave a more realistic estimate of the entire flux in the feature.

A simple graphical illustration of these three different types of features is shown in Figure 3. In the illustration, clumping identifies two large flux features, downhill finds three slightly smaller features, while curvature finds two small features; the right-hand peak is too flat to be detected by this method.

All of the magnetic features identified by the above methods are then associated and tracked in time (DeForest et al. 2007). A series of extra criteria may now be applied to determine which features are real. These include a minimum area and minimum lifetime criteria, as well as, in the case of clumping and downhill,



**Figure 3.** Illustration of the flux-identification methods showing two large flux massifs (yellow; clumping method) as contiguous regions above the lower cutoff (purple dashed line). Individual peaks within a flux massif (red diagonal lines; downhill method) are separated by saddle points. The convex cores of the flux summits (blue bricks; curvature method) are found when the summit is not too flat.

the requirement of a second (upper) cutoff above which the peak magnetic field of a feature must lie at some stage during its lifetime. The details of the specific criteria applied here to all data sets in the paper are as follows:

- 1. Before association and tracking:
  - (a) All pixels of any feature must be above a lower cutoff set at  $2\sigma$ .
  - (b) The area of all features much be greater than 4 pixels.
- 2. After association and tracking:
  - (a) The peak magnetic field strength of a feature must, at some stage during the life of the feature, be above an upper cutoff set at  $3\sigma$ .
  - (b) The lifetime of any feature must be at least 4 frames.

Clearly, these three different kinds of feature-identification methods will produce different flux distributions. For instance, flux peaks are commonly found to be distributed in the form of a log normal: the downhill approach was used in three of the active-region studies (e.g., Bogdan et al. 1988; Abramenko & Longcope 2005; Canfield & Russell 2007), while another study used the curvature approach (Schrijver et al. 1997a), which also carves up flux massifs into smaller features. The quiet-Sun papers, which applied either curvature (Schrijver et al. 1997b; Hagenaar et al. 2003) or clumping (Parnell 2002) techniques to MDI HR data, found distributions covering barely 1–2 decades of flux, so it is not surprising that these results were inconclusive.

In this paper, only one feature-identification method, the clumping approach, is applied to find features of all sizes in each of the data sets studied. This approach counts flux massifs. Sample frames with all the identified flux massifs colored are shown in Figure 4. These frames are the same as those shown in Figure 2.

The reason the clumping approach is chosen, instead of either of the other two, is because it has been shown to be far less sensitive to changes in pixel resolution and sensitivity which occur when data sets from different instruments are compared (Parnell et al. 2008; C. E. Parnell et al. 2009, in preparation). Over the range of sizes and fluxes considered here, an increase in resolution and sensitivity leads to more substructure, or "summits," observed within the flux features. Both the downhill and curvature methods are very sensitive to the number of "summits" that are observed. This means as the resolution/sensitivity improves these two methods find fewer large peaks or cores, but more smaller ones. Hence, the observed flux distributions from the same area of the photosphere would be different if two different instruments were used (Parnell et al. 2008; C. E. Parnell et al. 2009, in preparation). However, it



Figure 4. Same magnetograms as those in Figure 2 except, here, all flux features identified using the clumping method have been colored (positive flux massifs: greens/yellows/oranges/reds; negative flux massifs: black/purples/blues/cyans).



Figure 5. (a) Histograms of feature fluxes observed in the SOT data (blue), MDI high-resolution data (green), and 1998 May MDI full-disk data (red) using the clumping feature-identification method. (b) As (a), except here the lines are SOT data (blue)—for comparison—2001 December (orange) and 2007 December (cyan) MDI full-disk data. The dashed line in both graphs is a fit to the data in (a) and has slope -1.85.

has been found that, when counting flux massifs, large features maintain their integrity as the resolution/sensitivity increases. However, as one would expect, as the resolution/sensitivity increases, more smaller features are picked up, which simply extends the observed flux distribution to smaller scales. Thus, clumping appears to be the most robust approach in the situation where flux distributions from different instruments are to be compared, as is the case here.

#### 4. DISTRIBUTION OF FLUXES

The characteristics of the flux massifs found in each of the data sets are given in Table 2. Clearly, the mean fluxes of the features decrease as the resolution of the instruments increases. However, there is a wide spread in mean fluxes for the full-disk features. The 2007 MDI FD data set is very quiet with no active regions present, but the 2001 MDI FD data set has many active regions. It is therefore no surprise that in 2001 the mean flux of massifs is more than seven times that in 2007.

Parnell et al. (2008) and C. E. Parnell et al. (2009, in preparation) found that flux massifs observed in a pair of simultaneous data sets taken on 2007 June 24 by MDI HR and SOT/NFI followed a single power law over 3 orders of magnitude with a power-law index of -1.85. Here, we consider not only the distribution of the fluxes of flux massifs found in new SOT/NFI and MDI HR data sets, but also look at the distribution of flux from larger features using MDI FD data to determine if the power law found by Parnell et al. (2008) and C. E. Parnell et al. (2009, in preparation) is general.

 Table 2

 Characteristics of the Magnetic Features Detected in Each Data Set

Instrument	Time of Cycle	Total No. of Features	No. per Frame	Mean Flux $(\times 10^{18} \text{ Mx})$	Mean Area (Mm <sup>2</sup> )
SOT/NFI 07	Min	251,205	1262	0.33	0.72
MDI HR 05	Fall/min	71,652	355	4.90	15.04
MDI FD 98	Min/rise	429,256	3328	101.13	1515.43
MDI FD 01	Max	482,279	3132	128.05	2139.81
MDI FD 07	Min	315,989	4647	18.20	915.41

Histograms of the feature fluxes from the SOT/NFI, MDI HR and MDI FD 98 data sets are plotted in Figure 5(a). Amazingly, all three distributions appear to follow a single power law, although the low-flux tail of each distribution falls off. One of the limitations of all the flux-identification methods is that they underestimate the amount of flux in small features because flux from pixels below the artificially imposed lower cutoff is ignored. Furthermore, small-scale features are also more likely to fail the stringent criteria that all features must meet to be counted. If a feature fails just one of the criteria, then it and its associates are ignored. This means that the low-flux tail of the distribution of feature fluxes falls off artificially and must be discarded if the true distribution of fluxes is sought. Furthermore, large-scale fluxes in each of the instruments may also be underestimated due to limitations of either the observed area or the duration of observation, or both. In particular, observations of sunspots are problematic with the possibility of under-reporting of their true fluxes by the MDI magnetograph, as discussed by Liu et al. (2007).

So if these low-flux tails are ignored, then these three data sets suggest that the fluxes of flux massifs in the solar photosphere follow a single power law that extends over more than 5 orders of magnitude in flux and 10 orders of magnitude in frequency. The dashed line fitted to the data has the form

$$N(\phi) = N_f \phi^{-1.85} \text{ Mx}^{-1} \text{ cm}^{-2},$$

where  $N(\phi)$  is the frequency of occurrence of massifs with flux  $\phi$  and  $N_f = 3 \times 10^{-4}$ . This line appears to fit all observed flux values from small-scale intranetwork fluxes with just a few times  $10^{17}$  Mx up to sunspots and large regions of plage with  $10^{23}$  Mx.

Since it is well known that solar magnetic fields follow a cycle, a key question to answer is: how does the distribution of feature fluxes vary over the 11-year solar cycle? Features detected from the MDI HR data reliably span little more than one decade in flux, so unfortunately it is not possible to determine the distribution of small-scale fluxes using these data. Furthermore, SOT has only been operational for about two years, so it is only possible to determine reliably the distribution of small-scale magnetic features during this solar minimum. MDI FD data, though, cover a much greater range of feature fluxes: 3-4 decades (red line in Figure 5(a)) when sunspots exist. So we consider additional full-disk data sets taken near solar maximum (2001 December) and solar minimum (2007 December). Histograms of the massif fluxes found in these data sets are plotted on a graph with the SOT histogram that was found earlier (Figure 5(b)). The same dashed line from Figure 5(a) is plotted on this graph too for comparison purposes. The 2001 distribution again appears to follow a power law over four decades with a similar slope to that of the SOT data and the fitted dashed line. In contrast, the 2007 flux histogram seems to only follow the -1.85 power-law slope up to a few times  $10^{20}$  Mx, after which the number of features drops off rapidly. This is not surprising since no sunspots are present in this data set.

Histograms have many inherent problems and are, therefore, not ideal for quantifying the nature of the distribution. In particular, by varying the bin size and/or using different weighted or unweighted fitting methods, a wide variety of power-law indices may be derived for the distribution considered (e.g., Parnell 2004). In order to avoid bias and to determine accurately the power-law indices of these flux distributions we use maximum likelihood (e.g., Parnell & Jupp 2000; Parnell 2002; Clauset et al. 2007). To determine the true slope of the observed flux distribution, we choose only fluxes above a specified minimum flux  $\phi_0$  which is chosen for each data set such that all the fluxes in the low-flux tail of the distribution are discarded.

The form of the power-law probability density function (PDF) is

$$f(\phi; \alpha) = \frac{(\alpha - 1)}{\phi_0} \left(\frac{\phi}{\phi_0}\right)^{-\alpha}$$

where  $\phi$  are the observed fluxes ( $\phi \ge \phi_0$ ) and  $\alpha$  (where  $\alpha > 1$ ) is the index of the power law. By definition the integral of a PDF over all  $\phi$  must be one. Hence, if one wishes to determine the true frequency of feature fluxes, the function  $f(\phi; \alpha)$  must be multiplied by a factor  $f_0$ , where  $f_0$  is the number of features per Maxwell per cm<sup>2</sup> per frame.

Finding an accurate estimate,  $\hat{\alpha}$ , of the power-law index,  $\alpha$ , of the best-fitting power law for a given minimum flux  $\phi_0$  is very

 Table 3

 Maximum Likelihood Parameters for Each Data Set

Instrument	М	$\phi_0$	â	$f_0$	K-S Stat.
		$(\times 10^{18} Mx)$		$(\times 10^{-37} \text{ Mx}^{-1} \text{ cm}^{-2})$	
SOT/NFI 07	50,359	0.2	-1.86	36.64	0.02
MDI HR 05	15,547	2.0	-1.83	2.47	0.08
MDI FD 98	276,461	11.0	-1.86	1.50	0.06
MDI FD 01	175,324	18.0	-1.84	0.75	0.03
MDI FD 07	145,065	10.0	-2.12	1.44	0.07

straightforward using maximum likelihood, and is simply given by

$$\hat{\alpha} = 1 - \frac{M}{M \log \phi_0 - \sum_{i=1}^M \log \phi_i}$$

where the *M* observed fluxes,  $\phi_i$ , are ordered such that  $\phi_0 \leq \phi_1 \leq \cdots \leq \phi_i \leq \cdots \leq \phi_M$ . The particular parameter values found for the best-fitting power laws, the  $f_0$  factors, and the chosen  $\phi_0$  values are given in Table 3.

The errors on the  $\hat{\alpha}$  estimates are very small, < 0.01, but these errors simply reflect the fitting of the power law. One source of error not accounted for in that number is the error resulting from the choice of  $\phi_0$ . For each data set, except MDI FD 07, varying the truncation value  $\phi_0$  used in the fit leads to derived exponents that range over 1.74-1.97; but going too low includes too many low-flux features near the observational-limit rollovers, while going too high excludes too many low-flux features to the right of the rollovers. For the MDI FD 07 data set the exponent ranges between 1.99 and 2.23 as the truncation value  $\phi_0$  is varied. The truncation value for each data set was chosen so that we achieved a balance between maximizing the numbers of feature fluxes included and at the same time maintaining a power-law distribution, i.e., minimizing the goodness-of-fit statistic. The test of goodness of fit is explained below after a discussion of the systematic errors.

Another source of the error in the  $\hat{\alpha}$  estimates comes from systematic errors due to feature algorithm, instrumental, and observational effects (e.g., feature-identification threshold, instrumental biases, cadence, area, and duration of data). Since small-flux features lie nearer the detection limit, their occurrence frequencies are much more susceptible to systematic biases than large-flux features. Comparisons of flux distributions from different algorithms applied to the same data by DeForest et al. (2007) demonstrated that algorithmic differences can lead to differences of  $\sim 3$  in feature fluxes at the low-flux end of the distribution. Since these differences arose from the details of how flux from weak-field pixels was treated, we hypothesize that systematic errors from (nonalgorithmic) instrumental and observational effects, which also primarily affect weak-field pixels, can also cause variations by a factor of about 3 in the numbers of small-flux features found. For our study, which only employed a single algorithm, we therefore estimate an uncertainty of  $\sim 0.5$  a decade in feature frequency over five decades in flux, leading to an uncertainty in estimated slope of  $\pm 0.1$ . Our derived indices, apart from the MDI FD 07 index, are sufficiently close to be indistinguishable at this level of uncertainty. By including the assumed uncertainty introduced by algorithmic choices, we calculate a total uncertainty of 0.14 in our estimate of the true power-law slope.

Maximum likelihood does not test whether a power-law distribution is a good fit to the data. It simply determines the most probable power-law distribution that fits the data. In order



Figure 6. P–P plots testing goodness of fit for the five data sets: SOT/NFI 2007 (blue), MDI HR 2005 (green), MDI FD 1998 (red), MDI FD 2001 (orange), and MDI FD 2007 (cyan).

to test whether a power-law distribution is a good fit to the data, we use the Kolmogorov–Smirnov (K–S) goodness-of-fit test. The K–S statistic D of each of the power-law fits to the data sets are determined (Table 3), where

$$D = \max_{\phi_0 < \phi < \infty} |F(\phi; \hat{\alpha}) - F_{\text{emp}}(\phi)|$$

Here,  $F(\phi; \hat{\alpha})$  is the cumulative distribution function, while  $F_{\text{emp}}(\phi)$  is the empirical distribution function. These functions are defined as

 $F(\phi_i; \hat{\alpha}) = 1 - \left(\frac{\phi_i}{\phi_0}\right)^{1-\hat{\alpha}},$ 

and

$$F_{\rm emp}(\phi_i) = \frac{i - 0.5}{M},$$

where  $\phi_i$  is the *i*th largest of the *M* ordered fluxes in the data set.

Clearly, the K–S statistic is just a one-number summary of the plot of  $F(\phi; \hat{\alpha})$  versus  $F_{emp}(\phi)$ . Such a plot is known as a P–P plot. So to get a better handle on the goodness of fit of our power-law distributions, we plot their P–P plots (Figure 6). A perfect fit would simply lie along the line x = y. Obviously, since we have observational data, we do not expect to get a perfect fit, but the SOT/NFI and MDI FD 01 distributions clearly lie very close to this line over their entire range. Thus, for these data sets the power-law distributions found are good fits. The MDI FD 98 distribution of fluxes is not fitted as well by a power law, although a power law is not an unreasonable fit to the distribution.

Both the MDI HR and MDI FD 07 distributions are clearly not particularly well fitted by power law. This is not surprising since this was also clear from the histograms. From Figure 5(a), it is apparent that the MDI HR data fall off more quickly than the other distributions from the power law at low fluxes. The reason for this rapid and extended fall off is possibly because MDI is not optimized for high-resolution mode observations, therefore requiring a high degree of post-processing to extend the distribution of fluxes to the faintest features possible. This post-processing seems to have succeeded down to a certain level, but there are clearly still many faint features that have been missed. This means the distribution bends away from the more accurate distribution over this range given by the SOT line. As already mentioned, the solar minimum MDI FD 07 data set also has a well defined dog-leg due to a significant lack of fluxes above  $10^{20}$  Mx and so, not surprisingly, these data are not well described by a single power-law distribution. However, from Figure 5(b), it appears as if below  $10^{20}$  Mx the MDI FD 07 fluxes do seem to follow the same power law as the SOT/NFI 2007 data (as one would expect). Hence, we suspect that flux features of all scales actually do follow a single power law, but the lack of sunspots at solar minimum creates a cutoff in flux at about  $10^{20}$  Mx.

Finally, we address the issue of whether the slightly different preparation methods for the three data sets has a significant impact on the distributions we have determined. We think this is unlikely since our main result is consistent across all three data sets and preparation methods, which would be highly unlikely if the particulars of any "reasonable" data preparation method significantly affected the flux features detected. Indeed, Parnell et al. (2008) and C. E. Parnell et al. (2009, in preparation) also identified flux massifs in a pair of simultaneous SOT/NFI and MDI HR data sets, and found power-law distributions with the same -1.85 slope that we report here, although those data were not prepared in the same way as the data in this paper. In particular, those SOT/NFI data were not deconvolved with a PSF, and the MDI HR data were not filtered in the same manner.

#### 5. CONCLUSION AND DISCUSSION

A single power law of flux features over all currently observable scales (more than 5 orders of magnitude in flux and 10 orders of magnitude in frequency) strongly suggests that the mechanism generating magnetic features of all scales is the same. Two possibilities arise. Either (1) all magnetic features are created by a solar dynamo that acts in the same way on all scales, or (2) after their emergence into the solar atmosphere, all magnetic features are dominated by surface processes, described below, which somehow creates a single distribution of feature fluxes. These two possible scenarios are discussed below.

Solar magnetic fields are known to be created by dynamo action (Parker 1955, 1979; Moffatt 1978; Choudhuri 1998; Dikpati & Gilman 2006; Weiss & Thompson 2009) which must occur in or just below the convection zone. Theoretical modeling has established that it is not possible for a dynamo acting throughout the convection zone to produce sunspots, since strong magnetic fields rise too rapidly due to magnetic buoyancy (Parker 1984; Choudhuri & Gilman 1987). Instead, these features are most likely created by a dynamo situated around the base of the convection zone (Spiegel & Weiss 1980; Parker 1993). This idea was strengthened by the discovery of the tachocline, a shear layer just below the base of the convection zone (Christensen-Dalsgaard & Schou 1988; Schou 1991; Parker 1993; Schou et al. 1998; Hughes et al. 2007).

Where do the observed small-scale magnetic features come from? Some arise from the decay of sunspots and active regions (Martinez Pillet 2002), but not all can be explained by this mechanism. Instead, many small-scale magnetic fields emerge as ephemeral regions, small bipoles, with unsigned fluxes of  $10^{19}$  Mx. Hagenaar (2001) showed that there were enough ephemeral regions emerging to replace all the quiet-Sun flux within 14 hr. It is possible that these small-scale bipoles are generated by a second (local) dynamo just below the photosphere (Cattaneo 1999; Cattaneo & Hughes 2001; Hagenaar 2001). In this scenario, dynamo action is predominantly driven

by turbulent convection flows, although the tachocline may play a role (Corbard & Thompson 2002).

Our observation of a single power law over all scales does not appear consistent with the idea that two separate dynamos are dominating the flux distribution, one in the tachocline and one at the surface. Instead, it suggests one of the two possibilities. The first is that the flux distribution is caused directly by the dynamo and that, in addition to the solar-cycle dynamo at the tachocline, turbulent dynamo action occurs continuously over a range of scales throughout the convection zone, from the tachocline right up to the surface. The largest-scale flux features (greater than  $10^{20}$  Mx), created in the tachocline, would then be choked off at solar minimum, but smaller features (less than  $10^{20}$  Mx) would continue to be produced in the convection zone throughout solar maximum and solar minimum. This idea has some support from the recent numerical convection simulations of Stein et al. (2009), which show that convection does not occur at two discrete scales (granulation and supergranulation), but rather that it occurs at a continuum of scales whose scale length increases with depth.

The second possible explanation for the single power law is that the magnetic field is fed in at most at a few specific scales, but that the reprocessing of the flux via several physical processes dominates the observed flux distribution. The behavior of magnetic fields in the solar surface is dominated by the continual convective motion of the plasma on the surface. New flux emerges as a cluster of features whose total flux is zero—i.e., equal amounts of positive and negative flux emerge. Flux emerges near cell centers and is swept to the edges of cells. From the moment they emerge, flux features may undergo three important processes (e.g., Schrijver et al. 1997b). They may encounter other features of either the same or opposite polarity. In the former case, the feature will then coalesce to form a larger feature, while in the latter it will cancel, removing equal amounts of positive and negative flux from the canceling features. Furthermore, convective motions can also break up magnetic features, a process known as fragmentation. This type of behavior is very common at both large and small scales, and is one of the mechanisms by which sunspots are dispersed.

It is generally believed that fragmentation produces a lognormal distribution, which is true if the distribution is formed by the breakup of a single large feature, but that is not the case on the Sun. Instead, the features come from a range of sources, and a combination of emergence, coalescence, cancellation, and fragmentation may well produce a power-law distribution. Schrijver et al. (1997b) used the magnetochemistry equations and a given set of assumptions to show that an exponential distribution of fluxes was achievable. Parnell (2002) used the same equations, but different assumptions, to achieve a Weibull distribution. So it would not be unreasonable to expect that a third set of assumptions might produce a power-law distribution, as we have found here.

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