

Analysis of the Hungaria asteroid population

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ARTICLE INFO

Article history:

Received 17 December 2008

Revised 29 May 2009

Accepted 3 June 2009

Available online 14 June 2009

Keywords:

Asteroids
Photometry

ABSTRACT

There are approximately 5000 known asteroids in the Hungaria orbital space, a region defined by orbits with high inclination ($16^\circ < i < 34^\circ$), low eccentricities ($e < 0.18$), and semi-major axes $1.78 < a < 2.0$ AU. We argue that this region is populated by a large number of asteroids formed after a catastrophic collision involving (434) Hungaria, the presumptive largest fragment of the Hungaria collisional family. The remaining objects form a background population that share orbital characteristics with the family members. Due to the general dynamic stability of the region, it is likely that most asteroids in Hungaria space (the Hungaria “group”) have been in this region since the formation of the Solar System or at least since the planets assumed their current orbital configuration. Our examination of the Hungaria group included comparing rotation rates, taxonomic classification, and orbital dynamics to determine the characteristics of the family and background populations. We first found there is an excess of slow rotators among the group but, otherwise, the distribution of spin frequencies is essentially uniform, i.e., that a plot of the cumulative number of objects over the range of $1 \text{ d}^{-1} < f < 9 \text{ d}^{-1}$ is nearly a straight line or, put another way, if the distribution over the range is binned by equal intervals of $f(1-2 \text{ d}^{-1}, 2-3 \text{ d}^{-1}, \text{etc.})$, the number of objects in each bin is statistically the same.

There is a distinct family within the Hungaria group, centered at a semi-major axis of 1.940 AU, with a dispersion range that increases with decreasing size of members, as expected of an evolved collisional family. The larger members with well-determined taxonomic class, including (434) Hungaria itself, have flat spectra, mostly likely type E or similar. The degree of spreading versus size of family members is consistent with that expected from Yarkovsky thermal drift in roughly 0.5 Gyr, suggesting that age for the family. The Asteroid (434) Hungaria is displaced in semi-major axis by 0.004 AU from the center of the Hungaria family. The collision event that produced the family should not have left the largest body displaced by more than 0.001 AU from the original orbit, thus we infer that the displacement of (434) Hungaria is mainly due to Yarkovsky drift, and is consistent with the expected drift for that size body in ~ 0.5 Gyr. Below ~ 1.93 AU heliocentric distance the Hungaria family is perturbed by at least two secular resonances, $2g - g_5 - g_6$ and one of the family of 4th or 6th order secular resonances near $s \sim -22.25$ "/year. Their combined effect results in larger inclination dispersion of the family members.

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1. Introduction

Before proceeding, it is important to differentiate between members of a *family* and of a *group* in the context of this work. Members of a group share common orbital characteristics, i.e., similar values of a , e , and i . They may or may not have similar taxonomy or albedos (e.g., Cellino et al., 2002). Family members,

however, share not only orbital characteristics but other traits as well, such as similar taxonomy and albedos, and thus appear to originate from a common parent asteroid through a catastrophic collision. We will use the terms “Hungaria group” and “Hungaria family”, as defined above, in the rest of this paper. If we use the term “Hungaria” or “Hungarias” without qualification, it should be taken to mean the Hungaria *group*. We will also use the term “background” and take it to mean those asteroids in the Hungaria *group* minus those in the Hungaria *family*.

Hungaria orbital space is bounded by the v_5 and v_{16} secular resonances, and by Mars-crossing in (a, e) space (Gradie et al., 1979). The zone is roughly defined by semi-major axis a ($1.78 < a < 2.0$ AU), eccentricity e ($e < 0.18$), and inclination i ($16^\circ < i < 34^\circ$). With their,

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on average, high albedos ($p_V \sim 0.3\text{--}0.4$), this makes the Hungarias among the smallest main belt objects that can be readily studied with modest-size instruments. Since they are also not generally subject to tidal encounters with the terrestrial planets, the Hungarias provide a control set within the main belt to compare against the near-Earth asteroids (NEAs) for such characteristics as rotation rate distribution and binary population. Studies along those lines (discussed in more detail below) have shown some striking similarities between the NEAs and Hungarias. This suggests that forces other than tidal encounters were responsible for the binary and/or paired asteroids in the Hungaria group.

It is likely that, because of their dynamical stability, the objects trapped in Hungaria space have resided there since the planets assumed their final configuration. This makes the asteroids in the region prime targets for studies concerning the formation and evolution of the Solar System in terms of orbital dynamics, space weathering, and YORP/Yarkovsky forces, among others. The large surveys coming on-line or planned for the near future, e.g., Pan-STARRS, LSST, and the Discovery Channel Telescope (DCT), will be able to provide a wealth of information that will supplement and extend the current state of knowledge about the Hungarias. In this work examine what is known about the Hungarias in terms of rotation rates, taxonomic makeup, and orbital dynamics.

It appears that most, but not all, of the smaller objects in the Hungaria group were likely produced by the catastrophic disruption of the Hungaria parent body and so are members of the Hungaria family. Fig. 1 shows all currently known objects in Hungaria space using absolute magnitude (H) as an indicator of size versus the semi-major axis. It is easy to see the large concentration of Asteroids around (434) Hungaria (dark triangle) that fall within the V-shaped zone (curved lines) characteristic of a collisionally-born cluster of asteroids (see, e.g., Zappalà et al., 2002). The vertical line represents the center of the V-shape. This figure and, in particular, the apparent offset of (434) Hungaria from the center of the “V” will be discussed in detail in Section 4.

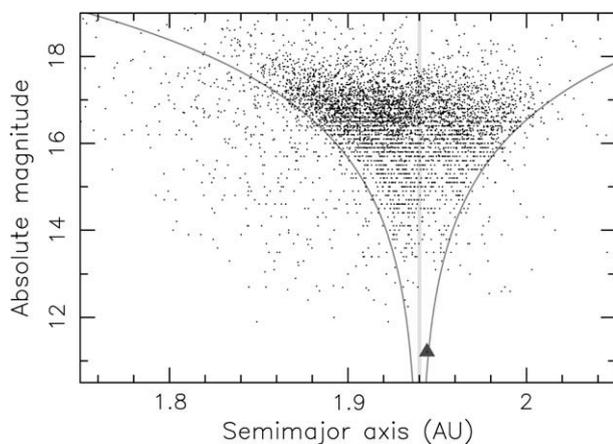


Fig. 1. Hungaria-population asteroids projected onto the plane of absolute magnitude H versus proper semi-major axis a (~ 5000 objects). The curved lines show our assumed Hungaria-family zone where the highest concentration of the objects is located; it also correlated with the concentration of objects in Fig. 5. To define the family zone in this diagram we used a canonical form $0.2 H = \log((a - a_c)/C)$, where $a_c = 1.94$ AU and $C = 3 \times 10^{-4}$ AU, discussed by Vokrouhlický et al. (2006b). (434) Hungaria itself (dark triangle) is displaced outward with respect to the symmetry axis of the zone delimited by the curves by about 0.0045 AU. Yet, with the estimated 10 m/s characteristic dispersal velocity, we would expect it that it would initially fall within the ± 0.001 AU distance from the center (vertical gray zone). We interpret this displacement as an accumulated drift due to Yarkovsky forces on (434) Hungaria.

2. Lightcurve and rotation rate analysis

There are approximately 5000 known asteroids in Hungaria orbital space, as determined from pseudo-proper elements in the AstOrb data file (Bowell, 2008). Asteroids in the Hungaria group have been the subject of a concentrated study the past few years by the authors. That program has produced one or more lightcurves for more than 100 Hungarias and determined statistically-useful rotation rates and amplitudes for almost all observed objects. Those lightcurve observations have also led to the discovery of seven binary asteroids. Since the Hungarias are not subject to tidal encounters with Mars or Earth, the discovery of binaries in this population requires that other forces were involved in binary formation, likely the YORP (Yarkovsky-O'Keefe-Radzievskii-Paddack) effect (Warner and Harris, 2007; Bottke et al., 2006). YORP is the thermal re-radiation of sunlight that creates a torque on smaller ($D < 50$ km) irregularly-shaped bodies that can cause the rotation rate to increase or decrease. In the case of the former, the parent body can eventually reach a critical spin rate where it sheds mass in order to prevent total break up. That mass can reform into a separate body that goes into orbit about the parent, becoming a satellite (Pravec and Harris, 2007; Walsh et al., 2008), or the two bodies can escape from one another to become separate asteroids with very similar heliocentric orbits (Vokrouhlický and Nesvorný, 2008, 2009).

It has been suggested that some binaries in a collisional family may be created during the family-forming event, i.e., they are escaping ejecta binaries or EEBs (see Durda et al., 2004). However, all of the binary asteroids found to date among the Hungarias have fast-rotating primaries, which suggests that they were more likely formed by YORP spin up.

Warner and Harris (2007) showed that the spin rates among the Hungarias have a relatively uniform distribution down to a frequency of $\sim f = 1 \text{ d}^{-1}$ and that below that ($f < 1 \text{ d}^{-1}$) there is an excess of slow rotators. The bottom part of Fig. 2 shows the frequency distribution for the 129 Hungarias with reliable lightcurve periods in the Asteroid Lightcurve Database (LCDB, Warner et al., 2008) as of 2008 December. The excess of slow rotators is clearly seen, amounting to approximately 28% of the total number of asteroids. Pravec et al. (2008) did an extended study that included small main belt and Mars-crossing asteroids ($3 < D < 15$ km) as well as a subset of the Hungaria lightcurves and found similar results, including the excess of slow rotators (approximately 21% of the total set). Rossi et al. (2009) show that this excess is a direct result of YORP action, and, in fact, they suggest that the observed excess should be even greater. This leads us to suspect that the excess we observe in the Hungarias may be closer to the actual excess in both populations rather than any difference between the populations.

The top section of Fig. 2 shows the spin distribution for the range of $0 \text{ d}^{-1} < f < 2 \text{ d}^{-1}$. The slight “spike” for $0.25 < f < 0.5 \text{ d}^{-1}$ is also seen in the Pravec et al. data set. A larger data set of slow rotators is needed to determine if the effect is statistically significant. The mostly uniform distribution of the spin rates $f > 1 \text{ d}^{-1}$ means that the number of objects at any given spin rate is approximately the same as at any other spin rate over the range of $1 \text{ d}^{-1} < f < 9 \text{ d}^{-1}$. This also appears to be the result of the YORP effect (Pravec et al., 2008).

3. Taxonomic classification

Until recently, only a small number of the Hungaria group had been taxonomically classified. Tholen (1987) included only 25 group members while Bus and Binzel (SMASS II, 2002a,b) listed only 18. These numbers are hardly sufficient to draw conclusions about the group, let alone the family. The release of the Sloan Digital Sky Survey Moving Object Catalog (MOC, Ivezić et al., 2001) provided the

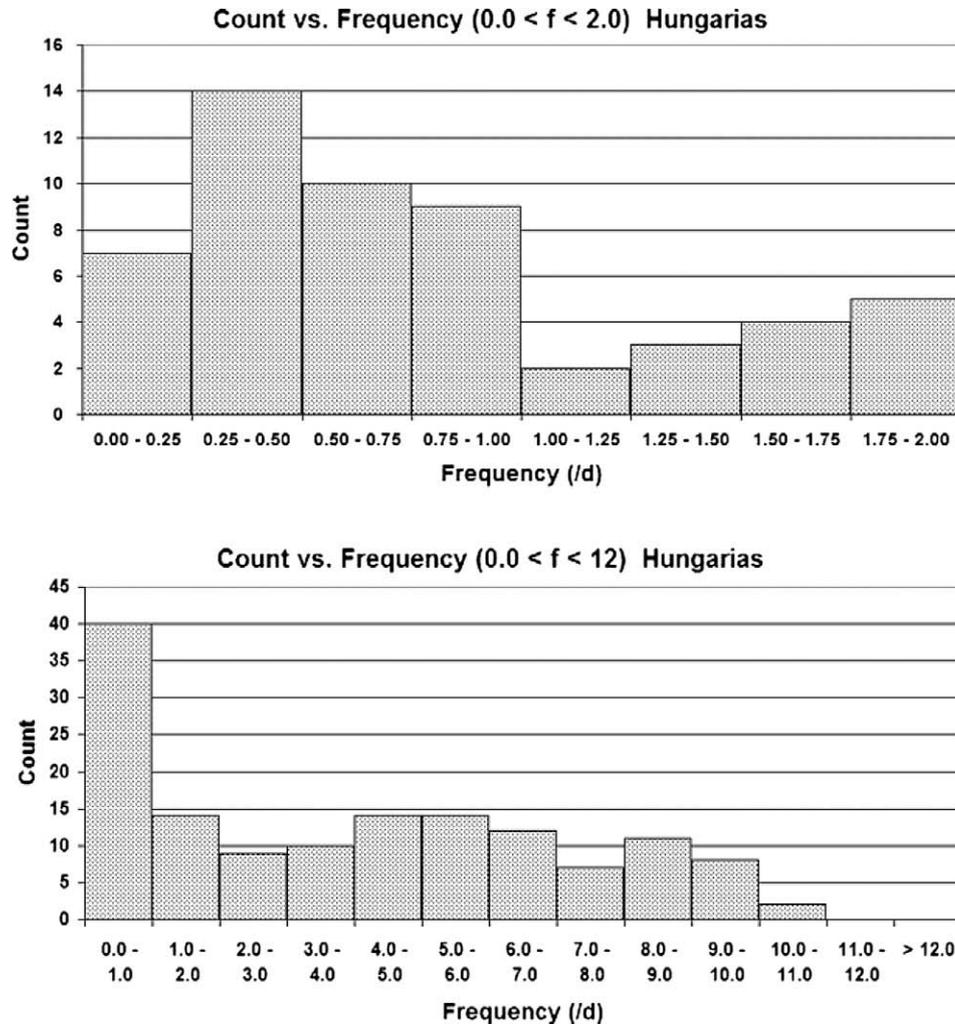


Fig. 2. The bottom plot shows spin rate distribution of the 129 Hungaria asteroids in the LCDB that have statistically-useful rotation rates. An excess of slow rotators ($f < 1 \text{ d}^{-1}$) stands out from the rest of the population, which otherwise shows a relatively uniform distribution through $\sim f = 10 \text{ d}^{-1}$. The top plot is a close-up of the range $f < 2 \text{ d}^{-1}$ to show a small spike in the range $0.25 \text{ d}^{-1} < f < 0.50 \text{ d}^{-1}$ for which there is currently no explanation.

opportunity to explore family membership in greater detail. Nesvorný et al. (2005) used Principal Component Analysis (PCA) to map SDSS *ugriz* magnitudes onto the SMASS II taxonomic system that proved very effective in distinguishing among S, C, and X-type asteroids. The X-type is degenerate in that it encompasses other types (E, M, and P), with accurate albedo information required to remove the ambiguity. The recent fourth release of the MOC (ADR4, Parker et al., 2008) allowed us to take another look at the Hungaria population using the original Nesvorný PCA method.

The *u* magnitudes in the SDSS catalog are often very noisy and so, despite having data on more than 1000 Hungarias in ADR4, only 362 could be reliably classified. Even so, the results were strongly conclusive. Of the 362 asteroids, 22 (6.0%) were type C, 63 (17.2%) were type S, and 282 (76.8%) were type X. As will be shown in subsequent sections, there is convincing evidence of a collisional family in Hungaria space and that (434) Hungaria is the largest fragment of that family. That asteroid is known to be type E, which is characterized by a flat “X-type” spectrum. In the case of (434) Hungaria itself, we also know its albedo, $p_V = 0.38$ (Morrison and Zellner, 1979).¹ It is a reason-

able assumption that other members of the family will be of that type and, indeed, most of those with spectral information do show a flat spectrum.

Justification for this assumption is based on a review of more than 3000 asteroids with known or SDSS-derived classes where we counted taxonomic classification versus semi-major axis. Only eight asteroids with $a < 2.2 \text{ AU}$ have flat spectra. Of those, only two have known low albedos ($p_V < 0.10$) and, more important, are near-Earth asteroids (NEA), not main belt asteroids (MBA), and so their provenance is unknown. Similar checks using the SIMPS catalog (Tedesco et al., 2002) and Asteroid Lightcurve Database (LCDB, Warner et al., 2008) also found very small numbers of flat spectra, low albedo MBAs with $a < 2.2 \text{ AU}$. These reviews strongly suggest that a flat spectrum implies something other than type C in the inner reaches of the main belt.

As a side note, we did look at two other SDSS mapping schemes, those being Masi et al. (submitted for publication) and Parker et al. (2008). The former did not make a distinction between X and C type asteroids, though assuming the Masi C types (flat spectrum) to be X/E, we found similar results for the Hungaria family and background populations as when using the PCA method. Parker et al. did not distinguish between C and X and the distinction between S and C was such that we found an unjustifiable number of background objects in Hungaria space.

¹ The Morrison and Zellner albedo was based on the $V(1, 0)$ system. The value used here, $p_V = 0.38$, is on the $H-G$ system and was derived using the original albedo and H and G values from the Asteroid Lightcurve Database (LCDB, Warner et al. 2008).

We also did a search correlating SIMPS (Tedesco et al., 2002) and AstOrb data to look for Hungaria asteroids common to both and which had low albedos ($p_V < 0.2$). We found a total of twelve common objects with five matching the criteria: (1656) Suomi, $p_V = 0.157$; (3353) Jarvis, $p_V = 0.074$; (5384) Changiangcun, $p_V = 0.072$; (17,657) 1996 VO4, $p_V = 0.031$; and (21,688) 1999 RK37, $p_V = 0.050$. It should be noted that even these five objects do not have firmly established albedos because of uncertainties in the SIMPS program and so may or may not actually have low albedos. Ideally, the values should be confirmed with modern-day instruments and measurements.

These assumptions, i.e., that supposed C- and X-type asteroids within the Hungaria population, more so for the *family*, are most likely actually E-type is supported by other work (e.g., see Carvano et al., 2001) but contradicts some conclusions recently stated by Assandri and Gil-Hutton (2008) where they used SDSS colors on 334 Hungaria asteroids. In part, they claim that 59% of the asteroids are of type X, 26% are type C, and 9% are type S. They further assert that the (presumed) relatively large number of C-type asteroids, since they formed in other regions, may indicate a dynamical mechanism that transports objects from the main belt to the inner Solar System.

We would argue that if the C-type population actually accounted for ~25% of the Hungarias, the SIMPS survey should have found many more objects in Hungaria space, not just the five given above. It is also important to appreciate that X-type asteroids do not have an albedo. This is a broad classification for objects with spectra similar to C, M, or E. It is only after an albedo is found that an object is formally classified as one of these three types. In a similar vein, it is not appropriate to classify an object as type C based only on spectral analysis, but wait until an albedo is available. There are “suggestive” slight differences that Bus classifies as Xe and so on, but the success rate of Xe objects being confirmed as type E is poor. Differentiating between X and C classes without an albedo amounts to probabilistic guesswork. We make the assumption that $X = C$ in the outer main belt ($a > 2.6$ AU or so) and $X = E$ in Hungaria space because they are not contrary cases in their respective zones as confirmed by actual albedos.

A recent polarimetric study (Gil-Hutton et al., 2007) also suggests a larger than expected population of darker asteroids in the Hungaria population. First, this work does not distinguish between the Hungaria *group* versus *family*. Only a third of the objects for which a taxonomic class was reported is within the Hungaria *family* as defined by being within the “V” in Fig. 1. We fully expect asteroids outside the family to be of various taxonomic types, including E and, mostly, S.

Finally, we point out that whether X-type asteroids among the Hungarias are actually type C, E, or even M, is largely irrelevant to the main points of our paper. We are using the colors only in the sense as “fingerprints” to separate the two populations, group versus family, and not to say definitely the actual taxonomic type of each asteroid.

Fig. 3 shows an a - H plot of the 367 Hungaria group asteroids for which a PCA taxonomic classification was available. Small dark squares represent C-type asteroids, dark circles represent the X/E-type asteroids, and the small gray triangles represent type S asteroids. The same distinctive V-shape is seen among the Hungarias, with the X/E-type asteroids dominating the “V” and having only a few outliers. The distribution of the S-type asteroids is much more random. Of interest is that all but one of the type C asteroids fall within the collisional family zone. Of the 367 asteroids represented in Fig. 3, only 41 (11%) have lightcurve rotation rates listed in the LCDB. These are shown in the figure as larger circles (X/E) and larger triangles (S). Not included in Fig. 3 are four other asteroids that have “unusual” classes. Three are type A (1600 Vyssotsky, 4713 Steel, and 5641 McCleese) and one is type V (Vestoid, 4483

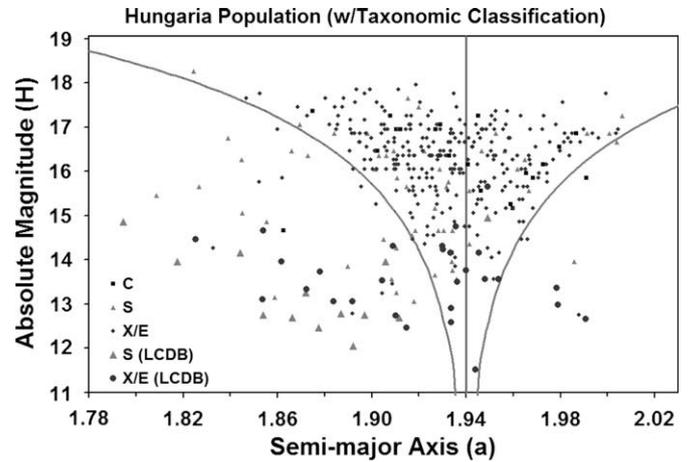


Fig. 3. Similar to Fig. 1, this plot includes only those asteroids with a taxonomic classification based on Tholen, SMASS II, or Nesvorný et al. 367 Asteroids are included, with 77% of them being of type X/E (dark circles), 17% being type S (gray triangles), and 6% of type C (dark squares). The vertical line represents the center of the collisionally-derived family, the largest member of which, (434) Hungaria ($a \sim 1.944$, $H = 11.46$), is located never the vertex of the “V” but displaced slightly outward. The larger symbols (almost all $H < 14$) represent those asteroids for which rotation rates have been determined and included in the Asteroid Lightcurve Database (LCDB).

Petofi). Objects in both of these classes have moderate to high albedos, which fits with the type E class, but given the significant difference in spectra, it is likely that these four asteroids are interlopers.

Looking outside the Hungaria family zone defined by the curved lines, it appears that the number of X/E and S type asteroids is about the same. This, perhaps, is to be expected since it would be unlikely that the parent body that lead to the Hungaria family was the only X/E-type asteroid in Hungaria space some 0.5 Gyr ago. This leads to the interpretation that those X/E-type asteroids outside the family zone are background objects and so may or may not follow the expectation that, having $a < 1.94$ AU, they are all in retrograde rotation.

As might be expected, the number of asteroids with known lightcurves is dominated by objects with both smaller H and semi-major axes, i.e., the brighter members of the group. This goes hand-in-hand with the fact that most of the lightcurve rotation rates have been determined using modest “backyard telescopes” of 0.35–0.5 m size. Appeals have and are being made to observers with larger telescopes to start working the fainter members of the group, in particular the X/E-type asteroids within the “V”. Anyone able and willing to participate in the Hungaria observing program in order to reduce the existing observing biases is urged to contact the corresponding author.

Fig. 4 is another frequency distribution plot similar to Fig. 2 except that it includes only the known S and X/E types and shows the combined totals. The distribution within the 41 asteroids is split about 2:1, with 26 type X/E, and 15 type S asteroids. For the most part, the distribution is similar between the two classes for each frequency bin. The notable differences of no fast rotators ($f > 9 \text{ d}^{-1}$) among the X/E-type asteroids and an excess of slow rotators among the X/E-type asteroids are most likely due to small number statistics rather than any real difference.

4. Dynamics of the Hungaria asteroid population

To set a stage for our work on the dynamics of the Hungaria group, we initially needed the best characterization of the orbital

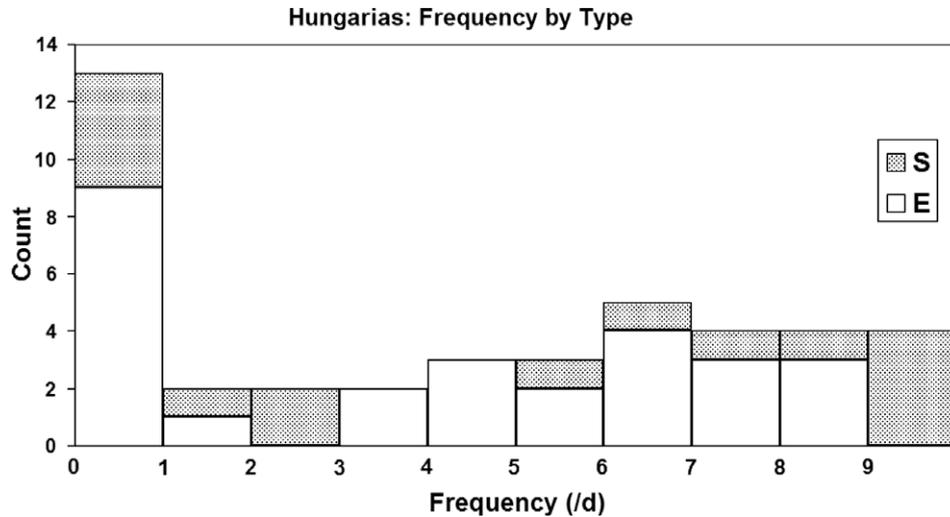


Fig. 4. A plot of the number of objects with a given rotation frequency (d^{-1}) based on the two broad spectral classes shows little differences and the same excess of slow rotators seen in Fig. 2. Since there are only 42 asteroids included, the sample is too small to draw any reasonable conclusions.

distribution of its members and a glimpse into their size–frequency distribution. First we note that there is no set of proper elements for the Hungarias available on-line. The AstDyS web site (<http://newton.dm.unipi.it>) does provide synthetic proper elements for high-inclination objects but always with semi-major axis >2 AU. However, Milani and Knežević (in preparation) are working toward providing a set of proper elements for the Hungarias.

The semi-numerical theory of Lemaître and Morbidelli (1994) developed for high-inclination orbits is applicable in this case. See, for example, Lemaître (1994) where the author noticed clustered values of proper inclination for several members of the Hungaria group and hypothesized that they are fragments from a common parent objects much like members in other main belt families. Since we do not have the software to apply the Lemaître–Morbidelli theory to the currently available (much larger) sample of Hungaria-type asteroids, we adopted a more simple, but acceptable, numerical approach. We extracted all potential Hungaria group asteroids from the AstOrb file (July 2008), some 5150 asteroids, and numerically integrated their orbits for 10 Myr. This integration contained perturbations from all planets (we used SWIFT-RMVS integrator, a time step of 10 days, and output sampling of 500 years). 72 objects were discarded before the integration ended because our initial filter in semi-major axis also included some Mars-crossing objects with very high eccentricities. These are obviously not related to the Hungaria group and so they were rejected. The remaining 5074 objects survived the 10 Myr integration and are likely members of the Hungaria group. We then computed the mean values of the semi-major axis, eccentricity, and inclination of these orbits. These results were used to approximate proper orbital elements for this work.

Fig. 5 shows the results of our calculations. We notice a strong concentration of objects in the range of $20\text{--}21^\circ$ inclination, and somewhat less clustered in the eccentricity values. This is a dynamical mark of the Hungaria collisional family. We should note that we also ran a series of tests using other quantities for the proper elements such as maxima of osculating eccentricity and minima of osculating inclination and achieved similar results. We do not detail those calculations here since the work by Milani and Knežević (in preparation) will lead to actual proper elements for the Hungaria family.

It is also interesting to remark that the Hungaria collisional family does not entirely fill the long-term stable zone of the Hungaria phase space. This fact not only speaks in favor of the collisional origin of this group but also points out to a smaller dimension of the parent object

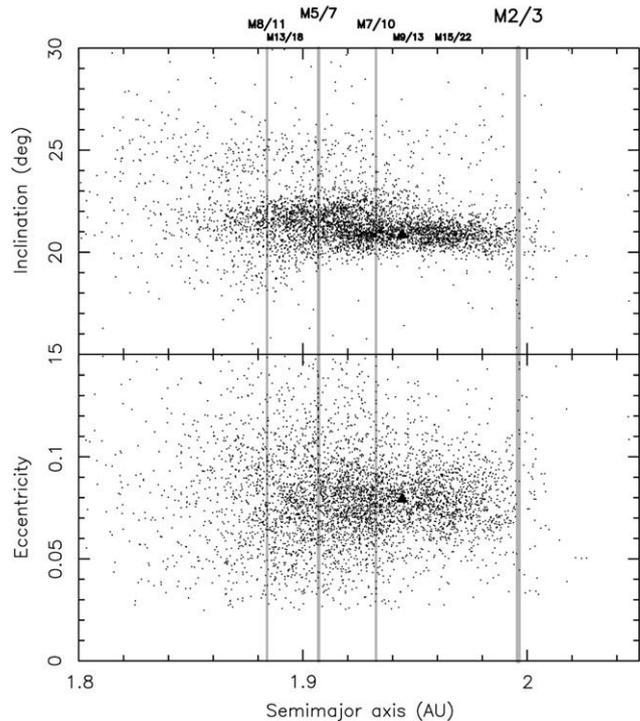


Fig. 5. Pseudo-proper orbital elements for about 5000 Hungaria-type asteroids determined as mean values of the osculating elements over the integrated 10 Myr time interval. The members of the Hungaria family are best seen as a dense concentration of asteroids around the inclination value of $\sim 20.5^\circ$. (434) Hungaria is shown as a large dark triangle at $a \sim 1.944$ AU. The gray vertical lines show major mean motion resonances in this heliocentric zone, all exterior resonances with Mars (see labels above the top panel). The most important is M2/3 which seems to terminate the Hungaria family (and the whole group). The powerful jovian resonances J4/1 and J5/1 are outside the scale of this figure; the higher order jovian resonances (e.g., J9/2 at ~ 1.908 AU or J13/3 at ~ 1.957 AU) or the three body resonances are overall less important. The orbital space occupied by the Hungaria asteroids is also crossed by a number of high-order secular resonances (e.g., Figs. 8 and 9); because of their complicated three-dimensional structure in (a, e, i) it is difficult to show their location in this figure. The major secular resonances surrounding the Hungaria region, ν_5 and ν_{16} are outside the scale of the figure.

(see below). To probe, but not prove, such conclusions we continued our 10 Myr integration of the multi-opposition Hungaria asteroids to 100 Myr. Some objects near the Mars-crossing limit (pericenter dis-

tance ≤ 1.67 AU) as well as those near and beyond the M2/3 exterior mean motion resonance with Mars were either eliminated or showed signs of instability. There are, however, many objects outside the region of the Hungaria collisional family that remain perfectly stable over this integration timespan (see also McEachern et al. 2009 for independent tests of the long-term stability in the Hungaria region).

Returning to Fig. 1, we again note the large concentration of Asteroids around (434) Hungaria (dark triangle) that fall within the V-shaped zone (curved lines) characteristic to the collisionally-born clusters of asteroids. We also point out that the family members dominate the whole population since, for example, there are 2589 objects within the curved lines in the range $15.5 < H < 17.0$ while there are only 232 of those outside the lines. If we consider without prejudice all objects within the region defined by the curved lines and assume that they all have a geometric albedo value of 0.38, the same as (434) Hungaria, their collective volume is equivalent to that of a ~ 26 km body. This implies that these are rather small objects, including the parent body, (434) Hungaria, which has $D \sim 11$ km.

Note that in Figs. 1 and 3 the largest member of the family, (434) Hungaria, is located outward of the center of the cluster by ~ 0.004 AU. We would expect a shift in the semi-major axis of ≤ 0.001 AU with respect to the center of the family after the initial disruption event. The escape velocity from the parent body is $v_{\text{esc}} \sim 16$ m/s, which itself means $\delta a \sim 2 v_{\text{esc}}/v_{\text{orb}} \sim 1.5 \times 10^{-3}$ AU (here $v_{\text{orb}} \sim 21$ km/s is the characteristic orbital velocity at 2 AU). Assuming even up to 10% asymmetry in the transversal velocities of the ejection field of all other members in the family, we would expect (434) Hungaria was launched with a speed $\cong 0.1 \times [(27/13)^3 - 1] \times 16 \sim 11$ m/s implying $\delta a \sim \pm 0.001$ AU from the center-of-mass of the family (the vertical line in Fig. 1). Yet, the observed offset of (434) Hungaria from the family center is much larger, $\sim 4.4 \times 10^{-3}$ AU. We thus postulate that a major part of this offset was not due to the initial velocity of this asteroid but was slowly accumulated over time due to Yarkovsky forces. The fact that this displacement puts (434) Hungaria near the edge of the family drift zone for its size is consistent with this hypothesis.

We checked the hypothesis with the following. It has been shown (e.g., Bottke et al., 2006) that objects spinning in a prograde sense experience outward drift while those with retrograde spin drift inward. (434) Hungaria has a rotation period of 26.5 h and an estimated pole position of $(\lambda_1, \beta_1) = (159^\circ, 65^\circ)$ or $(\lambda_2, \beta_2) = (117^\circ, 64^\circ)$ (Đurech, 2006). These correspond to small obliquities of $\sim 28^\circ$ or $\sim 13^\circ$ with the orbital plane of (434) Hungaria and are, in either case, prograde. This implies that (434) Hungaria should indeed migrate outward from the Sun by the Yarkovsky effect. Its long rotation period suggests that it is near the asymptotic state of the YORP evolution (e.g., see Čapek and Vokrouhlický, 2004). Interestingly, the characteristic timescale of YORP evolution for an asteroid of its size and heliocentric distance is ~ 0.5 Gyr. Smaller asteroids in the Hungaria family should have even shorter time scales of spin evolution, as we see from the nearly flat spin rate distribution. That same timescale (0.5 Gyr) is needed to acquire the suggested 0.002–0.003 AU outward drift of (434) Hungaria with respect to the center-of-mass position of the family. These independent lines of evidence suggest that the Hungaria family is about 0.5 Gyr old. It is thus tempting to link its birth to other major disruption events in the inner part of the main asteroid belt, possibly the birth of the Flora family (e.g., Nesvorný et al., 2002, 2007).

5. Analysis of the size-frequency distribution

Here we analyze the size-frequency distribution (SFD) of Hungaria group, especially its two distinct parts: the Hungaria

family and the background population (the Hungaria group minus the Hungaria family). We do this by examining the absolute magnitude distribution of both populations. With the limited means described above to distinguish family members from the background, we decided to construct a Monte Carlo code to explore how randomly changing the taxonomic type of different asteroids would modify the family SFD (size-frequency distribution). In this code, we assumed the Hungaria family objects were delimited by the curved lines in the a – H projection in Fig. 1. To determine the background population for a given value of H , we looked at the number of objects well outside the family zone in 0.5 magnitude wide strips and determined the number of interlopers per AU for each strip. This number was applied against the width, in AU, of the given strip within the family zone to find the estimated percentage of interlopers within that strip. As a result, we found that for $H > 15$ the interloper contamination is only 1–3%.

After finding the number of interlopers for each 0.5 magnitude strip, we then ran 20 different test cases to find the SFD, each time using the calculated number of interlopers to pick family members at random and move them to the background population. The top panel of Fig. 6 shows the results for the background population

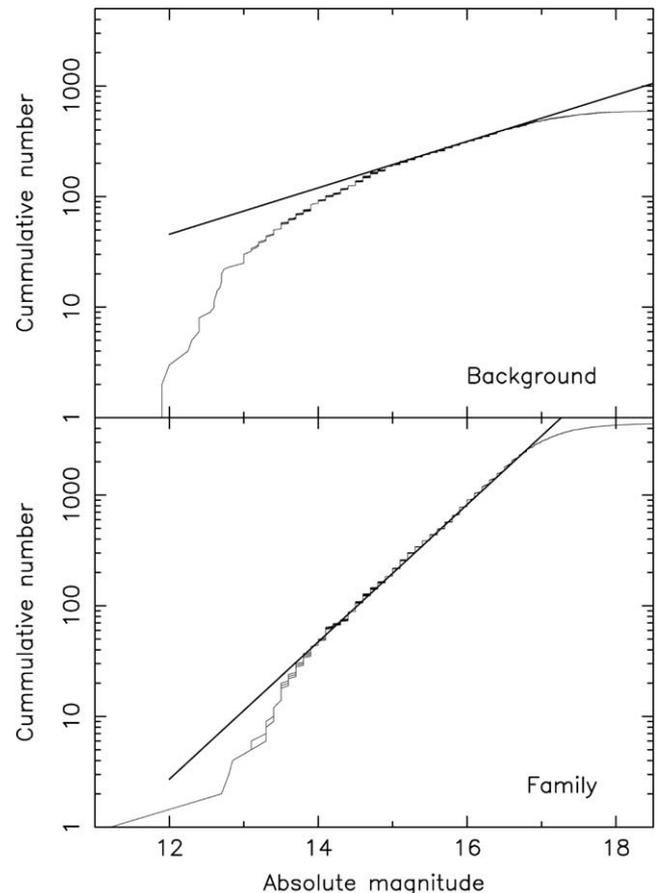


Fig. 6. Cumulative magnitude distribution $N(<H)$ for modeled background population in the Hungaria region (top) and the Hungaria family (bottom). We picked the interloper background objects using a Monte Carlo scheme using 20 different simulations for each case. See the text for details on how the background population count was determined. Approximating $N(<H)$ using a power-law model $N(<H) \cong 10^{\gamma H}$, the straight lines in both panels, we obtain approximate values of the γ parameter. For the Hungarias, we consistently obtain $\gamma \cong 0.62$ in the range $14 < H < 17$, independent of the interloper percentage. This is fairly a well-established value for many asteroid families (steeper than Dohnanyi slope). Since the background is a minor component compared to the Hungaria family, the assumed interloper fraction affects the slope value, which we found to be 1–3% for $H > 17$. Our simulations lead to $\gamma \cong 0.21$ for the background population.

while the bottom panel shows the results for the family. Since the family population dominates the zone between the curved lines in Fig. 1, the results for all of our cases are similar in character and so the individual SFD curves overlap to the point to where they usually cannot be distinguished from one another. The straight lines in each panel represent power-law estimates in the interval of $15 < H < 17$ for the background population and $14 < H < 17$ for the family population. From this analysis we obtain an approximate power-law distribution of the form $N(<H) \sim 10^{\gamma H}$ with $\gamma = 0.21$ for the background population and $\gamma = 0.62$ for the family. Translating the latter value for γ to a cumulative power-law SFD, this yields a power-law exponent of approximately -3.1 [$N(>D) \sim D^{-\alpha}$, $\alpha \sim 3.1$]. This slope is slightly steeper than a power-law SFD in collisional equilibrium as described by Dohnanyi (1969), where the slope would be $\alpha \sim 2.5$. However, the steep slope is expected as evidenced by similar slopes among many other families and it also fits into the hypothesis of the relatively young age of the Hungaria family.

Our analysis of the background population is more complicated because the Hungaria family dominates the background population. The general result mentioned above can be seen by the fact that the number of family members goes up faster with increasing H than the number of background objects. Confirmation of the background results will require additional spectra data. The value of $\gamma = 0.21$, which corresponds to a surprisingly shallow SFD slope of -1.05 , is only slightly shallower than $\gamma = 0.26$, which has been repeatedly found in the SFD of the inner main belt as one moves from multi-kilometer objects to kilometer and sub-kilometer objects. For example, the 3rd and 4th releases of the SDSS catalog find this value for $H > 15.5$ and $H > 16.0$, respectively (e.g., Ivezić et al., 2001; Parker et al., 2008). Assuming an average albedo of $p_V = 0.18$ among inner main belt asteroids, the slope change at $15.5 < H < 16$ corresponds to $D = 2.0$ – 2.5 km diameter asteroids.

This “bump” or slope change in the SFD is thought to be a byproduct of collisional evolution (see Davis et al., 2002, for a review). As asteroids increase in size, they move from the strength-to the gravity-scaling disruption regimes, with the transition occurring near $D = 0.2$ km. Because objects slightly larger than 0.2 km are more difficult to disrupt, more of them survive, which in turn creates an excess number of projectiles capable of disrupting still larger asteroids. This perturbation launches a wavy pattern into the asteroid SFD and creates a bump near $D = 2$ – 3 km.

For Hungaria background objects, the slope change to $\gamma = 0.21$ occurs at $H \sim 15.0$, a somewhat brighter value than those in the inner main belt. However, using the average albedo of the Hungaria family, $p_V = 0.4$, we find this value corresponds to a $D = 2.1$ km asteroid, nearly the same as than those found in the inner main belt. The fact that inner main belt and Hungaria background asteroids have “bumps” at similar diameters suggests the disruption properties of typical objects in both populations are similar to one another. We hesitate to take this further, given the uncertainties in using average albedos to characterize different populations, but if the bump in the Hungaria background SFD is indeed at a slightly larger diameter than the bump in the inner main belt SFD, it could indicate that typical Hungaria group asteroids are more difficult to disrupt than typical inner main belt objects.

The Hungaria group is small enough that it probably could not develop a bump by itself if it were placed in isolation. Instead, we suspect the bump developed via collisions between Hungaria group and inner main belt asteroids. While proving this is beyond the scope of this paper, we did perform the following suggestive calculation. Using the collision probability code described in Bottke et al. (1994), we computed the “intrinsic collision probability” (i.e., the probability that a single member of the impacting population will hit a unit area of the target body in a unit of time) and impact velocity between a representative object of each population: (434) Hungaria

and (8) Flora. We found that a typical Flora object is three times as likely to strike a typical Hungaria object as a typical main belt object is to strike another main belt object ($9.24 \times 10^{-18} \text{ km}^{-2} \text{ year}^{-1}$ versus $2.85 \times 10^{-18} \text{ km}^{-2} \text{ year}^{-1}$). Moreover, Hungaria asteroids also hit inner main belt objects at nearly twice the velocity as standard main belt collisions (9.1 km/s vs. 5 km/s). All of this points in the direction of the inner main belt population playing an important role in the evolution of the Hungaria asteroids over the last several Gyr.

Hungaria asteroids can also be dynamically lost from their zone by Yarkovsky thermal drift forces and resonances working in tandem. This should affect the Hungaria SFD by preferentially depleting smaller objects compared to larger ones. The difference may not be severe, however, because the mobility of smaller objects may be obstructed by the YORP cycles (e.g., Bottke et al., 2006). YORP cycles, which are fairly short for kilometer-size and smaller Hungarias, effectively randomize the asteroid spin axes and thereby change Yarkovsky drift directions from a slow steady march toward or away from the Sun to a random walk. At the extreme limit, when all Yarkovsky effects are random walking with time steps given by the YORP cycle length DT_{YORP} , the effective Da/Dt becomes $(da/dt)/(DT_{\text{YORP}}/T)^{1/2}$. Since both numerator and denominator here are $1/D$, the effective Da/Dt becomes size independent. There is no clear evidence that the limit has been reached for the Hungaria family, though there are hints of it at $H \sim 17$ and greater. Conclusive evidence awaits additional data from upcoming surveys that will reach to larger values of H (smaller diameters).

6. The role of Yarkovsky migration for structure of the Hungaria family

Fig. 5 shows that the Hungaria family has been possibly perturbed below ~ 1.92 AU heliocentric distance. Most strikingly, the tight confinement in the pseudo-proper inclination value for $a = 1.92$ AU disappears and the bulk of the family population is shifted by nearly one degree toward large inclination values. There is also a weaker “stream” of particles going down to smaller values of the pseudo-proper inclination up to ~ 17 – 18° . Either (i) the population of small asteroids migrating by the Yarkovsky forces interacted with weak mean motion or secular resonances to produce this structure (in a similar way as in the Koronis and Eos families; see Bottke et al., 2001; Vokrouhlický et al., 2006a), or (ii) the cluster of asteroids with $a = 1.92$ AU form a separate collisional family. Since there is no clear separate V-shape structure in the a – H projection (Fig. 1) we *a priori* tend to prefer the first possibility. In order to get a glimpse whether the low- a end of the Hungaria family is indeed dynamically less stable and if there are dynamical pathways to disperse the inclination values below 1.92 AU, we constructed the following numerical experiment.

We selected asteroids in the Hungaria family zone and semi-major axis values in the range of 1.925 – 1.945 AU. We numerically integrated their orbits using the SWIFT-RMVSYS integrator (see, e.g., Brož, 2006). We applied a constant Yarkovsky force equivalent to a 0.1 – 0.2 km object (to speed up the computation) and let the population evolve for 30 Myr. Figs. 7 and 8 summarize sample orbits that demonstrate the ability of the evolving orbits with initial semi-major axes of $1.925 < a < 1.94$ AU to populate both higher- and lower-inclination zones and they migrated inward. We did not integrate outward since the primary goal of the exercise was to see how the high- i , low- a zone could be populated. Approximately 40% of the asteroids did not significantly change orbital inclination. About 40% (shown in Fig. 7) showed a modest increase in inclination while the rest (about 20%, Fig. 8) showed a significant decrease in orbital inclination.

In an attempt to understand the situation somewhat more closely, we computed resonant angles of several putative high-order

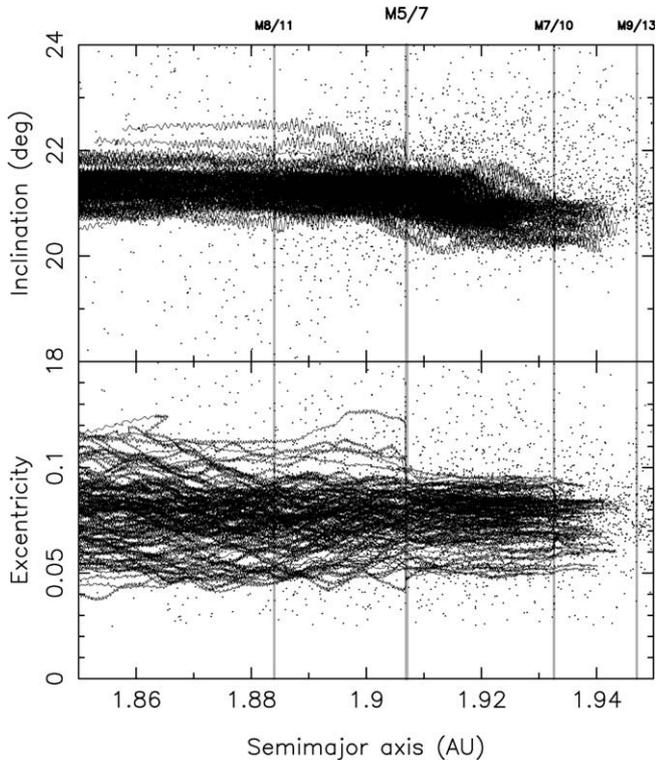


Fig. 7. Evolutionary tracks of 114 Hungaria-family asteroids projected onto the planes of pseudo-proper orbital elements (dark lines; mean values computed in a 1 Myr running window). The black dots in the background are asteroids in the observed Hungaria family. The integrated objects were selected such that they initially started with semi-major axis values of $1.925 < a < 1.945$ AU in the Hungaria family region. We applied a constant Yarkovsky force corresponding equivalently to a 0.1–0.2 km object (to speed up the computation) and let the population evolve for 30 Myr. The selected orbits, with initial pseudo-proper inclination values of 20° – 21° , increased by about 1° , i.e., to 20.75° – 21.75° .

secular resonances. Toward that end we first analysed Fourier spectra of the non-singular eccentricity and inclination vectors to obtain their proper frequencies g and s . We found that the Hungaria collisional family forms a dense cluster in the proper-frequencies plane in between 14 to 16 "/year in g and -22 to -23.5 "/year in s (and would have been also easily identifiable by clustering methods in the frequency space, e.g., Carruba and Michtchenko, 2007, 2009). This zone is delimited or crossed by a couple of potentially important secular resonances such as $2g - g_5 - g_6 = v_5 + v_6$, $s + 2g - 2g_5 - s_7 = 2v_5 + v_{17}$ and a clump of s -related resonances near $s \sim -22.2$ to -22.3 "/year value (Milani, personal communication). Interestingly, the first has already been anticipated by Morbidelli and Henrad (1991). Candidate cases for the s -resonances may be of the 4th order (e.g., $s + g_5 - g_8 - s_3$ or $s - g_5 + g_8 - s_6$) or some of the plethora of the 6th order resonances (e.g., $s - g_6 + g_8 - 2s_6 + s_7$, $s - g_5 + g_6 + s_7 - 2s_8$, $s + g_4 - g_6 - 2s_4 + s_7$, $s + 2g_4 - g_5 - g_6 - s_3$, etc.). None of these resonances is capable of producing a major orbital instability, such as the mean motion resonances like M2/3 (Fig. 5) or overall shape the family structure, such as the z_1 resonance in the case of Agnia and Padua families (Vokrouhlický et al., 2006c; Carruba, 2009), but they can serve as a temporary pathway for orbits undergoing long-term migration due to the thermal forces in a way documented in other asteroid families (e.g., Bottke et al., 2001; Vokrouhlický et al., 2006a). This effect is most strikingly seen in Fig. 8 where it results in macroscopic decrease of the mean orbital inclination value. We found that the example orbits in this figure temporarily interact with one of the $s \sim -22.2$ to -22.3 "/year resonances at ~ 1.93 AU heliocentric distance (Fig. 9). However, this

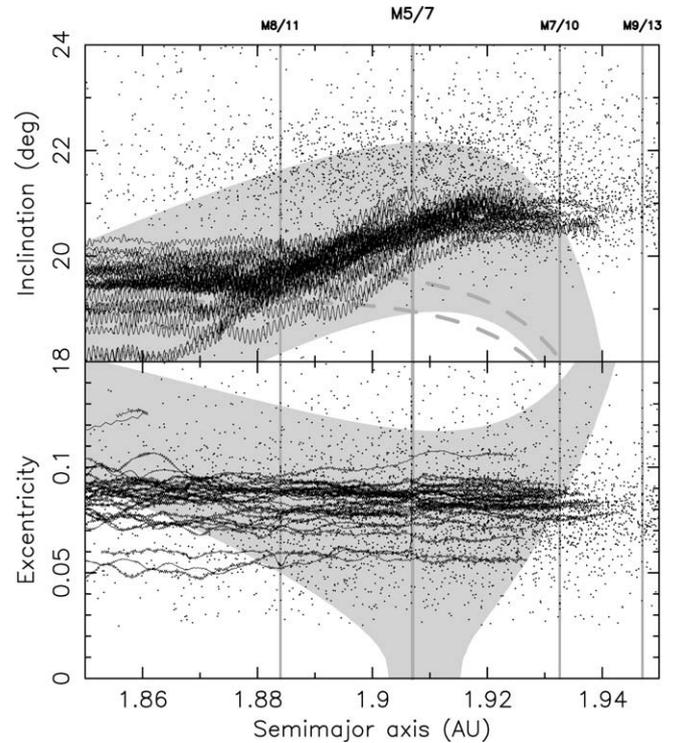


Fig. 8. The same as in Fig. 7 but now the 30 displayed orbits follow an evolution that drags their final pseudo-proper inclination values into the 18° – 20° range. Analysing the orbits in some detail, we found they interact with some of the $s \sim -22.2$ to -22.3 "/year high-order secular resonances at ~ 1.93 AU and by following the resonance they significantly decrease their mean inclination value. At ~ 1.88 – 1.89 AU they become released from this resonance which moves to higher eccentricity zone of the phase space. Some may become captured in the crossing $2g - g_5 - g_6$ high-order resonance (± 0.5 "/year border of this resonance is shown by the dashed gray lines in the top panel). The gray zone shows ± 0.5 "/year region around the position of the $s = -22.25$ "/year denominator related to a number of possible high-order secular resonances: (i) at 0.08 eccentricity fixed in the top panel, and (ii) at 20.5° inclination fixed in the bottom panel (because of their complicated three-dimensional structure we have to use position of the resonance as it intersects a plane of the constant value of the complementary orbital element; we used *secr8* software provided at the AstDyS site, see Milani and Knežević (1992, 1994)).

resonance is crossed by yet another secular resonance $2g - g_5 - g_6$ below ~ 1.9 AU and simultaneously moves to higher eccentricity zone of the Hungaria orbital phase space. The migrating orbits are thus released from the temporary resonant lock, some having even a chance to get temporarily trapped in the $2g - g_5 - g_6$ resonance (Fig. 9). Using the same techniques, we found that the inclination perturbation of orbits in Fig. 8 is also produced by temporary interaction with the s -type resonances. A combination of secular resonant effects and thermal-forces-driven orbital changes thus produce the perturbation of the inclination confinement of the Hungaria-family members below ~ 1.92 AU. Our integration also roughly reproduces relative density of orbits in the different inclination values: there are 5–6 times more orbits lifting their inclination value than those sliding to lower-inclination values, which is in agreement with the much more populated cluster in Fig. 5. In addition, we find that about one-third of the orbits did not significantly change in inclination while migrating through the 1.92 AU threshold. We believe that this can be reconciled by the fact that the small asteroids ($0.5 < D < 2$ km) that dominate this population should have undergone a number of YORP cycles in the estimated age of ~ 0.5 Gyr. As such, they would go through periods of random walking up-and-down over the dynamical structures below 1.92 AU and would likely diffuse to the higher inclination zone. Therefore, we conclude that the perturbed structure of the Hungaria family below ~ 1.92 AU heliocentric

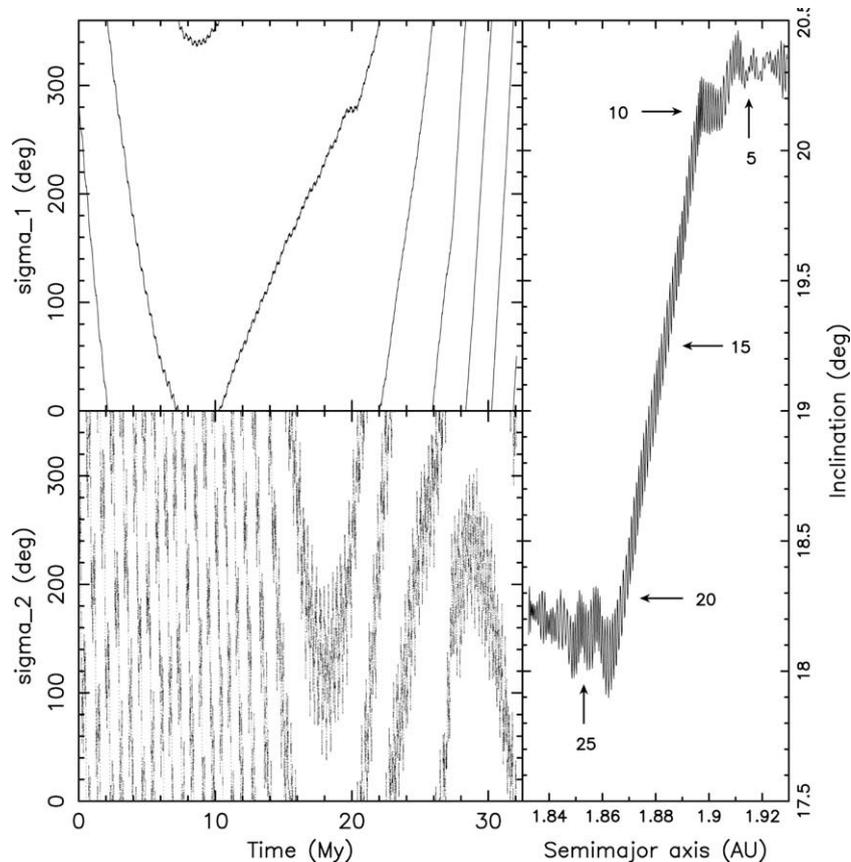


Fig. 9. Left panels show resonant angles σ_1 and σ_2 of the high-order secular resonances $s - g_6 + g_8 - 2s_6 + s_7$ and $2g - g_5 - g_6$ for one of the orbits shown in Fig. 8 (the first is an example of the $s \sim -22.25$ "/year group of resonances). Reversal of their circulation pattern indicates periods when the orbits interact with the respective resonance. The right part of the figure shows the orbit in the projection of 1-Myr running window averaged semi-major axis a and inclination i as in Fig. 8; the labels and arrows point the position of the orbit at different times, namely 5, 10, 15, 20 and 25 Myr. These tic-marks allow localizing time through the $a-i$ evolution and associate it with the corresponding phase of the resonant confinement. Note that between 15 and 20 Myr the orbit resides in both resonances, which is why its lock in the $s \sim -22.25$ "/year resonance becomes destabilized.

distance can be reconciled with a long-term dynamical evolution and does not require an ad hoc hypothesis of a second collisional family.

We also find it interesting that our long-term integration of the Hungaria asteroids without Yarkovsky forces included (mentioned in Section 4) indicates that objects inside and beyond the M2/3 mean motion resonance reside on unstable orbits. This would imply they must have acquired their current orbits only recently, within the last few hundreds of Myr, most likely by heliocentric outward drift due to the Yarkovsky forces.

7. Conclusions

A study of the Hungaria orbital space, located at the inner main belt and characterized by orbits with high inclinations and low eccentricities shows many characteristics common to the NEA population, e.g., both show a relatively flat distribution of spin rates but with an excess of slow rotators. The excess is likely due to YORP causing an asteroid to spin down to this state and the excessive amount of time required for YORP to reverse the trend and cause the asteroid to spin up again. The binary population is approximately 15% in both populations. Since the Hungarias are not influenced by tidal encounters from the terrestrial planets, the mechanism for binary formation is now thought to be primarily YORP forces that cause an asteroid to spin up to where it sheds mass that subsequently becomes a satellite or, in an extreme case,

a pair of asteroids that share nearly identical heliocentric orbits but are not gravitationally bound to one another. It is possible that some of the binaries within the Hungarias are the result of the family-forming event but, given that all binaries found to date have fast-rotating primaries, we believe that YORP spin up is the primary formation mechanism.

More than 5000 known asteroids occupy the Hungaria region with ~ 370 (14%) of them having been roughly classified as either type X/E (77%), S (17%), or C (6%). Claims of a large significant C-type or other low albedo population in the region are not supported by the available albedo data, which shows only five for fewer asteroids with confirmed low albedos in Hungaria space. Those classified as X-type may be any one of three spectrally degenerate classes: E, M, or P. Also, the few type C objects found in Hungaria space using the Principal Component Analysis (PCA) method may yet prove to be type E; independent albedo data are required to remove the ambiguities. In the meantime, a flat spectrum is considered to be type E in Hungaria space since it is consistent with existing albedo data.

The taxonomic data help confirm a true family in Hungaria space. A plot of semi-major axis (a) versus absolute magnitude (H) shows a distinctive "V" shape indicative of a family that has been influenced by Yarkovsky spreading where smaller asteroids have migrated inward or outward more than larger asteroids. Asteroids in prograde rotation migrate outward and those in retrograde rotation migrate inward. The dispersion width in semi-major axis of the collisional family, and the outward displacement of (434) Hungaria, known to be in prograde rotation, from the center

of the “V” indicates an approximate age of 0.5 Gyr for the Hungaria family.

We used pseudo-proper elements to examine the Hungaria members and found strong evidence of a true family, as evidenced by the slope of the cumulative count versus absolute magnitude for those asteroids presumed to be of type E. The “background” asteroids outside the zone of the collisional family (Fig. 1) show a much shallower slope, indicative of them being “random” occupants of Hungaria space. Additional study of the orbital evolution for the Hungarias indicates that there has been relatively little mixing since the Solar System assumed its current configuration and so we draw the conclusion that it is very likely that the Hungaria population is now essentially what it was at that time.

While a significant amount has been learned about the Hungarias, there is much work yet to be done. First, many more lightcurves are needed to obtain a better picture of the rotation rate distribution. Those lightcurves can be applied to shape and spin axis modeling. Given the “V” shape a - H diagram, we would expect to find that prograde rotation is dominant among the Hungarias with $a < 1.940$ AU and retrograde rotation dominant for those with $a > 1.940$ AU. The lightcurves can also be used to look for additional binaries within Hungaria space to confirm the percentage of binaries in comparison to the NEA population. Data on at least another 75–100 Hungarias is required to form a solid statistical sample.

However, that number presumes data without observational biases. The current data set of lightcurves is biased towards brighter (larger and/or closer) members of the Hungarias. We are trying to coordinate with other observers who have access to larger telescopes in order to reduce the size-brightness bias. We also need to assure that long-period objects are not bypassed because of time constraints. While backyard astronomers often have the luxury of time, those with access to larger telescopes often do not and so cannot do sufficient follow up on more difficult targets. We urge those with access to “mini- and micro-telescopes” (≤ 2 m) to contact the corresponding author in order to coordinate efforts to obtain lightcurve data as quickly and efficiently as possible while maintaining quality and bias control over the data.

Additional taxonomic and/or albedo data are needed to confirm assumptions based on SDSS-to-SMASS spectroscopy. When surveys such as Pan-STARSS and LSST accumulate sufficient data, it may be possible to expand the number of classified Hungarias considerably using either an approach similar to Nesvorný et al. or other methods and so do a more thorough analysis of taxonomic and dynamic characteristics.

The Hungarias may represent one of the best “laboratories” for exploring the formation and evolution of the Solar System. What we have presented here reflects just some of what has been learned so far. However, there are still many discoveries to be made about both the Hungarias themselves and what they can tell us about the Solar System as a whole.

Acknowledgments

The authors thank A. Morbidelli and V. Carruba (reviewer) for his helpful insights and discussions. We also thank M. Brož for allowing us to use his linux scripts to plot high-order secular resonances in the Hungaria region and A. Milani for confirming the role of the s -related secular resonances from his independent analysis. Funding for B.D.W. and A.W.H. was provided by NASA Planetary Astronomy Grant NNG 06G132G and National Science Foundation Grant AST-0607505. Additional funding for B.D.W. was provided by a Shoemaker NEO grant from the Planetary Society. The work of D.V. was supported by Czech Grant Agency (Grant 205/08/0064) and the Research Program MSM0021620860 of the Czech Ministry of Education. The work of D.N. and W.F.B. was supported by NASA’s Planetary Geology and Geophysics Program.

References

- Assandri, M.C., Gil-Hutton, R., 2008. Surface composition of Hungaria objects from the analysis of the Sloan Digital Sky Survey colors. *Astron. Astrophys.* 488, 339–343.
- Bottke, W.F., Nolan, M.C., Greenberg, R., Kolvoord, R.A., 1994. Velocity distributions among colliding asteroids. *Icarus* 107, 255–268.
- Bottke, W.F., Vokrouhlický, D., Brož, M., Nesvorný, D., Morbidelli, A., 2001. Dynamical spreading of asteroid families by the Yarkovsky effect. *Science* 294, 1693–1696.
- Bottke, W.F., Vokrouhlický, D., Rubincam, D.P., Nesvorný, D., 2006. The Yarkovsky and YORP effects: Implications for asteroid dynamics. *Ann. Rev. Earth Planet. Sci.* 34, 157–191.
- Bowell, E., 2008. The asteroids orbital elements database (AstOrb). <ftp://ftp.lowell.edu/pub/elgb/astorb.html>.
- Brož, M., 2006. Yarkovsky Effect and the Dynamics of the Solar System. PhD Thesis, Charles University. <<http://sirrah.troja.mff.cuni.cz/yarko-site/>>.
- Bus, S.J., Binzel, R.P., 2002a. Phase II of the small main-belt asteroid spectroscopic survey: The observations. *Icarus* 158, 106–145.
- Bus, S.J., Binzel, R.P., 2002b. Phase II of the small main-belt asteroid spectroscopic survey: A feature-based taxonomy. *Icarus* 158, 146–177.
- Čapek, D., Vokrouhlický, D., 2004. The YORP effect with finite thermal conductivity. *Icarus* 172, 526–536.
- Carruba, V., 2009. The (not so) peculiar case of the Padua family. *Mon. Not. Roy. Astron. Soc.* 395, 358–377.
- Carruba, V., Michtchenko, T.A., 2007. A frequency approach to identifying asteroid families. *Astron. Astrophys.* 475, 1145–1158.
- Carruba, V., Michtchenko, T.A., 2009. A frequency approach to identifying asteroid families. II. Families interacting with nonlinear secular resonances and low-order mean-motion resonances. *Astron. Astrophys.* 493, 267–282.
- Carvano, J.M., Lazzaro, D., Mothé-Dinz, T., Angeli, C.A., 2001. Spectroscopic survey of the Hungaria and Phocaea Dynamical Groups. *Icarus* 149, 173–189.
- Cellino, A., Bus, S.J., Doressoundiram, A., Lazzaro, D., 2002. Spectroscopic properties of asteroid families. In: Bottke, W.F., Cellino, A., Paolicchi, P., Binzel, R.P. (Eds.), *Asteroids III*. University of Arizona Press, Tucson, pp. 633–643.
- Davis, D.R., Durda, D.D., Marzari, F., Campo Bagatin, A., Gil-Hutton, R., 2002. Collisional evolution of small-body populations. In: Bottke, W.F., Cellino, A., Paolicchi, P., Binzel, R.P. (Eds.), *Asteroids III*. University of Arizona Press, Tucson, pp. 545–558.
- Dohnanyi, J.W., 1969. Collisional models of asteroids and their debris. *J. Geophys. Res.* 74, 2531–2554.
- Durda, D.D., Bottke, W.F., Enke, B.L., Asphaug, E., Richardson, D.C., Leinhardt, Z.M., 2004. The formation of asteroid satellites in catastrophic impacts: Results from numerical simulations. *Icarus* 167, 382–396.
- Đurech, J., 2006. Asteroid Spin States and Coarse Shape Models from Sparse Photometric Data and Ordinary Lightcurves, Poster Presented at the IAU236 Meeting in Prague.
- Gil-Hutton, R., Lazzaro, D., Benavidez, P., 2007. Polarimetric observations of Hungaria asteroids. *Astron. Astrophys.* 468, 1109–1114.
- Gradie, J.C., Chapman, C.R., Williams, J.G., 1979. Families of minor planets. In: Gehrels, T. (Ed.), *Asteroids*. University of Arizona Press, Tucson, pp. 359–390.
- Ivezić, Ž., and 31 colleagues, the SDSS Collaboration, 2001. Solar System objects observed in the Sloan Digital Sky Survey commissioning data. *Astron. J.* 122, 2749–2784.
- Lemaître, A., 1994. Hungaria: A potential new family. In: Kozai, Y., Binzel, R.P., Hirayama, T. (Eds.), *Seventy-five Years of Hirayama Asteroid Families*. ASP Conference Series, pp. 140–145.
- Lemaître, A., Morbidelli, A., 1994. Proper elements for highly inclined asteroidal orbits. *Celest. Mech. Dyn. Astron.* 60, 29–56.
- Masi, G., Foglia, S., Binzel, R.P., submitted for publication. Search for unusual spectroscopic candidates among 40313 minor planets from the 3rd release of the Sloan Digital Sky Survey Moving Object Catalog. *Astron. Astrophys.*
- McEachern, F.M., Čuk, M., Stewart, S.T., 2009. Dynamical evolution of the Hungaria asteroids. LPSC (abstract 2554). <<http://www.lpi.usra.edu/meetings/lpsc2009/pdf/2554.pdf>>.
- Milani, A., Knežević, Z., 1992. Asteroid proper elements and secular resonances. *Icarus* 98, 211–232.
- Milani, A., Knežević, Z., 1994. Asteroid proper elements and the dynamical structure of the asteroid main belt. *Icarus* 107, 219–254.
- Morbidelli, A., Henrard, J., 1991. Secular resonances in the asteroid belt – Theoretical perturbation approach and the problem of their location. *Celest. Mech. Dyn. Astron.* 51, 131–167.
- Morrison, D., Zellner, B., 1979. Polarimetry and Radiometry of the asteroids. In: Gehrels, T. (Ed.), *Asteroids*. University of Arizona Press, Tucson, pp. 1090–1097.
- Nesvorný, D., Morbidelli, A., Vokrouhlický, D., Bottke, W.F., Brož, M., 2002. The Flora family: A case of the dynamically dispersed collisional swarm? *Icarus* 157, 155–172.
- Nesvorný, D., Jedicke, R., Whiteley, R.J., Ivezić, Ž., 2005. Evidence for asteroid space weathering from the Sloan Digital Sky Survey. *Icarus* 173, 132–152.
- Nesvorný, D., Vokrouhlický, D., Bottke, W.F., Gladman, B.J., Haggström, T., 2007. Express delivery of fossil meteorites from the inner asteroid belt to Sweden. *Icarus* 188, 400–413.
- Parker, A., Ivezić, Ž., Jurić, M., Lupton, R., Sekora, M.D., Kowalski, A., 2008. The size distribution of asteroid families in the SDSS moving object catalog 4. *Icarus* 198, 138–155.

- Pravec, P., Harris, A.W., 2007. Binary asteroid population I. Angular momentum content. *Icarus* 190, 250–259.
- Pravec, P., and 31 colleagues, 2008. Spin rate distribution of small asteroids. *Icarus* 197, 497–504.
- Rossi, A., Marzari, F., Scheeres, D.J., 2009. Computing the effects of YORP on the spin rate distribution of the NEO population. *Icarus* 202, 95–103.
- Tedesco, E.F., Noah, P.V., Noah, M., Price, S.D., 2002. The supplemental IRAS minor planet survey. *Astron. J.* 123, 1056–1085.
- Tholen, D.J., 1987. Asteroid taxonomic classifications. In: Binzel, R.P., Gehrels, T., Matthews, M.S. (Eds.), *Asteroids II*. University of Arizona Press, Tucson, pp. 1139–1150.
- Vokrouhlický, D., Nesvorný, D., 2008. Pairs of asteroids probably of common origin. *Astron. J.* 136, 280–290.
- Vokrouhlický, D., Nesvorný, D., 2009. The common roots of Asteroids (6070) Rheinland and (54827) 2001 NQ8. *Astron. J.* 137, 111–117.
- Vokrouhlický, D., Brož, M., Morbidelli, A., Bottke, W.F., Nesvorný, D., Lazzaro, D., Rivkin, A.S., 2006a. Yarkovsky footprints in the Eos family. *Icarus* 182, 92–117.
- Vokrouhlický, D., Brož, M., Bottke, W.F., Nesvorný, D., Morbidelli, A., 2006b. Yarkovsky/YORP chronology of asteroid families. *Icarus* 182, 118–142.
- Vokrouhlický, D., Brož, M., Bottke, W.F., Nesvorný, D., Morbidelli, A., 2006c. The peculiar case of the Agnia asteroid family. *Icarus* 183, 349–361.
- Walsh, K.J., Richardson, D.C., Michel, P., 2008. Rotational breakup as the origin of small binary asteroids. *Nature* 454, 188–191.
- Warner, B.D., Harris, A.W., 2007. Lightcurve studies of small asteroids. *BAAS* 39, 432 (13.03).
- Warner, B.D., Harris, A.W., Pravec, P., 2008. Asteroid Lightcurve Database (LCDB). <<http://www.MinorPlanetObserver.com/astlc/default.htm>>.
- Zappalà, V., Cellino, A., Dell'Oro, A., Paolicchi, P., 2002. Physical and dynamical properties of asteroid families. In: Bottke, W.F., Cellino, A., Paolicchi, P., Binzel, R.P. (Eds.), *Asteroids III*. University of Arizona Press, Tucson, pp. 619–633.