**THE EVECTION RESONANCE AND THE ANGULAR MOMENTUM OF THE EARTH-MOON SYSTEM.** Wm. R. Ward and R. M. Canup, Southwest Research Institute, Boulder, CO 80302

Recently, Cuk and Stewart [1] have suggested that the Earth-Moon system may have had its angular momentum modified by a solar resonance. As the early Moon's orbit expands due to tidal interaction with the Earth (mass *M*) it can be captured into the evection resonance, which occurs when the precession period of the Moon (mass *m*) equals the orbit period of the Earth [2]. Capture excites the Moon's orbital eccentricity and drains angular momentum from the Earth-Moon system. If the angular momentum loss is substantial as recently advocated by Cuk and Stewart - who numerically integrated the evolution of the lunar orbit with an ersatz version of a constant *Q* tidal model - this could allow for a broader range of lunar forming impact scenarios than previously considered viable. Here we further examine this possibility by using complimentary semi-analytical methods in the context of a popular tidal model due to Mignard [3] that assumes a constant time lag, $\Delta t$, of a body's response to tidal distortion.

The equations for the tidal evolution of the lunar semi-major axis $a$ and eccentricity $e$ vs. $\tau \equiv t/t_T$ due to *Earth* tides read

$$a^{-1}da/d\tau|_{\oplus}=\left[(s/n)f_1(e)-f_2(e)\right](R/a)^8$$

$$e^{-2}de^2/d\tau|_{\oplus}=\left[(s/n)g_1(e)-g_2(e)\right](R/a)^8$$

where $s$ is the Earth's spin rate, $n$ is the Moon's mean motion, $f_1$, $f_2$, $g_1$ and $g_2$ are functions of $e$ [4], $R$ *is* the Earth's radius, and $t_T$ is a characteristic tidal time scale. Since tides conserve angular momentum, we can set $C_\oplus ds/d\tau=-dL_o/d\tau|_\oplus$, where $C_\oplus$ is the Earth's spin moment of inertia and $L_o\equiv m(GMa)^{1/2}(1-e^2)^{1/2}$ is the Moon's orbital angular momentum. The corresponding evolution expressions due to *satellite* tides are then given by using the lunar spin rate, $s_M$, in place of $s$ in the $\dot{a}$ and $\dot{e}$ expressions and then multiplying the RHS by

$$A\equiv\frac{k_M}{k}\frac{\Delta t_M}{\Delta t}\left(\frac{M}{m}\right)^2\left(\frac{R_m}{R}\right)^5\simeq 10\frac{k_M}{k}\frac{\Delta t_M}{\Delta t},$$

which is a ratio of physical parameters of the two bodies that scales the relative strength of tides on the Moon to tides on the Earth, with $R_m$, $k_M$ and $\Delta t_M$ being the Moon's radius, tidal Love number, and lag time, respectively and $k$, $\Delta t$ are the corresponding quantities for the Earth. The lunar spin rate is then found from setting $C_M ds_M/d\tau=-dL_o/d\tau|_M$, where $C_M$ is the lunar moment of inertia.

In addition to the tidal rates, we need the Lagrange equations associated with the evection resonance, *viz.,*

$$\frac{de}{d\tau}=\frac{15}{4}e(1-e^2)^{1/2}(\frac{a}{R})^{3/2}(\frac{n_\odot}{n_R})(n_\odot t_T)\sin 2\varphi$$

$$\frac{d\varphi}{d\tau}=[\frac{\Lambda^2(s/n_R)^2}{(1-e^2)^2(a/R)^{7/2}}-1$$

$$+\frac{3}{4}(1-e^2)^{1/2}(\frac{a}{R})^{3/2}(\frac{n_\odot}{n_R})(1+5\cos 2\varphi)]n_\odot t_T$$

where $n_\odot$ is the mean motion of the Earth about the Sun and $n_R\equiv(GM/R^3)^{1/2}$. The resonance angle $\varphi$ measures the difference between the solar longitude, $\lambda_\odot$, and the position of the moon's perigee, $\varpi$, as seen from the Earth. In writing $\dot{\varphi}$, we have set $J_2=J_*(s/n_R)^2$ with $J_*=0.315$ to take into account the effect of the Earth's spin rate on its oblateness, and then defined $\Lambda\equiv(3J_*n_R/2n_\odot)^{1/2}=54.2$.

To evolve the Earth-Moon-Sun system, we use a 2nd order Runge-Kutta routine to integrate the tidal and evection equations together. The simulations are started with the Moon in a nearly circular orbit inside the evection resonance location. Tides push the orbit outward until the resonance is encountered, at which time the eccentricity starts to increase rapidly. Eventually, *e* becomes so large that the tidal expansion of the orbit stalls in this tidal model. Past this point both *a* and *e* begin to decrease. The system angular momentum, $L = C_{\oplus} s + C_M s_M + L_o$ also decreases. Just how much angular momentum is drained by the Sun depends on the duration of resonance occupancy.

In the left-hand figure below, with initial values *a/R* = 4, *e* ~ 0, $2\pi / s = 2.5hrs$, $\varphi = \pi/2$ and an *A* of 9, the Moon escapes the evection resonance very soon after the semi-major axis stalls. Once the Moon is out, its semi-major axis resumes an outward migration (dashed line = perigee distance), while the eccentricity undergoes a slow increase. This is qualitatively similar to some earlier investigations of this mechanism [4]. The bottom panel shows the normalized system angular momentum, $L' \equiv L / C_{\oplus} n_R$, which has a starting value of 0.64, but is only slightly changed by the time of escape and remains constant thereafter.

By contrast, the right-hand figure shows the evolution with the same initial conditions except for the starting value of the resonance angle, $\varphi = 0$. Here, resonance escape occurs late in the evolution, after the orbit has contracted back to *a* ~ 6*R*. By then, significant angular momentum has been removed from the system, leaving *L* pretty close to its current value of 0.35. This is qualitatively similar to the results of Cuk and Stewart [1]. Both types of outcomes appear possible with this tidal model.

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References. [1] Cuk & Stewart, *Science*, 10, 1126, 2012; [2] Kaula, W. M. & C. F. Yoder, *Lunar Planet Sci. Conf.* XVII, 440, 1976; [3] Mignard, F., *Moon Planets* 23 185, 1980; [4] Touma, J. & J. Wisdom. *Astron. J.* 115, 1653, 1998.