A HYBRID FLUID/N-BODY MODEL FOR LUNAR ACCRETION. R. M. Canup[1] and W. R. Ward[1], [1]Southwest Research Institute; 1050 Walnut Street, Ste. 426; Boulder, CO 80302; robin@boulder.swri.edu

Introduction: Recent simulations of lunar accumulation from an impact-generated disk have utilized direct N-body integrations to describe the evolution of protolunar disk material [1,2]. These models have been generally successful in producing a single moon just exterior to the classical Roche limit for silicate density material, where $a_R \equiv 2.9R_{Å}$. Predicted accretion time scales are as short as a month to a year, and the mass of the resulting moon is found to be a function of the initial disk angular momentum [1,2].

However, to date such models have not accounted for the thermal evolution of accreting material. As we discuss in more detail below, the disk spreading rates implied by the purely dynamical simulations are so rapid that thermal considerations cannot be neglected, and may indeed significantly influence the evolution of the disk, *e.g.* [3, 4]. In addition, even the highest resolution N-body models ([2]) describe the disk inside the Roche limit as a collection of ~ 100-km diameter bodies. Such massive bodies would not have been able to form within a_R to begin with. Instead, the inner protolunar disk would have been a semi-continuous distribution of much smaller debris and vapor. Such a disk would respond collectively to perturbations from moons accreting outside the Roche limit. Here we describe developments in a new "hybrid" accretion model that treats the Roche interior disk as a fluid disk, while material exterior to the Roche limit is followed using a standard N-body orbital integration.

Interior to the Roche limit, the protolunar disk's tendency to clump due to its own self-gravity is frustrated by planetary tides. Temporary local instabilities form, but are soon sheared apart, leading to enhanced collision rates and an effective viscosity given by

$$n_{WC} \sim \left({}^{M_d}\!\big/_{M_\oplus} \right) r_d^2 \Omega_d \qquad (1)$$

where M_d and r_d are the disk mass and radius, and W_d is orbital frequency. This viscosity, first identified by Ward and Cameron [5], has been directly observed in recent N-body simulations [2]. Associated with (1) is a disk spreading time

$$t_{spread} \sim (r_d^2 / n) \sim 100 (M_L / M_d) T_K \,,$$

where T_K is the orbital period, M_L is a lunar mass, and t_{spread} ~ a month. In contrast, the disk cooling time is ~ 50-100 years [3,4]. The energy released per unit mass of disk material due to spreading is

$$\frac{dE}{dt} \times t_{spread} \sim \left(a \, \frac{d\Omega}{da} \right)^2 r_d^2 \quad (2),$$

or ≈ (a_R / r_d) 5 x 10^{11} ergs/g. Since for a ~ lunar-mass disk $t_{spread} << t_{cool,}$ radiative losses are negligible, and the energy release per gram of disk material implied by (2) exceeds the latent heat of vaporization (~ 2 x 10^{11} ergs/g). Thus the rapid spreading rates of the Roche interior disks predicted by the N-body simulations are unphysical. Even if the disk were to start out in a cold state, material would be heated and soon vaporized by the rapid energy release implied by (2). Vaporization would cause the rate of disk spreading to slow, until a balance between energy released due to spreading and radiative cooling was achieved [3]. This balance would regulate the inner disk spreading to be:

$$t_{spread} \approx 50 \left({}^{a_R}\!\big/_{r_c} \right)^3 \left({}^{T}\!\big/_{2000K} \right)^{-4} \left({}^{M_d}\!\big/_{M_L} \right) yr \,, \qquad (3)$$

orders of magnitude slower than that implied by the N-body simulations (here T is the disk photospheric temperature) [3].

Exterior to the Roche limit, N-body simulations predict accretion of the Moon in ~ months. Whether this short time scale is physically plausible depends on the initial distribution of material in the disk. If the Moon must accrete from material that is initially interior to a_R (as suggested by [2]), the accretion time scale will be governed by the radiation-limited spreading time of the inner disk, from (3) ~ 10^2 years. Vaporization during disk spreading of the great majority of material that ends up in the Moon in this case may occur.

If instead the Moon forms primarily from material that was placed into orbit beyond the Roche limit by the impact event, it may accrete much more rapidly. The primary energy source in this case is the accretional energy, $3(GM_L/R_L)/5$ ~ 2 x 10^{10} ergs/g (where R_L is a lunar radius). The accretional energy is sufficient to vaporize about 10% of a lunar mass in the limit that the material is at the boiling point as it begins to accrete, and ignoring radiative cooling losses. Thus material in the outer disk could accrete on ~ month – year time scales, with the accretional energy budget accommodated by melting and vaporization of some small fraction of the lunar material, *e.g.* [4]. Since $t_{acc} << t_{spread}$ in this scenario, the accreting protomoon will likely co-exist with a massive Roche interior disk for some time, which may also offer a natural explanation for the Moon's initial 10° inclination. [6]

Recent SPH simulations of the impact event [7] find that disks containing sufficient angular momentum to yield a lunar-mass moon just outside the Roche limit typically contain ~ 2 lunar masses, with ~ $1M_L$

outside a_{Roche}. Here we consider this as a basic starting condition for the protolunar disk. We assume that the Roche interior disk quickly heats itself via instability enhanced viscous spreading until it is partially vaporized and its spreading time becomes radiation limited to ~ 10^2 years. The Roche exterior material accumulates rapidly, maintaining a mainly molten and solid composition. As bodies in the exterior disk accrete, they gravitationally interact with the inner disk through resonant torques that transfer angular momentum from the disk (which contracts) to the exterior moonlets (whose orbits expand), *e.g.* [8]. The time scale for recoil from the disk due to resonant torques, t_{recoil}, becomes comparable to the accretion time, t_{acc} when moonlets achieve masses $\geq 0.2M_L$. Once bodies achieve this size, the Roche interior disk and the exterior accreting material will begin to repel one another and decouple. Interestingly, SPH simulations find that intact fragments survive from the impact event with of order this mass ([7]), in which case, the resonant driven inward recoil of the inner disk could commence very soon after the impact event.

Here we are developing a "hybrid" accretion model, in which the Roche interior disk is modeled analytically as a fluid disk, while Roche exterior material is tracked using an N-body accretion model. The N-body method utilized is "SyMBA" [9], which has been modified to include the effect of the zeroth-order disk torques (*i.e.*, due to the 2:1, 3:2, 4:3, etc. mean-motion resonances) on each of the orbiting bodies. At every time step, a "kick" of magnitude $\Delta t \times (\vec{F}_d / m) = \Delta v$ (where Δt is the time step, *m* is the orbiting body's mass, and *Dv* is the change in the velocity) is applied to each orbiting body. The force is given by $v \bullet \vec{F}_d = \Omega_S T$ and $\vec{r} \times \vec{F}_d = T\hat{k}$, where Ω_s is the orbital frequency of the orbiting body, *v* is its velocity, and $T\hat{k}$ is the total torque from the zeroth-order resonances. *T* for a given body goes to zero when the ratio of the body's orbital radius to this disk's outer edge, (a/r_d), is > 1.7, or when the 2:1 resonance is no longer in the disk. The total angular momentum transferred to all of the Roche exterior material is deducted from that of the disk. Also included is a simple model for the viscous spreading of the disk, assuming a constant disk viscosity.

In a preliminary set of ~ 15 simulations we have utilized both the new hybrid model and a standard model using an N-body simulation to model the entire disk. Generally, the accretion times, as well as the masses and orbital properties of the final moons are fairly similar in both cases, and resemble those found in [1]. The largest moon formed in each of our initial simulations had $3.35R_{Å} < a < 5.7R_{Å}$, with $\langle a \rangle = 4.0R_{Å}$, $0.06 < e < 0.3$ with $\langle e \rangle = 0.12$, and $0.2° < I < 2.8°$ with $\langle I \rangle = 1.62°$. The fraction of the total initial mass (inside and outside a_R) incorporated into the largest moon (or sometimes largest two moons) ranged from 0.3 to 0.465, again consistent with [1].

The rapid month-year accretion times predicted by previous N-body simulations are thus most plausible for an initial protolunar disk that contains ~ a lunar mass of material exterior to the Roche limit initially, the case which we have investigated here. However, we find that even in this case, the hybrid and the pure N-body results differ greatly in their predictions for the fate of the Roche interior disk. In the N-body simulations, rapid spreading and scattering clear the Roche interior of all material in a few months to a year. In the hybrid model, a radiation-limited spreading time for the inner disk (~10-100 years) translates to a longer disk lifetime. In the time required for a moon to accrete, the inner disk edge r_d retreats to ~ 2-$2.4R_{Å}$ from an assumed initial value at $a_R = 2.9R_{Å}$ (due to resonant interactions with the accreting moons) but still contains nearly all of its initial mass.

The next scenario we plan to investigate is the evolution of a protolunar disk initially contained within the Roche limit. In this case, the final accretion time scale will be much longer than that predicted by the N-body simulations, as delivery of material into the Roche exterior region will be regulated by the ~10^2 year spreading time of the disk. The accretion dynamics may also be quite different. The recoil time of a moon from a $2M_L$ disk due to zeroth-order resonant torques is comparable to the 10^2-year disk spreading time once moonlets reach masses of $10^{-4}M_L$. Small moons will thus accrete just outside a_R, but will experience significant recoil due to tidal interaction with the disk well before they reach a lunar mass. This case may then involve the short-term state of a system of multiple moonlets that later mutually collide to yield a single Moon.

References:

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