Collisional Origin of Families of Irregular Satellites

David Nesvorný¹

Cristian Beaugé²

and

Luke Dones³

(1) Department of Space Studies, Southwest Research Institute, 1050 Walnut St., Suite 400, Boulder, CO 80302, USA

(2) Observatorio Astronómico, Universidad Nacional de Córdoba, Laprida 854, (X5000BGR) Córdoba, Argentina

ABSTRACT

A distinctive feature of the irregular moons of the giant planets is their orbital groupings. Previously, the prograde and retrograde groups of irregular moons at Jupiter were believed to be groups of fragments produced by the disruption of two large moons (Colombo & Franklin 1971, Pollack et al. 1979). More recently, we have shown that the retrograde group has not one, but probably four or more parent bodies (Nesvorný et al. 2003a). We also found that two of the four identified parent moons were catastrophically disrupted, producing two clusters of irregular moons with members of each group having similar orbits. Named the Ananke and Carme families, these two groups consist of seven and nine known member moons, respectively. The origin of this orbital clustering is unknown. Current rates of catastrophic collisions among satellites in the retrograde group are too low to explain them (Nesvorný et al. 2003a). Disruptions by cometary impacts are even less likely (Zahnle et al. 2003). Groups of irregular satellites with similar inclinations at Saturn are also yet to be explained (Gladman et al. 2001a).

It is conceivable that the satellite families are remnants from early epochs of Solar System formation when impactors were more numerous. In this paper we investigate the possibility that satellite families formed via collisions between large parent moons and stray planetesimals. We find that the Ananke and Carme families at Jupiter could indeed have been produced by this mechanism unless the residual disk of planetesimals in heliocentric orbit was already severely depleted when the irregular satellites formed. Conversely, we find that formation of the Himalia group of prograde jovian satellites by the same mechanism is unlikely unless a massive residual planetesimal disk was still present when the progenitor moon of the Himalia group was captured. We place
constraints on the mass of the residual disk (i) when satellites were captured, and (ii) when the Ananke and Carme families formed. These values sensitively depend on the assumed size-frequency distribution of planetesimals.

1. Introduction

The irregular satellites of the jovian planets are those moons which are far enough from their parent planet that the precession of their orbital plane is primarily controlled by the Sun (see Burns 1986). According to this definition, as of September 15, 2003 we know of fifty-three irregular satellites at Jupiter (Sheppard & Jewitt 2003), fourteen at Saturn (Gladman et al. 2001a), six at Uranus (Gladman et al. 1998, 2000), and four at Neptune (Kuiper 1949, Holman et al. 2003)\(^1\).

The irregular moons appear to show an interesting hierarchy of orbits. For example, Jupiter’s irregulars can be divided into two groups: seven prograde and forty-six retrograde moons. The retrograde bodies, moreover, show two sub-groups of tightly clustered orbits (the Carme and Ananke families, Nesvorný et al. 2003a, Sheppard & Jewitt 2003). These satellite groups are reminiscent of the distribution of orbits in the main asteroid belt, where disruptive collisions between asteroids produced groups of asteroid fragments sharing similar orbits (the so-called asteroid families, Hirayama 1918, Zappalà et al. 1994).

Using this analogy, we may ask whether disruptive collisions between irregular moons may explain their orbital groupings. The answer is: probably not, or at least they cannot explain all the observed groups. Nesvorný et al. (2003a) calculated the rates of disruptive collisions between irregular moons. They found that \(\approx 1\) collision per 1 Gy occurs between known moons in the prograde group at Jupiter. Conversely, the retrograde group of jovian irregular satellites has a much lower rate of collisions due to the longer orbital periods of these moons and due to the large volume of space occupied by their orbits. The current impact rate on these moons from km-sized comets and escaped Trojan asteroids is also negligible (Zahnle et al. 2003). It then seems likely that the origin of the Carme and Ananke families (and also of some other groups of irregular moons, Nesvorný et al. 2003a) dates back to

\(^1\)See [http://ssd.jpl.nasa.gov/sat_elem.html](http://ssd.jpl.nasa.gov/sat_elem.html) for an up-to-date list of orbital elements for all known planetary satellites. [http://ssd.jpl.nasa.gov/sat_discovery.html](http://ssd.jpl.nasa.gov/sat_discovery.html) lists the provisional and IAU-adopted names for the irregular satellites, and lists the publications in which the discoveries were originally reported. Since these publications include more than twenty different IAU Circulars, we have not referenced them all here, but refer the reader to the JPL Solar System Dynamics web pages listed above. Note that our definition of irregular satellites excludes Neptune’s large moon Triton and Saturn’s moon Iapetus.
early epochs of the Solar System when impactors were more numerous.

To show that this scenario is plausible, we crudely estimate the number of disruptive collisions of irregular moons at Jupiter. We assume that a 100 $M_{\oplus}$ residual planetesimal disk in the jovian planets region, which we take to originally extend from 10 to 35 AU, contains $\sim 3 \times 10^{10}$ planetesimals with diameters $\gtrsim 10$ km and a 1 g cm$^{-3}$ bulk density (§4). Beaugé et al. (2002) estimated that a planetesimal has on average 27 encounters within one Hill radius of Jupiter before it is ejected from the Solar System (see §7). If the cumulative number of encounters within a distance $R$ scales as $R^2$, a 50-km-radius moon of Jupiter (Himalia has a mean radius $R \sim 70$ km, Elara has $R \sim 35$ km) suffers $27 \times 3 \times 10^{10} \times [(50+5)/(5 \times 10^7)]^2 \approx 1$ disruptive collision. Here, $R_H = 5 \times 10^7$ km is the Hill radius of Jupiter. Similar estimates yield $\sim 3$ collisions for Saturn’s moon Phoebe, and $\sim 7$ collisions for Neptune’s moon Nereid. In these latter cases, however, impacts of 10-km-diameter planetesimals on the moons are sub-catastrophic owing to the large sizes of Phoebe and Nereid ($R \sim 110$ km and $R \sim 170$ km, respectively) and because the typical impact velocities are lower (§7). Nevertheless, collisions with external impactors may have been important for shaping the size and orbital distributions of smaller moons at these planets.

In this paper, we study the scenario in which the parent moons of satellite families became disrupted via impacts from stray planetesimals. We show that the observed satellite families such as the Ananke and Carme groups (§2) may be outcomes of such events. We also determine the range of planetesimal disk masses that is compatible with the observed satellite families. To this end, we calculate the rates of collisions between bodies on elliptic and hyperbolic orbits (§3). To calculate the probability that a moon with a given orbit and diameter was disrupted during early stages, we generate the size-frequency distribution (SFD) of planetesimal impactors (§4), their orbital distribution (§5), and make use of standard scaling laws for impacts (§6). Results are presented in §7.

### 2. Observed satellite families

To determine which satellites have similar orbits and may thus share a common origin, Nesvorný et al. (2003a) computed the average orbital elements of irregular moons by averaging their orbital elements over $10^8$ years. These average values are plotted in Fig. 1 for the irregular moons of Jupiter. The prograde group of irregular moons around Himalia (Themisto excluded, Figs. 1(a) and 1(b)) is more compact than the retrograde group (Figs. 1(c) and 1(d)). Assuming that these groups were formed by two collisional breakups (Colombo & Franklin 1971), we find that the collision that formed the prograde group was less energetic.
than the one that formed the retrograde group. We calculate from the Gauss equations\textsuperscript{2} that

\[ 50 \lesssim \delta V \lesssim 400 \text{ m/s} \text{ for the prograde group, and } 300 \lesssim \delta V \lesssim 500 \text{ m/s} \text{ for the retrograde group, where the } \delta V \text{ are the ejection velocities of individual fragments. Curiously, both these velocity ranges (especially the one for the retrograde group) are inconsistent with the velocity dispersion of multi-km collisional fragments derived for catastrophic collisions by other means. For example, laboratory impact experiments, where cm-sized projectiles are shot into targets, and numerical hydro-code experiments, which are capable of simulating hypervelocity collisions among large bodies, both indicate that the mean and median ejection velocities from impacts are on the order of several 10 m s\textsuperscript{-1} (Benz & Asphaug 1999, Michel et al. 2001). Similarly small ejection velocities were found for asteroid families that have not yet been dispersed by thermal forces (such as the Karin, Veritas and Iannini families, Nesvorný et al. 2002, 2003b).

We are thus left with a contradiction: either we invoke some mechanism which further disperses orbits in addition to the velocity spread expected from their formation, or we should reject the scenario in which the two satellite groups formed by collisional breakups of two precursor bodies. A closer inspection of the retrograde satellite group shows that there seem to exist several sub-clusters (Fig. 1; see also Nesvorný et al. 2003a, Sheppard & Jewitt 2003). The average orbits of S/2000 J2, S/2000 J4, S/2000 J6, S/2000 J9, S/2000 J10, S/2001 J6, S/2001 J8, and S/2001 J11 cluster tightly around the average orbit of J11.

\textsuperscript{2}We use the Gauss equations to relate the size of a satellite family in the average orbital elements space \((\delta a, \delta e, \delta i)\) with a selected velocity impulse \((\delta V)\):

\[
\frac{\delta a}{a} = \frac{2}{na(1-e^2)^{1/2}} \left[ (1+e \cos f)\delta V_T + (e \sin f)\delta V_R \right], \\
\delta e = \frac{(1-e^2)^{1/2}}{na} \left[ \frac{e+2 \cos f + e \cos^2 f}{1+e \cos f} \delta V_T + (\sin f)\delta V_R \right], \\
\delta i = \frac{(1-e^2)^{1/2} \cos(\omega + f)}{na} \frac{\delta V_W}{1+e \cos f}.
\]

Here, \(a, e, \text{ and } i\) are the semimajor axis, eccentricity, and orbital inclination of a satellite prior to an impact; \(\delta a, \delta e, \text{ and } \delta i\) are the changes in these elements due to the impact; \(\delta V_T, \delta V_R, \text{ and } \delta V_W\) are components of \(\delta V\) along the direction of the orbital motion, in the radial direction, and perpendicular to the orbital plane, respectively. Assuming that a satellite family originated by a collisional disruption, \(f\) and \(\omega\) are the true anomaly and the perihelion argument of the disrupted parent body at the instant of the impact. If fragments are isotropically ejected from the breakup site with velocities \(V_{ejc}\) exceeding the escape velocity \(V_{esc}\) by \(\delta V = \sqrt{V_{ejc}^2 - V_{esc}^2} < V_{max}\), the Gauss equations show that their osculating orbital elements will be located within an ellipsoid centered at the parent body’s initial \((a, e, i)\) orbit. The size, shape and orientation of the ellipsoid are determined by \(V_{max}, f\) and \(\omega\).

From the Gauss equations we find $5 \lesssim \delta V \lesssim 50$ m/s for the group of Carme (which we call the Carme family) and $15 \lesssim \delta V \lesssim 80$ m/s for the group of Ananke (the Ananke family). These velocities are more compatible with a $\delta V$ expected from simple collisional breakups than the velocities computed for the whole retrograde group. Based on these results, Nesvorný et al. (2003a) proposed that the retrograde group of jovian irregular moons witnessed a more complicated collisional history than thought before. In particular, it seems likely that we see fragments from at least two distinct precursor moons. The spectral differences between Carme and Ananke suggest that these immediate precursor bodies correspond to two captured satellites rather than having a common ancestor (Luu 1991, Sykes et al. 2000, Brown 2000, Rettig et al. 2001).

Gladman et al. (2001a) classified the irregular moons of Saturn into groups of similar average orbital inclinations: the 1st satellite inclination group (S/2000 S4, S10, and S11), the 2nd satellite inclination group (S/2000 S2, S3, S5, and S6), and a rather loosely clustered Phoebe group (Phoebe, S/2000 S1, S7, S9, and S12). The moons in each of these inclination groups probably do not have a common collisional origin, unless (i) asymmetric and large-magnitude ejection velocity fields occurred, and/or (ii) collisions occurred early and some subsequent primordial mechanism modified the semimajor axes. Otherwise, it is hard to reconcile the magnitude and components of $\delta V$ computed from the Gauss equations with the current understanding of collisional breakups (Nesvorný et al. 2003a).

Due to the small number of uranian and neptunian irregular moons known at this moment, it is impossible to tell whether or not their orbits can be grouped in some way. S/1997 U1 Caliban and S/1999 U2 Stephano may be linked in some way, but this association is yet to be demonstrated. In fact, the orbital distribution of Uranus’s and Neptune’s irregular moons is statistically indistinguishable from a random distribution of orbits within stability limits.

Here we concentrate on the Ananke and Carme families at Jupiter because the $\delta V$ values determined for these two groups are compatible with impact-generated structures. If, indeed, the Ananke and Carme families were produced by collisions, we find that cratering impacts on Ananke and Carme can best explain them. To show this we calculate the total volume of ejecta and compare it to the sizes of the parent moons. First, we convert the
magnitudes of the member moons into sizes using albedo $A = 0.04$ (Rettig et al. 2001). We then combine the volumes of the satellites within each group. We find that the total volumes of ejecta are $\sim 4.7 \times 10^{17}$ cm$^3$ and $\sim 4.3 \times 10^{17}$ cm$^3$ for the Ananke and Carme families, respectively. These volumes correspond to only 4% and 1% of the volumes of the parent moons, respectively. Using a 0.2 depth-to-diameter-ratio (Schenk et al. 2003), we estimate that in both cases the crater diameters corresponding to these volumes are roughly 18 km. By comparison, Ananke and Carme are $\sim 28$ and $\sim 46$ km across. Thus the putative family-forming impacts should have formed large craters on Ananke and Carme, but did not catastrophically disrupt the parent moons.

To relate the amount of ejecta to the impactor size, we use the scaling suggested by Schmidt & Housen (1987) (see Eq. (20) in our §5). With a 1 g/cm$^3$ density and a 1.25 km/s impact velocity equal to the mean collision velocity between the known retrograde irregular moons; see Nesvorný et al. 2003a), we find that a $\sim 1.5$-km-diameter impactor is required to produce the Ananke family and that a $\sim 1.65$-km-diameter impactor is required to produce the Carme family. With $N(> D) \sim 100 \times (D/1$ km$)^{-2.5}$ (Sheppard & Jewitt 2003), where $N(> D)$ is the number of moons larger than diameter $D$, we find that $N(> 1.5$ km$) \approx 36$ and $N(> 1.65$ km$) \approx 29$. Nesvorný et al. (2003a) found that the probabilities of collision of Ananke and Carme with the other retrograde moons are $2.8 \times 10^{-15}$ km$^{-2}$ yr$^{-1}$ and $2.5 \times 10^{-15}$ km$^{-2}$ yr$^{-1}$, respectively. Taken together, these numbers suggest cratering rates of $2 \times 10^{-3}$ and $6 \times 10^{-3}$ per impactor per 4.5 Gyr on Ananke and Carme, respectively.

With 36 and 29 impactors in the required size range and assuming Poisson statistics, we find only 7% and 15% probabilities that the Ananke and Carme families were produced by satellite-satellite collisions. We conclude that it is difficult to explain these satellite families by satellite-satellite collisions unless (i) more $D \gtrsim 1$-km moons exist than the number suggested by Sheppard & Jewitt (2003) (these authors suggest that this number is known to within a factor of two), and/or (ii) the population of retrograde moons at Jupiter was much larger in the past. On the other hand, our estimate for the ejecta volume was conservative because we derived it from the observed, incomplete population of family members. Larger ejecta volumes and larger impactor sizes are probably more plausible. If so, it becomes even more difficult to explain the Ananke and Carme families by collisions between retrograde irregular moons.

We favor a scenario in which these satellite families were produced during early epochs of the Solar System when heliocentrically orbiting impactors were more numerous. To investigate this scenario in detail we must: (1) calculate collision rates between these impactors and moons, (2) model the size-frequency and orbital distributions of impactors, and (3) use scaling laws to determine collision outcomes. Sections 3–6 address these issues.
3. Collision rates between moons and planetesimals

To compute the rate of collisions between moons and planetesimals, we first recall that a population of satellites moving in keplerian ellipses around planets with the same semimajor axis \( a \), eccentricity \( e < 1 \), inclination \( i \) and random \( \lambda, \varpi, \Omega \), has a space density distribution given by (Kessler 1981, Nesvorný et al. 2003a):

\[
P_1(r, \beta) = \frac{1}{2\pi^3 a^2 r} \frac{1}{\left[ e^2 - (\frac{r}{a} - 1)^2 \right]^{1/2} \left[ \cos^2 \beta - \cos^2 i \right]^{1/2}}
\]

with the limits

\[
a(1 - e) \leq r \leq a(1 + e),
-\beta \leq \beta \leq i.
\]

Here, \( r = \sqrt{x^2 + y^2 + z^2} \) and \( \beta = \arcsin(z/r) \), where \( x, y, z \) are the Cartesian coordinates. The angular variables introduced prior to equation (1) are the mean longitude \( \lambda \), longitude of pericenter \( \varpi \) and longitude of the ascending node \( \Omega \). We normalized the above distribution to the total number of one body in the population, thus \( P_1(r, \beta) \Delta x \Delta y \Delta z \) is the probability that the body is located within a box \( \Delta x \times \Delta y \times \Delta z \) centered at \((x, y, z)\).

Planetesimals that enter the Hill sphere of a planet generally move in hyperbolic orbits in a reference frame centered on the planet. Assuming a population of planetesimals with the same (planetocentric) \( a, e > 1, i \) and random \( \lambda, \varpi, \Omega \), the space density distribution of unbound orbits can be determined in a similar manner and yields:

\[
P_2(r, \beta) = \frac{K}{2\pi^3 a^2 r} \frac{1}{\left[ \left( \frac{r}{a} + 1 \right)^2 - e^2 \right]^{1/2} \left[ \cos^2 \beta - \cos^2 i \right]^{1/2}}.
\]

Once we normalize the distribution in Eq. (3) to contain one body within a sphere of radius \( R \), the constant \( K \) becomes:

\[
K = \pi \left[ \sqrt{A^2 - e^2} - \ln \left( \frac{a}{e} \left( \sqrt{A^2 - e^2} + \frac{A}{a} \right) \right) \right]^{-1},
\]

where \( A = R/a + 1 \).

Given these functions, the probability of collision per unit time between a body on an elliptic orbit with elements \( a_1, e_1, i_1 \) and a body on a hyperbolic orbit with orbital elements \( a_2, e_2, i_2 \) is:

\[
P_{\text{col}} = \pi (R_1 + R_2)^2 \times 2\pi \int_{r_{\text{MIN}}}^{r_{\text{MAX}}} \int_{b_{\text{MIN}}}^{b_{\text{MAX}}} P_1(r, \beta) P_2(r, \beta) V_{\text{col}}(r, \beta) r^2 \cos \beta \, dr \, d\beta,
\]
where $P_1$ and $P_2$ are the probability distributions (1) and (3) with $a_1, e_1, i_1$ and $a_2, e_2, i_2$, respectively. The other quantities are defined as: collision velocity $V_{\text{col}} = |\vec{V}_1 - \vec{V}_2|$, where $\vec{V}_1$ and $\vec{V}_2$ are the orbital velocities of the two bodies; $r_{\text{MIN}} = \max[a_1(1-e_1), a_2(e_2-1)]$, $r_{\text{MAX}} = \min[a_1(1+e_1), R_1]$, $\beta_{\text{MAX}} = \min[i_1, i_2, 180^\circ - i_1, 180^\circ - i_2]$, and $\beta_{\text{MIN}} = -\beta_{\text{MAX}}$; $\sigma = \pi(R_1 + R_2)^2$ is the effective cross-section of the two bodies, with $R_1$ and $R_2$ being their effective radii. The orbital velocities $\vec{V}_1$ and $\vec{V}_2$ at $x, y, z$ are computed from $a_1, e_1, i_1$ and $a_2, e_2, i_2$, respectively.

The integrals in equation (5) are not trivial but they can be evaluated numerically with little difficulty. Using this new algorithm we verified that our collision probabilities and velocities agree within 1% with values reported by Bottke & Greenberg (1993) and Manley et al. (1998) for their test cases of asteroidal and cometary orbits. We also confirmed that our algorithm produces the expected result in a special case in which the moon’s orbit is circular (Öpik 1951, Shoemaker & Wolfe 1982).³

### 4. Size-frequency distribution of planetesimals

Let the differential and cumulative size-frequency distributions of heliocentric planetesimals be $N(d)$ and $N(> d)$, respectively, where $d$ is the planetesimal’s diameter. We use $N(d) = f \times N_{\text{EC}}(d)$, where $N_{\text{EC}}(d)$ is the differential size distribution of the present-day ecliptic comets (primarily derived from crater records on the galilean satellites and Triton; Zahnle et al. 2003), and $f$ is a multiplication factor. We will vary $f$ over the range that

³Assuming a circular orbit for a moon, the impact probability per one planetesimal encounter is

$$P_i = \frac{R_{\text{sat}}^2}{a_{\text{sat}}^2} \left( 1 + \frac{v_{\text{esc}}^2}{v_{\text{orb}}^2 U^2} \right) \frac{U}{|U_x|} \frac{1}{\pi \sin i},$$

where $R_{\text{sat}}$ is the satellite radius and $v_{\text{orb}}$ and $v_{\text{esc}}$ are the satellite’s orbital velocity and the escape velocity from its surface. $U$ and $U_x$ are the encounter velocity and the radial component of the encounter velocity in units of the satellite’s orbital velocity $v_{\text{orb}}$ (Öpik 1951). Following Öpik’s formulae as written for hyperbolic orbits (Shoemaker & Wolfe 1982), we have:

$$U^2 = 3 - (1 - e)/q - 2 \cos i \sqrt{q(1+e)} ,$$
$$U_x^2 = 2 - (1 - e)/q - q(1 + e) ,$$
$$v_i^2 = v_{\text{esc}}^2 + v_{\text{orb}}^2 U^2 ,$$

where $q$, $e$, and $i$ are the planetocentric orbital elements of a stray planetesimal, and $v_i$ is the impact velocity. Because $v_{\text{esc}} \ll v_{\text{orb}} U$ in our case, the effects of gravitational focusing in the above equations can be neglected.
gives plausible values for the total mass of the planetesimal disk (Hayashi et al. 1985).

It is notable that most small craters on Jupiter’s moons appear to be secondaries, indicating a relative paucity of small impactors (Bierhaus et al. 2001, 2003, Schenk et al. 2003), while small craters on Triton imply a relatively abundant population of small impactors. However, it is unclear whether the craters on Triton are of heliocentric or planetocentric origin (Croft et al. 1995, Stern & McKinnon 2000, Zahnle et al. 2001). We therefore, like Zahnle et al. 2003, present two cases, a Case A, depleted in small impactors, in which the SFD is like that at Jupiter, and a Case B in which small objects follow a distribution as at Triton. At large sizes, the SFD is constrained by observations of Kuiper belt objects: Gladman et al. (2001b) and Trujillo et al. (2001) found that $N(>d) \propto d^{-3.4}$ for $d > 50$ km and $N(>d) \propto d^{-3.0}$ for $d > 100$ km, respectively. We take the average of the two power indices, i.e., an index of 3.2 at large sizes.

Following Zahnle et al. (2003), our Case A and B distributions are (here $d$ is measured in km):

$$N_A(>d) = N_{\text{cal}} \times \begin{cases} 
(d/1.5)^{-1}; & d < 1.5 \text{ km} \\
(d/1.5)^{-1.7}; & 1.5 \text{ km} \leq d < 5 \text{ km} \\
(1.5/5)^{1.7}(d/5)^{-2.5}; & 5 \text{ km} \leq d < 30 \text{ km} \\
(1.5/5)^{1.7}(5/30)^{2.5}(d/30)^{-3.2}; & d \geq 30 \text{ km}.
\end{cases}$$

(7)

and

$$N_B(>d) = N_{\text{cal}} \times \begin{cases} 
(d/1.5)^{-1.7}; & d < 1.5 \text{ km} \\
(d/1.5)^{-2.5}; & 1.5 \text{ km} \leq d < 30 \text{ km} \\
(1.5/30)^{2.5}(d/30)^{-3.2}; & d \geq 30 \text{ km}.
\end{cases}$$

(8)

We also consider, like Zahnle et al. (2003), as Case C a distribution that is suggested by studies of the formation of Kuiper Belt objects in situ (Stern 1995, Kenyon & Luu 1998, Kenyon 2002). This distribution assumes that the SFD of bodies with $d < 6.3$ km has Dohnanyi’s equilibrium slope (Dohnanyi 1972) and the measured slope for Kuiper Belt Objects at larger sizes:

$$N_C(>d) = N_{\text{cal}} \times \begin{cases} 
(1.5/6.3)^{2.5}(d/6.3)^{-2.5}; & d < 6.3 \text{ km} \\
(1.5/6.3)^{3.2}(d/6.3)^{-3.2}; & d \geq 6.3 \text{ km}.
\end{cases}$$

(9)

Here, $N_{\text{cal}}$ is a calibration constant (equal to the number of planetesimals with $d > 1.5$ km). Assuming a $1 \text{ g cm}^{-3}$ bulk density for a planetesimal, $N_{\text{cal}} = 1.5 \times 10^{12}$ gives a total mass of $100 \ M_\oplus$ for Case A, $40 \ M_\oplus$ for Case B, and $10 \ M_\oplus$ for Case C. Hahn & Malhotra (1999) suggested that a disk mass of order $50 \ M_\oplus$ is required to expand Neptune’s orbit by $\Delta a \sim 7$ AU, in order to explain the eccentricities of Pluto and its cohort of Kuiper
Belt objects at Neptune’s 3:2 mean motion resonance. We will examine cases in which the residual planetesimal disk has a mass $M_{\text{disk}}$ ranging from 10 to 200 $M_\oplus$.

5. Orbital distribution for encounters

We analyzed the planetesimal encounters with the outer planets simulated by Beaugé et al. (2002) for the *in situ* formation of Uranus and Neptune and designed an “encounter generator”, which is a fast code with several adjustable parameters (such as $N(D)$) that mimics the orbital distribution of encounters obtained in their realistic numerical integrations, but is not limited to a small number of planetesimals. This was accomplished following a similar route as Zahnle et al. (1998, 2001, 2003) in their studies of cratering rates on the regular satellites of the jovian planets.

As expected, we found no preferred values in the distributions of the planetocentric angular variables $\varpi$ (longitude of pericenter), $\Omega$ (longitude of node) and $\lambda$ (mean longitude). We therefore assumed that these angles have uniform random values between zero and $2\pi$. We found that the distribution of inclination $i$ with respect to the planet’s equator can be described by (also see Zahnle et al. 1998):

$$N(< i) = \frac{1}{2} (1 - \cos i).$$

(10)

This distribution corresponds to an isotropic distribution of velocities of planetesimals as seen by the planet.

Beaugé et al.’s simulations also yielded information about the distributions of planetocentric pericentric distances $q$ of planetesimals. From their results we found that $N(< q) \propto q^2$ for $q > 0.06R_H$ and every outer planet. This is consistent with a regime in which high-velocity hyperbolic encounters abound. For $q < 0.06R_H$, most of the encounters occur in quasi-parabolic orbits, and consequently, the square-law approximation ceases to be precise. This is not a problem, however, because we are primarily interested in the impact rates on irregular satellites, which are dominated by planetesimals with $q > 0.06R_H$.

5.1. Distribution of Jacobi constant

In addition to $\varpi$, $\Omega$, $\lambda$, $i$ and $q$, we need one last variable for the complete description of an encounter. Instead of the planetocentric eccentricity, we use the heliocentric Jacobi constant $C$ because it provides an interesting link between local (planetocentric) variables and the global heliocentric distribution of the disk planetesimals. A detailed construction
of the relationship between $C$ and $e$ is given in the Appendix. There we show that in planetocentric orbital elements, the Jacobi constant $C$ is given by:

$$C = c_0 + c_1 h + c_2 h^2,$$

where $h = \sqrt{GM_p q(1 + e)}$ is the planetocentric angular momentum per unit mass of the incoming body. The coefficients $c_i$ are given by:

$$
c_0 = \frac{2a_p G(M_\odot + M_p)}{\sqrt{q^2 + a_p^2 + 2qa_p \cos \theta}} - G(M_\odot + M_p) + 2GM_p \left(\frac{a_p}{q} - 1\right)
+ 2a_p n_p \sqrt{G(M_\odot + M_p)a_p} - 2a_p \left(\frac{GM_p q}{a_p^2} + n_p^2 qa_p\right) \cos \theta,
$$

$$
c_1 = 2a_p n_p \cos i,
$$

$$
c_2 = -\frac{a_p}{q^2},
$$

where $G$ is the gravitational constant, $M_\odot$ and $M_p$ are the Sun’s and the planet’s masses, $a_p$ is the planet’s semimajor axis, and $n_p$ is the planet’s orbital frequency. Equation (11) with coefficients (12) is valid for any type of conic, be it elliptic ($e < 1$), parabolic ($e = 1$) or hyperbolic ($e > 1$). The coefficient $c_0$ depends on the phase angle $\theta$; the quantity $(\pi - \theta)$ gives the relative position of the pericenter with respect to the Sun in the planetocentric reference frame. We assume that $\theta$ is a random variable with uniform distribution between zero and $2\pi$.

We find that the distribution of $C$ in Beaugé et al. (2002)’s numerical simulations bears a close resemblance to a log-normal distribution in $C_{\text{max}} - C$, where $C_{\text{max}}$ is the maximum value of $C$ detected for encounters with a given planet. We can thus approximate the differential distribution of $C$ via:

$$N(C) = \frac{1}{\sqrt{2\pi}|C_{\text{max}} - C|S} \exp \left[ -\frac{(\log(C_{\text{max}} - C) - M)^2}{2S^2} \right],$$

where $\exp$ is the exponential function, and $M$ and $S$ are the mean value and standard deviation, respectively. These parameters were determined numerically from Beaugé et al.’s data (Table 1).

A comparison between the determined log-normal distributions of $C$ and the distributions of $C$ obtained numerically by Beaugé et al. (2002) is shown in Figure 2. The numerical distribution for Jupiter shows some peaks and valleys that are not reproduced by the log-normal distribution. In addition, the log-normal distribution shows somewhat larger values...
at $C \sim 2.9–3.0$ than the numerical data. The errors introduced by this compromise are small in the context of this work.

The cumulative distribution for a log-normal function is given by:

$$N(> C) = \frac{1}{2} + \frac{1}{2} \text{erf}\left[\frac{\log (C_{\text{max}} - C) - M}{\sqrt{2}S}\right],$$

where erf is the error function. Taking $x = N(> C)$, with $x \in [0, 1]$ and inverting equation (15), we obtain:

$$\log (C_{\text{max}} - C) = M + \sqrt{2S} \text{ erf}^{-1}(2x - 1).$$

Thus, in terms of a uniform random variable $x \in [0, 1]$, the distribution of $C$ is given by:

$$C = C_{\text{max}} - \exp[M + \sqrt{2S} \text{ erf}^{-1}(2x - 1)].$$

### 5.2. Distribution of eccentricities

So far, and apart from the trivial angular variables, we obtained the encounter distributions in terms of $(C, q, i)$. Inverting equation (11), we determine the angular momentum as $h = h(C, q, i, \theta)$, where $\theta$ is a random angle. Finally, using the relationship between angular momentum and eccentricity, we get:

$$e = \frac{h^2}{qGM_p} - 1.$$

Figure 3 shows the eccentricity distributions for Jupiter and Neptune for $q = 0.06, 0.6$ and $1.2 \ R_H$. The agreement is not good for $q = 0.06 \ R_H$, where the model distribution is deficient in quasi-parabolic orbits (i.e., orbits with $e \sim 1$). As we look toward larger values of $q$, the agreement between model and numerical distributions becomes better. With $q > 0.1 \ R_H$ the model distribution reproduces the numerical data with errors smaller than $\sim 20\%$. Errors of this order will not affect our results because uncertainties in other model parameters (such as, for example, the SFD of planetesimals) are much larger.

### 6. Scaling for impacts

We will study two cases: (i) impacts that lead to catastrophic disruptions of moons; and (ii) large cratering impacts. While (i) is important to understand issues related to satellite survival, (ii) is more relevant for satellite families at Jupiter.
For (i), we use the threshold for a catastrophic disruption determined by Benz & Asphaug (1999). They used a smoothed particle hydrodynamics method to simulate colliding rocky and icy bodies in an effort to self-consistently define the threshold for catastrophic disruption \( Q_D^* \). This threshold is defined as the specific energy required to shatter the target body and disperse the fragments into individual but possibly re-accumulated objects, the largest one having exactly one half the mass of the original target. The functional form of Benz & Asphaug (1999)'s law is given by:

\[
Q_D^* = Q_0 (R_{PB})^a + B \rho (R_{PB})^b \text{ erg g}^{-1},
\]

where \( R_{PB} \) is the radius of the parent body (in cm), \( \rho \) is the density of the parent body (in g cm\(^{-3} \)), and \( Q_0 \), \( B \), \( a \), and \( b \) are constants. This functional form represents two distinct regimes dominated by: (i) material strength (first term, \( a < 0 \)), and (ii) self-gravity (second term, \( b > 0 \)). Because self-gravity dominates for \( R_{PB} \gtrsim 100-200 \) m (Benz & Asphaug 1999), we concentrate on the second term in (19).

We will assume that irregular moons are primarily composed of ice. Benz & Asphaug (1999) calculate coefficients \( B \) and \( b \) for ice and two different impact velocities: 0.5 and 3 km s\(^{-1} \). For 0.5 km s\(^{-1} \), \( B = 2.1 \) erg cm\(^3\) g\(^{-2} \) and \( b = 1.19 \). For 3 km s\(^{-1} \), \( B = 1.2 \) erg cm\(^3\) g\(^{-2} \) and \( b = 1.26 \). Because impact velocities (\( V_{imp} \)) between planetesimals and satellites range between 0.5 and 10 km s\(^{-1} \) (Beaugé et al. 2002; Table 2 of this paper), we interpolate/extrapolate from \( B \) and \( b \) given by Benz & Asphaug (1999) to obtain \( Q_D^* \) for a given velocity value.

For cratering impacts we use the scaling law derived from laboratory experiments of impacts into sand and from large explosions. Schmidt & Housen (1987, supplemented with the angular dependence assumed by Zahnle et al. 2003) suggest the following relation for the volume of a simple crater:

\[
V = 0.13 \left( \frac{m_i}{\rho_i} \right)^{0.783} g^{-0.65} \left( \frac{\rho_t}{\rho_i} \right)^{0.217} v_i^{1.3} \cos \alpha \text{ cm}^3,
\]

where the impactor has mass \( m_i \), density \( \rho_i \), and velocity \( v_i \), and where the surface gravity is \( g \) and the target density is \( \rho_t \). All quantities in Eq. (20) have to be evaluated in cgs units. The incidence angle \( \alpha \) is measured from the zenith. The mean and median value for the incidence angle for isotropic velocities is 45°. We will assume \( \alpha = 45^\circ \).

7. Results

We calculate the rates of disruptive and cratering collisions of real and fictitious irregular moons. For each real irregular moon, we assume that its orbital elements \( a, e, i \) are fixed and
equal to the average values determined by Nesvorný et al. (2003a). We neglect gravitational focusing by the moons, which is negligible in the regime of sizes and encounter velocities investigated here.

Table 2 shows the mean collision probabilities per encounter per area \( \pi P_{\text{col}}/\sigma \) and mean impact velocities \( V_{\text{col}} \) for selected irregular moons\(^4\). For Jupiter, we list only the largest moons of every group. \( P_{\text{col}} \) and \( V_{\text{col}} \) were computed by averaging over \( 10^5 \) planetesimal encounters within one Hill radius of a planet. This number of encounters is large enough to attain the convergence limit to within a 1\% precision. The encounters were generated using the recipe explained in §5.

In Table 2, the collision velocities \( V_{\text{col}} \) are generally larger for moons that are closer to a planet. This is expected because the orbital velocities of satellites and planetocentric velocities of planetesimals are both larger at smaller distances from the planet (Beaugé et al. 2002). Also the collision probabilities \( P_{\text{col}} \) are larger at smaller distances. What we see here is probably the effect of gravitational focusing by the parent planet. In the absence of gravitational focusing, \( P_{\text{col}} \) would be roughly constant with planetocentric distance because all locations within a Hill sphere would be receiving roughly the same number of planetesimals. In fact, the difference in \( P_{\text{col}} \) between the innermost and outermost irregular satellites of a given planet is not large. The effects of gravitational focusing by a planet are more important for the regular moons that orbit at smaller \( a \) than for the irregular satellites (Zahnle et al. 2003).

Because \( P_{\text{col}} \) and \( V_{\text{col}} \) do not vary much among the irregular moons of planet \( j \) (\( j = 5 \) to 8 from Jupiter to Neptune), we calculate their average values \( \langle P_{\text{col}} \rangle_j \) and \( \langle V_{\text{col}} \rangle_j \) (Table 2) and use \( \langle P_{\text{col}} \rangle_j \) and \( \langle V_{\text{col}} \rangle_j \) to discuss the disruption and cratering rates on the irregular moons of planet \( j \). The average values of \( \langle P_{\text{col}} \rangle_j \) and \( \langle V_{\text{col}} \rangle_j \) progressively decrease with the increasing semimajor axis of a planet, i.e., from Jupiter to Neptune. For this reason, we expect that the consequences of planetesimal bombardment were more severe at Jupiter than at Neptune. At a first glance, this result may be compatible with observations because the orbital distribution of jovian irregular satellites shows clear signatures of past collisions (e.g., the Ananke and Carme families), while the same structures are not observed among the irregular satellites of Saturn, Uranus and Neptune. Unfortunately, we do not yet know of enough irregular moons with 1–10-km diameters at Saturn, Uranus and Neptune to detect structures like the Ananke and Carme families at these planets. Moons of these sizes at Saturn, Uranus and Neptune are very faint and difficult to detect (see, e.g., Nesvorný &

\(^4\)The quantity listed as \( 10^{14} P_{\text{col}}/\sigma \) in Tables 5, 6, and 7 of Nesvorny et al. (2003a) should have been labeled \( 10^{14} \pi P_{\text{col}}/\sigma \). The calculations of impact rates in Nesvorny et al. (2003a) are correct.
We use the following procedure to calculate the probability that a diameter $D$ moon of planet $j$ is catastrophically disrupted. For $Q_L^j$ (Eq. 19) and the characteristic impact velocity $\langle V_{col}\rangle_j$ (Table 2), we calculate the diameter $d^*$ of the smallest impactor that can catastrophically disrupt the moon. The total population of impactors larger than this size is $N(>d^*)$ (Eqs. 7, 8, and 9). Beaugé et al. (2002) found that every planetesimal suffers on average $N_{enc}^j = 27.4$, 21.4, 31.9, and 51.2 encounters within one Hill radius of Jupiter, Saturn, Uranus, and Neptune, respectively, before it is removed from the Solar System. The disruption rate is then

$$x = N_{enc}^j \times N(>d^*) \times \langle \pi P_{col}/\sigma \rangle_j \times (D/2)^2,$$

where $\langle P_{col} \rangle_j$ is taken from Table 2. The probability $p(D)$ that the diameter-$D$ moon is catastrophically disrupted by impacts from the residual planetesimal disk of mass $M_{\text{disk}}$ is then

$$p(D) = 1 - \exp(-x)$$

(according to Poisson statistics), where $M_{\text{disk}}$ is calculated from Eqs. 7, 8, and 9 using a bulk density of 1 g cm$^{-3}$ for planetesimals.

Figures 4, 5, and 6 show $p(D)$ for our Case A, B and C size-frequency distributions of planetesimals, respectively. We first discuss these cases separately.

Figure 4 shows an interesting behavior of $p(D)$. Large moons ($D > 100$ km) are difficult to disrupt because large $d^*$ is needed and there are not enough impactors range with diameters $> d^*$ for plausible $M_{\text{disk}}$. On the other hand, $p(D)$ decreases with $D$ for $D < 10$ km because $N_A(>d)$ is very shallow for $d < 1$ km and there are just not enough impactors in this size range to compensate for the small cross-sections of $D < 10$-km moons. In effect, planetesimals with our Case A size-frequency distribution $N_A(>d)$ will preferentially disrupt satellites of intermediate sizes ($D \sim 10$–100 km).

Figure 5 tells a different story. Because $N_B(>d)$ is steeper than $N_A(>d)$ at small $d$, there are now enough small impactors to make the survival of small moons difficult. The transition from low to high probability of disruption generally occurs in the 10–100-km diameter range. The same transition is more abrupt in Case C because $N_C(>d)$ (Eq. 9) is steeper than $N_B(>d)$ down to smaller sizes (Fig. 6). With $N_C(>d)$, satellites with $D \lesssim 10$ km have disruption probabilities $\sim 1$ for even the smallest considered $M_{\text{disk}} (= 10M_\oplus)$. This makes the survival of early-captured $D \lesssim 10$ km moons difficult.

8. Discussion

To place Figs. 4–6 in the context of satellite formation, we describe the likely sequence of events that led to the capture of irregular satellites. The irregular satellites were probably captured via dissipation of their orbital energy in circumplanetary gas disks (Pollack et al. Dones 2002).
Indeed, an object that suffers a low-velocity encounter with a planet may be temporarily captured even in the absence of a disk (Kary & Dones 1996), then lose kinetic energy via aerodynamic drag in the residual circumplanetary gas disk, and eventually end up on a planet-bound orbit. These events probably occurred during the late phases of the circumplanetary disk’s lifetime, because otherwise the captured object would spiral into the planet by the effects of gas drag (Pollack et al. 1979, Čuk & Burns 2003). By contrast, in the late stages of formation of the jovian planets, the surface density in the circumplanetary disk is likely orders of magnitude smaller than in a “minimum mass” nebula, and the lifetime against gas drag, even for a km-sized irregular satellite, probably exceeds 1 My (Camp & Ward 2002, Mosqueira & Estrada 2003a,b).

During early epochs, satellites must have survived not only the effects of circumplanetary gas drag, but also a phase of heavy bombardment during which myriads of planetesimals were traversing the planets’ neighborhoods. To constrain the maximum mass of the planetesimal disk at the time of the irregular moons’ formation, we will require that the collision rates between moons and planetesimals were low enough to guarantee the moons’ survival. We will use Figs. 4, 5, and 6 to this end. It is also clear that the mass in the planetesimal disk was not strongly depleted when the irregular moons formed if the irregular moons are, in fact, captured planetesimals.

For the Case B and C distributions (Figs. 5 and 6), survival of an irregular satellite at Jupiter was unlikely unless: (i) the moon was large enough and/or (ii) the planetesimal disk was already partially depleted when the moon was captured. For example, the probability of catastrophic disruption \( p(D) < 0.5 \) for Case C for \( D \gtrsim 60 \) km and \( M_{\text{disk}} = 10M_\oplus \) and for \( D \gtrsim 80 \) km and \( M_{\text{disk}} = 50M_\oplus \). Because parent retrograde irregular satellites at Jupiter have \( 28 \lesssim D \lesssim 60 \) km, we require \( M_{\text{disk}} \lesssim 10M_\oplus \) for their survival if Case C applies. Most prograde irregular satellites at Jupiter probably derive from a single large parent body with \( D \sim 150 \) km (Čuk & Burns 2003). Interestingly, the breakup of this large parent moon by an external planetesimal impactor is unlikely for even large \( M_{\text{disk}} \) and independent of whether we use the Case A, B or C distribution. We discuss this interesting case below. Themisto, a \( D = 8 \) km prograde irregular moon of Jupiter, does not seem to be related to the Himalia family. Its survival is more problematic and requires small \( M_{\text{disk}} \) and/or the Case A distribution. In general, the Case A distribution poses weaker constraints on \( M_{\text{disk}} \) because \( p(D) < 0.7 \) at Jupiter for any \( D \) with \( M_{\text{disk}} \lesssim 50M_\oplus \) (Fig. 4a).

The constraints shown in Figs. 4–6 for satellites of Jupiter are shown in a different way in Table 3. (We focus on the jovian system because, since the reflected brightness of small bodies scales approximately as the inverse fourth power of heliocentric distance, much smaller irregular moons have been much discovered at Jupiter than around the other giant
planets. Thus jovian satellites provide the strongest observational constraint.) We list the range of satellite diameters $D_D$ (for Case A) or the maximum diameter (Cases B and C) for which disruption of a given moon is 50%, 95%, or 99% likely. Cases B and C constrain the mass of the protoplanetary disk to be less than $50M_\oplus$ (and for Case C, much less) at the time when the present-day irregular satellites formed. Jupiter has 20 known irregular moons with $D \leq 2$ km. It seems most unlikely that these are the surviving remnants of a population of 2,000 irregular moons (assuming 99% were destroyed). Thus we reject models in which $D_D(99%) > 2$ km. This criterion implies that $M_{\text{disk}} < 27M_\oplus$ for Case B and $M_{\text{disk}} < 0.4M_\oplus$ for Case C. These correspond to upper limits of $1 \times 10^{12}$ and $6 \times 10^{10}$ planetesimals with $d > 1.5$ km at the time the irregular satellites formed. (By contrast, if the “scattered disk” of comets beyond Neptune is the source of the Jupiter-family comets, the scattered disk must contain some $10^8$–$10^9$ such planetesimals at present, Bottke et al. 2002.) Thus at the time the irregular satellites formed, the protoplanetary disk would have been depleted compared to the original disk, but much more populous than at present.

Phoebe ($D = 240$ km), Sycorax ($D = 120$ km), and Nereid ($D = 340$ km) outlast the phase of heavy bombardment by planetesimals for any plausible $M_{\text{disk}}$. Other known irregular satellites of Saturn and Uranus range in size from $D \sim 7$ to 60 km. With the Case C distribution these smaller satellites require small $M_{\text{disk}}$. In particular, $p(D) < 0.5$ for $D > 30$ km with $M_{\text{disk}} = 10M_\oplus$ at Saturn (Fig. 6b). Thus, survival of many $D \lesssim 30$-km irregular moons at Saturn is problematic unless $M_{\text{disk}} \ll 10M_\oplus$, or the observed moons represent only a small fraction of the original population. Cases A and B (Figs. 4 and 5) place weaker constraints on $M_{\text{disk}}$.

Some time after their captures occurred, some of the moons suffered energetic (but sub-catastrophic) collisions (e.g., Ananke and Carme, §2). The tight clustering of orbits of the Ananke and Carme family members suggests that the circumplanetary gas envelope had already dissipated at the time when these collisions occurred. Otherwise, the size-dependent aerodynamic gas drag would disperse these clusters and sort them according to the moons’ diameters, which is not observed (e.g., Gladman et al. 2001a). Using Eq. (20), typical impact velocities, and ejecta mass (§2) for the Ananke and Carme families, we estimate the required size for a planetesimal impactor $d$. Using this size, the SFD of impactors, and impact probabilities, we then determine the minimum required $M_{\text{disk}}$ that yields cratering impacts of diameter $d$ planetesimals on Ananke and Carme.

With a 6.7 km s$^{-1}$ impact velocity (Table 2) for an impact on Ananke and Eq. (20), we find that a planetesimal impactor with $d \sim 0.53$ km and 1 g cm$^3$ bulk density would excavate and disperse enough material to produce the Ananke family. Similarly, a $d \sim 0.59$-km impactor is needed to create the Carme family if it collides with Carme at $\sim 6.6$ km
Both these collisions are sub-catastrophic, because $M_{lf}/M_{pb} \sim 0.96$ and 0.99, where $M_{lf}$ and $M_{pb}$ are the masses of the largest fragment and parent body, respectively. The specific energies of these impacts are $1.5 \times 10^6$ and $4.5 \times 10^5$ ergs g$^{-1}$ or only $\sim 2.3\%$ and $\sim 0.4\%$ of $Q_D^*$ defined by Eq. (19). We calculate that to have a $\gtrsim 50\%$ probability that an impact of this energy or greater occurred on Ananke, $M_{\text{disk}} \gtrsim 8M_\oplus$ with Case A, $M_{\text{disk}} \gtrsim 1.7M_\oplus$ with Case B, and $M_{\text{disk}} \gtrsim 0.2M_\oplus$ with Case C. For Carme, we find $M_{\text{disk}} \gtrsim 3.3M_\oplus$ with Case A, $M_{\text{disk}} \gtrsim 0.7M_\oplus$ with Case B, and $M_{\text{disk}} \gtrsim 0.1M_\oplus$ with Case C. The required values of $M_{\text{disk}}$ are very sensitive to the assumed size-frequency distribution of planetesimals because our Cases A, B and C have very different power indices at sub-km sizes of impactors. Nevertheless, all calculated values for $M_{\text{disk}}$ are plausible, making it conceivable that the Ananke and Carme families were, indeed, produced by impacts of sub-km planetesimals during early epochs. To summarize, to form the Ananke and Carme families by this mechanism, we require $M_{\text{disk}}$ of the contemporary disk that is only a small fraction of the mass of solids initially present (Hayashi 1981, Hahn & Malhotra 1999).

Another interesting and related issue is the provenance of the prograde Himalia group at Jupiter. We estimate from the observed members of the Himalia family that the prograde group progenitor was a body $\sim 150$ km across. If so, $M_{lf}/M_{pb} \sim 0.78$. This ratio is lower and requires a larger-scale impact than the one needed to explain the Ananke and Carme families. Using Eq. (20) and a 7.7 km s$^{-1}$ impact velocity (Table 2), we calculate that a $d \sim 13$-km planetesimal must have impacted the prograde group progenitor body. The specific energy of this impact is $2.1 \times 10^8$ ergs g$^{-1}$ or about 40% of $Q_D^*$ (Eq. 19), which is in perfect agreement with the scaling for sub-catastrophic collisions given by Benz & Asphaug (1999; their Eq. 8).

We calculate that to have a $\gtrsim 50\%$ probability that an impact of this energy occurred on the prograde group progenitor, $M_{\text{disk}} \gtrsim 70M_\oplus$ for any of our three SFDs. These values of $M_{\text{disk}}$ may be too large (Hahn & Malhotra 1999, Beaugé et al. 2002) unless Himalia was captured very early. In that case, it probably would have been swallowed by Jupiter due to gas drag or density wave torques. We thus believe that the Himalia group probably did not form by an impact of a stray planetesimal. Instead, we speculate that the progenitor of the Himalia family was hit and disrupted more recently by an impact of another irregular satellite of Jupiter. Nesvorný et al. (2003a) found that the expected number of impacts between Himalia and Elara was 1.46 and that such a collision would be catastrophic. It is difficult to characterize the disruption history of Himalia more precisely because satellites in the Himalia group probably suffered other collisions between themselves since their formation (Nesvorný et al. 2003a).

The Phoebe group at Saturn (Gladman et al. 2001a) poses another intriguing prob-
lem that requires explanation. If this group formed by a collision of Phoebe with a stray planetesimal, we calculate that a $d \sim 3$ km planetesimal impacting at $5.1 \text{ km s}^{-1}$ (Table 2) and excavating a $\sim 40$-km-diameter crater on Phoebe’s surface can best explain it (also see Nesvorný et al. 2003a). It is likely that such an event would occur (i.e., with $>50\%$ probability) if $M_{\text{disk}} \gtrsim 0.2$–1.4 $M_{\oplus}$ at the time when Phoebe was captured, with the exact value depending on the detailed profile of the planetesimals’ SFD. It is thus statistically plausible that the Phoebe group was created by such an event. What is less well understood is the unusually large $\delta V$ (100–400 m s$^{-1}$) calculated for the Phoebe group members from the Gauss equations (Nesvorný et al. 2003a). Even more striking is the fact that the outermost satellites of this group do not intersect the orbit of Phoebe (Čuk & Burns 2003). Possibly, some yet-to-be-identified mechanism dispersed fragments of the Phoebe group after its formation, or its members have other (possibly unrelated) origins.

9. Conclusion

We proposed that the Ananke and Carme families of irregular satellites at Jupiter formed during early epochs when Ananke and Carme were cratered by impacts of stray planetesimals from the residual protoplanetary disk. Conversely, we found that formation of the Himalia group by the same mechanism is unlikely unless a massive residual planetesimal disk was still present when the parent body of the Himalia group was captured. We speculated that the Himalia family formed more recently by a collision of its progenitor with another irregular satellite of Jupiter.

We placed constraints on the mass of the residual disk: (i) when satellites were captured, and (ii) when the Ananke and Carme families formed. These values depend sensitively on the assumed size-frequency distribution of planetesimals. For example, we require $M_{\text{disk}} \lesssim 10 M_{\oplus}$ to guarantee survival of the retrograde irregular satellites at Jupiter with $28 \lesssim D \lesssim 60$-km-diameters (Ananke, Carme, Pasiphae, and Sinope), if our Case C distribution applies. Similarly, we estimated that $M_{\text{disk}} \gtrsim 0.1$–8 $M_{\oplus}$ is required to produce the Ananke and Carme families by cratering impacts of planetesimals on Ananke and Carme.

Unfortunately, we cannot draw stronger constraints on $M_{\text{disk}}$ because of the uncertainty caused by the poorly known profile of the size-frequency distribution of planetesimals. By using three distributions, we made an effort to span a realistic range of SFDs. It may be that the Case C distribution is more realistic for the primordial planetesimal disk and the Case A and B distributions, which are more characteristic for ecliptic comets, have resulted from later collisional grinding in the Kuiper Belt. We do not know. Hopefully, future studies of these issues will help us to select a specific SFD and calibrate constraints generated by this
work on $M_{\text{disk}}$ to this distribution.

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Appendix: The Jacobi Constant in Planetocentric Elements

We consider a circular restricted three-body system consisting of the Sun, a planet and a planetesimal with infinitesimal mass in heliocentric orbit. In this system, the Jacobi constant $C$ is an integral of motion. Although this function is usually written in the Cartesian coordinates of a rotating reference frame, we can also express $C$ in terms of the coordinates of the fixed non-rotating system (Brouwer & Clemence 1961) as:

$$ C = \frac{GM_\odot}{r_s} + \frac{GM_p}{r_p} + n_0(x_s \dot{y}_s - y_s \dot{x}_s) - \frac{1}{2} V_s^2, \quad (21) $$

where $G$ is the gravitational constant, $M_\odot$ is the solar mass, $n_p$ is the mean motion of the planet, $x_s, y_s, z_s$ are the heliocentric Cartesian coordinates of the planetesimal, and $V_s$ is its heliocentric velocity. Finally, $r_s$ and $r_p$ are, respectively, the heliocentric and planetocentric distances of the planetesimal.

Denoting by $(x_0, y_0, z_0)$ the heliocentric coordinates of the planet, we can obtain the planetocentric coordinates of the planetesimal $(x_p, y_p, z_p)$ as:

$$
\begin{align*}
    x_p & = x_s - x_0 \\
    y_p & = y_s - y_0 \\
    z_p & = z_s - z_0.
\end{align*} \tag{22}
$$

Expressions for the velocities are analogous. Assuming $\dot{z}_0 \equiv 0$, we can write:

$$ V_s^2 = V_p^2 + V_0^2 + 2(\dot{x}_p \dot{x}_0 + \dot{y}_p \dot{y}_0) \tag{23} $$

Similarly, we have:

$$
\begin{align*}
(x_s \dot{y}_s - y_s \dot{x}_s) & = (x_p \dot{y}_p - y_p \dot{x}_p) + (x_0 \dot{y}_0 - y_0 \dot{x}_0) \\
& + (x_p \dot{y}_0 - y_p \dot{x}_p) + (x_0 \dot{y}_p - y_0 \dot{x}_p) \tag{24}
\end{align*}
$$

Because the orbit of the planet is assumed to be circular:

$$
\begin{align*}
    x_0 & = a_0 \cos \lambda_p & \dot{x}_0 & = -a_0 n_0 \sin \lambda_p \\
    y_0 & = a_0 \sin \lambda_p & \dot{y}_0 & = a_0 n_0 \cos \lambda_p,
\end{align*} \tag{25}
$$

where $\lambda_p$ is the mean longitude of the planet. Similarly, we can express the planetocentric coordinates of the planetesimal as:

$$
\begin{align*}
    x_p & = r_p \cos (f + \varpi) & \dot{x}_p & = \dot{r}_p \cos (f + \varpi) - r_p \dot{f} \sin (f + \varpi) \\
    y_p & = r_p \sin (f + \varpi) & \dot{y}_p & = \dot{r}_p \sin (f + \varpi) + r_p \dot{f} \cos (f + \varpi),
\end{align*} \tag{26}
$$
where this is written in terms of the *planetocentric* orbital elements. $f$ is the true anomaly and $\varpi$ is the longitude of the pericenter.

In the following we will write $C$ as a function of planetocentric orbital elements at pericenter. We require that

$$f = 0, \quad r_p = q, \quad \dot{r} = 0. \quad (27)$$

Introducing these expressions into (23) and (24), we obtain:

$$\begin{align*}
(x_s \dot{y}_s - y_s \dot{x}_s) &= (x_p \dot{y}_p - y_p \dot{x}_p) + (x_0 \dot{y}_0 - y_0 \dot{x}_0) \\
&\quad + r_p a_0 (n_p + \dot{f}) \cos (\varpi - \lambda_p) \\
V_s^2 &= V_p^2 + V_0^2 + 2r a_0 n_p \dot{f} \cos (\varpi - \lambda_p).
\end{align*} \quad (28)$$

Before introducing these new expressions into $C$, we can simplify them further. For example, we can use the following relationships:

$$\begin{align*}
(x_p \dot{y}_p - y_p \dot{x}_p) &= h \cos i \\
(x_0 \dot{y}_0 - y_0 \dot{x}_0) &= \left( G (M_{\odot} + M_p) a_0 \right)^{1/2} \\
V_0^2 &= \frac{G (M_{\odot} + M_p)}{a_0} \\
r_s^2 &= q^2 + a_0^2 - 2qa_0 \cos (\varpi - \lambda_p)
\end{align*} \quad (29)$$

where $h$ is the planetocentric angular momentum per unit mass of the planetesimal. Similarly, at pericenter, we have that:

$$\dot{f} = \frac{h}{q^2} \quad \text{and} \quad V_p = \frac{h}{q} \quad (30)$$

which allow us to write the angular and linear velocity in terms of the angular momentum.

Finally, introducing all these expressions into the equations for $C$ and grouping terms in powers of $h$, we obtain:

$$C = c_0 + c_1 h + c_2 h^2 \quad (31)$$

where the functions $c_i$ are given in Eq. (12). These expression constitute a precise relationship between the planetocentric elements of a planetesimal at pericenter and $C$. 
Table 1: Parameters of the log-normal distribution of the Jacobi constant for each planet. Columns are: maximum value of $C$ detected for encounters with a given planet ($C_{\text{max}}$), mean value of $C$ ($= M$), and standard deviation of $C$ ($= S$). These parameters are used by our encounter generator program to produce distributions of $C$.

<table>
<thead>
<tr>
<th>Planet</th>
<th>$C_{\text{max}}$</th>
<th>$M$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jupiter</td>
<td>3.039</td>
<td>-1.694</td>
<td>0.780</td>
</tr>
<tr>
<td>Saturn</td>
<td>3.017</td>
<td>-1.856</td>
<td>0.928</td>
</tr>
<tr>
<td>Uranus</td>
<td>3.005</td>
<td>-2.750</td>
<td>1.315</td>
</tr>
<tr>
<td>Neptune</td>
<td>3.006</td>
<td>-2.773</td>
<td>1.311</td>
</tr>
</tbody>
</table>

Table 2: Collision rates and impact velocities for irregular satellites. The columns are: satellite name, ‘intrinsic’ collision probability ($\pi P_{\text{col}}/\sigma$, from Eq. 5), mean collision velocity ($V_{\text{col}}$), average semimajor axis ($\langle a \rangle$), average eccentricity ($\langle e \rangle$), and average inclination ($\langle i \rangle$). The intrinsic collision probability is given in units of $10^{-16}$ km$^{-2}$ per 1 encounter of a planetesimal within 1 $R_H$ of a planet. The row denoted by $\langle \cdot \rangle_j$ gives the average $\langle \pi P_{\text{col}}/\sigma \rangle$ and $\langle V_{\text{col}} \rangle$ over all listed satellites of planet $j$. Average orbital elements were taken from Nesvorny et al. (2003). The sizes of most moons were calculated from their magnitudes using albedos $A = 0.04, 0.05, \text{ and } 0.07$ for satellites of Jupiter, Saturn, and Uranus, respectively. The sizes for Himalia and Nereid were taken from Porco et al. (2003) and Thomas et al. (1991).
<table>
<thead>
<tr>
<th>Satellite</th>
<th>$\pi P_{\text{col}}/\sigma$ ($10^{-16}$ km$^{-2}$)</th>
<th>$\langle V_{\text{col}} \rangle$ (km s$^{-1}$)</th>
<th>$\langle a \rangle$ (AU)</th>
<th>$\langle e \rangle$</th>
<th>$\langle i \rangle$ (deg)</th>
<th>$D$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jupiter’s Irregulars:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Themisto (S/1975 J1)</td>
<td>4.24</td>
<td>8.70</td>
<td>0.049422</td>
<td>0.2513</td>
<td>44.41</td>
<td>8</td>
</tr>
<tr>
<td>Leda</td>
<td>4.12</td>
<td>7.78</td>
<td>0.074448</td>
<td>0.1633</td>
<td>28.07</td>
<td>20</td>
</tr>
<tr>
<td>Himalia</td>
<td>4.12</td>
<td>7.73</td>
<td>0.076427</td>
<td>0.1591</td>
<td>28.59</td>
<td>120 × 150</td>
</tr>
<tr>
<td>Elara</td>
<td>4.10</td>
<td>7.65</td>
<td>0.078113</td>
<td>0.1158</td>
<td>27.63</td>
<td>86</td>
</tr>
<tr>
<td>Lysithea</td>
<td>4.10</td>
<td>7.70</td>
<td>0.078334</td>
<td>0.2126</td>
<td>28.05</td>
<td>36</td>
</tr>
<tr>
<td>Ananke</td>
<td>3.80</td>
<td>6.70</td>
<td>0.14067</td>
<td>0.2429</td>
<td>147.73</td>
<td>28</td>
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<tr>
<td>Carme</td>
<td>3.79</td>
<td>6.58</td>
<td>0.15450</td>
<td>0.2633</td>
<td>164.53</td>
<td>46</td>
</tr>
<tr>
<td>Pasiphae</td>
<td>3.77</td>
<td>6.50</td>
<td>0.15671</td>
<td>0.2967</td>
<td>157.39</td>
<td>38</td>
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<tr>
<td>Sinope</td>
<td>3.77</td>
<td>6.52</td>
<td>0.15811</td>
<td>0.2967</td>
<td>157.39</td>
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<td>⟨·⟩5</td>
<td>3.99</td>
<td>7.32</td>
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<td>Saturn’s Irregulars:</td>
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<tr>
<td>Kiviuq (S/2000 S5)</td>
<td>3.32</td>
<td>5.11</td>
<td>0.075561</td>
<td>0.3082</td>
<td>47.90</td>
<td>17</td>
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<tr>
<td>Ijiraq (S/2000 S6)</td>
<td>3.32</td>
<td>5.10</td>
<td>0.075914</td>
<td>0.3027</td>
<td>48.00</td>
<td>14</td>
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<tr>
<td>Phoebe</td>
<td>3.31</td>
<td>5.07</td>
<td>0.086478</td>
<td>0.1642</td>
<td>175.18</td>
<td>240</td>
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<tr>
<td>Paaliaq (S/2000 S2)</td>
<td>3.24</td>
<td>4.87</td>
<td>0.10035</td>
<td>0.3462</td>
<td>49.23</td>
<td>25</td>
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<tr>
<td>Skadi (S/2000 S8)</td>
<td>3.22</td>
<td>4.88</td>
<td>0.10412</td>
<td>0.2731</td>
<td>152.00</td>
<td>8</td>
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<tr>
<td>Siarnaq (S/2000 S3)</td>
<td>3.21</td>
<td>4.78</td>
<td>0.11739</td>
<td>0.3180</td>
<td>47.73</td>
<td>45</td>
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<tr>
<td>Erriapo (S/2000 S10)</td>
<td>3.21</td>
<td>4.74</td>
<td>0.11706</td>
<td>0.4690</td>
<td>37.49</td>
<td>10</td>
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<tr>
<td>Albiorix (S/2000 S11)</td>
<td>3.22</td>
<td>4.79</td>
<td>0.10949</td>
<td>0.4907</td>
<td>37.46</td>
<td>30</td>
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<tr>
<td>Tarvos (S/2000 S4)</td>
<td>3.21</td>
<td>4.77</td>
<td>0.12126</td>
<td>0.5178</td>
<td>38.07</td>
<td>16</td>
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<tr>
<td>Mundilfari (S/2000 S9)</td>
<td>3.20</td>
<td>4.76</td>
<td>0.12421</td>
<td>0.2079</td>
<td>167.14</td>
<td>7</td>
</tr>
<tr>
<td>Suttung (S/2000 S12)</td>
<td>3.18</td>
<td>4.67</td>
<td>0.12938</td>
<td>0.1155</td>
<td>176.05</td>
<td>7</td>
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<tr>
<td>Thrym (S/2000 S7)</td>
<td>3.18</td>
<td>4.69</td>
<td>0.13553</td>
<td>0.4709</td>
<td>175.56</td>
<td>7</td>
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<tr>
<td>Ymir (S/2000 S1)</td>
<td>3.14</td>
<td>4.63</td>
<td>0.15334</td>
<td>0.3368</td>
<td>173.06</td>
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<tr>
<td>⟨·⟩6</td>
<td>3.23</td>
<td>4.84</td>
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<tr>
<td>Caliban (S/1997 U1)</td>
<td>3.37</td>
<td>2.70</td>
<td>0.047900</td>
<td>0.1922</td>
<td>141.19</td>
<td>60</td>
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<tr>
<td>Stephano (S/1999 U2)</td>
<td>3.34</td>
<td>2.65</td>
<td>0.053133</td>
<td>0.2325</td>
<td>143.46</td>
<td>20</td>
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<tr>
<td>Sycorax (S/1997 U2)</td>
<td>3.23</td>
<td>2.44</td>
<td>0.081501</td>
<td>0.5197</td>
<td>156.93</td>
<td>120</td>
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<tr>
<td>Prospero (S/1999 U3)</td>
<td>3.19</td>
<td>2.38</td>
<td>0.10952</td>
<td>0.4378</td>
<td>149.32</td>
<td>30</td>
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<tr>
<td>Setebos (S/1999 U1)</td>
<td>3.16</td>
<td>2.36</td>
<td>0.11711</td>
<td>0.5776</td>
<td>153.58</td>
<td>30</td>
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<tr>
<td>⟨·⟩7</td>
<td>3.26</td>
<td>2.51</td>
<td></td>
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<tr>
<td>Neptune’s Irregulars:</td>
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<tr>
<td>Nereid</td>
<td>1.57</td>
<td>2.46</td>
<td>0.03690</td>
<td>0.7460</td>
<td>9.66</td>
<td>340</td>
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<tr>
<td>⟨·⟩8</td>
<td>1.57</td>
<td>2.46</td>
<td></td>
<td></td>
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</tbody>
</table>
Table 3: Constraints on the mass of the planetesimal disk from the known sizes of the jovian irregular satellites. For three assumed size distributions (Cases A, B, and C) and total heliocentric disk masses (10, 50, and 200\(M_\oplus\)), we list the range of jovian satellite diameters \(D_D\) such that 50, 95, and 99% of the satellites would have been catastrophically disrupted. The Case A size distribution has the fewest km-sized (and smaller) impactors for a given disk mass, while Case C has the most such impactors, so that satellite destruction probabilities are larger for Case C than for the other cases. For example, in Case A with a 10\(M_\oplus\) disk, all satellites with diameters \(D > 1\) km have better than a 50% chance of survival. In Case A with a 200\(M_\oplus\) disk, satellites with diameters between 10 and 66 km have at least a 95% chance of being destroyed, while both smaller and larger satellites are more likely to survive (see Fig. 4). For Cases B and C, the probability of destruction is a monotonically decreasing function of satellite diameter, so we only list one number, the upper limit on \(D\). For example, for Case C and a 10\(M_\oplus\) disk, satellites with \(D \leq 17\) km have a 99% chance of being catastrophically disrupted. We also indicate with bold face the cases that are inconsistent with the known sizes of the jovian irregular satellites.

<table>
<thead>
<tr>
<th>Case</th>
<th>(M_{\text{disk}})</th>
<th>(D_D(50%))</th>
<th>(D_D(95%))</th>
<th>(D_D(99%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>9–69</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>1–169</td>
<td>10–66</td>
<td>21–33</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>21</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>67</td>
<td>26</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>164</td>
<td>64</td>
<td>48</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>55</td>
<td>22</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>79</td>
<td>64</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>136</td>
<td>76</td>
<td>70</td>
</tr>
</tbody>
</table>
Fig. 1.— Orbits of the irregular satellites of Jupiter: (a,b) average orbits of the prograde moons; (c,d) average orbits of the retrograde moons. The average orbital elements were taken from Nesvorný et al. (2003). In (c,d), the average orbits of many irregular moons are tightly clustered around the average orbits of Ananke and Carme. The similarity of orbits of these moons suggest their common origin.
Fig. 2.— Differential distributions of the Jacobi constant from Beaugé et al. (2002) (solid lines) and our log-normal fits (dashed lines; Eq. 14). Parameters of the log-normal distribution are given in Table 1.
Fig. 3.— Comparison between model and numerical distribution of eccentricities for Jupiter (left-hand plots) and Neptune (right-hand plots) for $q = 0.2$, 0.4 and 0.8 $R_H$. Histograms in broken lines show numerical data obtained from Beaugé et al. (2002); histograms in continuous lines are the model distributions (see Eq. 17). The peak present for Neptune at $q = 0.2$ $R_H$ is probably caused by the temporary capture of one or a few planetesimals, and is thus beyond the scope of our model.
Fig. 4.— Disruption probability $p(D)$ of irregular satellites from impacts by stray planetesimals: (a) Jupiter, (b) Saturn, (c) Uranus, and (d) Neptune. The planets gradually eliminate the residual planetesimal disk at 10–35 AU and send numerous potential impactors into planet-crossing orbits. For impactors, we use $N_A(> d)$ (Eq. 7) normalized to $M_{\text{disk}} = 10$, 50 and 200 $M_\oplus$ (solid lines; from bottom to top). The sizes of selected irregular satellites are denoted by vertical dashed lines. Survival of these moons is likely only for low $p(D)$. From left to right we show: (a) Themisto, Ananke, Carme and Himalia at Jupiter; (b) smallest known irregular moons, S/2000 S3 (Siarnaq) and Phoebe at Saturn; (c) Stephano and Sycorax at Uranus; (d) and Nereid at Neptune.
Fig. 5.— The same as Fig. 4 but with $N_B(> d)$ (Eq. 8).
Fig. 6.— The same as Fig. 4 but with $N_C(>d)$ (Eq. 9).