The Generation and Propagation of Pc 3–4 ULF Waves at High Latitudes

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A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy.

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I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

______________________________
Tim Howard.
To absent friends.
In memory, still bright.
ACKNOWLEDGMENTS

Firstly to my supervisor, Assoc. Prof. Fred Menk, I would like to extend my deepest gratitude and respect for his endless patience and encouragement. Despite several periods of long-distance correspondence and many months of frustration and sacrifice we finally got there.

To Prof. Brian Fraser I also extend my thanks for his support and provision of the many wonderful opportunities which have presented themselves during the course of this project. My thanks also to the Australian Research Council (ARC) and the Cooperative Research Centre for Satellite Systems (CRCSS), for providing me with the time and resources needed to make this project happen.

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Finally, I would like to thank my family and friends, especially my father, Mr. Gary Howard, for his neverending love, support and understanding.
The mechanism by which Pc 3–4 (10-100 mHz) ULF wave energy propagates in the high-latitude magnetosphere remains unknown. At low latitudes, these waves are produced by field line resonances (FLR’s) but the resonant frequency at high latitudes does not lie in the Pc 3–4 frequency range. One possibility is that they may be caused by higher harmonics of FLR’s. Other suggestions made by previous workers include disturbances in the ionosphere caused by cusp-latitude precipitating electrons, and FLR harmonics driven by cavity modes in the dayside magnetosphere. Another possibility is the direct propagation of fast mode waves from the solar wind through the ionosphere, with wave refraction and diffraction occurring due to the varying refractive index of the magnetospheric medium. All of the above mechanisms are believed to be generated by the ion-cyclotron resonance mechanism in the upstream solar wind region immediately sunward of the magnetopause.

The work presented in this thesis includes a review on research efforts on Pc 3–4 waves as well as a detailed analysis on 170 events which were observed on the ground during January and March, 1998. We used the IMAGE magnetometer array in northern Scandinavia and developed profiles of amplitude, coherence and cross-phase with latitude and longitude. Using these profiles calculations of coherence length, azimuthal wavenumber, and ground speed and direction of propagation were made. We also compared the frequency of the events with the interplanetary magnetic field and solar wind cone angle using WIND satellite data and two models used to predict the upstream ion-cyclotron resonance mechanism. The spatial extent of these waves was investigated by comparing signals observed at IMAGE with those from the MACCS array in arctic Canada. Conjugate point properties were also studied with the magnetometers at Mawson and Davis in the Antarctic. Finally, mathematical models of the possibilities suggested above were compared with our results.

Our results suggest that the majority of the Pc 3–4 waves propagate poleward and away from local noon, with an average velocity of around 60 km/s N40°W. The average azimuthal wavenumber was low (≈ +3.8), suggesting the existence of a field-guided Alfvén wave. Average coherence length was 1530 km and virtually
no comparable wave activity was observed at the MACCS array. No conjugate point activity was observed. The relationship between the interplanetary magnetic field and the cone angle agreed with the predicted frequencies for generation by the ion-cyclotron mechanism.

Using the amplitude profiles we identified two classes of Pc 3–4 activity. One class occurred primarily in the daytime and displayed a peak at high latitudes near the cusp. When compared with mathematical models and DMSP satellite data which predict the location of the cusp, no convincing relationship was found between the two. The second class of events were in the local nighttime and displayed a peak at lower latitudes, near the plasmapause. However, no relationship was found between the location of this peak and that of the predicted region of the plasmapause. Furthermore the plasmapause seemed not to affect the Pc 3–4 waves at all.

From the mathematical models we offer the suggestion that around 20% of Pc 3–4 wave energy at high latitudes is attributed to by FLR harmonics. We could not identify the waves predicted by the precipitating electron beam model and believe these to be localised and not observed across the entire IMAGE array. No evidence of cavity mode waves was found. We believe that the majority of the waves are produced by one of two mechanisms. Either the waves are propagating directly through the magnetosphere as a fast mode or else the waves are coupling to field lines and travelling to the ground as a field-guided Alfvén wave. We propose the latter as a new model in this thesis. Incoming fast mode wave energy couples with geomagnetic field lines favourable for harmonic resonances. However, the field line oscillations are not sustained long enough to support a standing wave, so the energy propagates along the field line as a travelling Alfvén mode. Preliminary mathematical models of cross-phase agree well with the results observed but lack of wave conjugacy remains unaccounted for.
## CONTENTS

### ACKNOWLEDGEMENTS

### ABSTRACT

#### 1 INTRODUCTION

1.1 The Magnetosphere .............................................. 1  
1.2 Global Magnetospheric Phenomena ............................. 3  
1.3 Terminology and Coordinate Systems .......................... 5  
1.4 Pulsations in the Geomagnetic Field ........................... 8  
1.5 Project Objectives ............................................. 10

#### 2 ULF WAVES IN GEospace

2.1 History .......................................................... 13  
  2.1.1 Wave Modes ................................................. 13  
2.2 Generation Mechanisms of ULF Waves ........................ 15  
  2.2.1 Surface Waves on the Magnetosphere ...................... 16  
  2.2.2 Theory Associated with KHIs .............................. 17  
  2.2.3 Solar Wind Perturbations .................................. 19  
  2.2.4 B\textsubscript{IMF}, Cone Angle and Pulsation Frequency Relationship .... 21  
2.3 Propagation Mechanisms ....................................... 26  
  2.3.1 Shear Alfvén Modes: Travelling Waves and Field Line Resonances .......... 26  
  2.3.2 FLR Harmonics .............................................. 32  
  2.3.3 Azimuthal Wavenumber .................................... 34  
  2.3.4 Fast Modes: Travelling and Cavity/Waveguide Resonances ................. 34  
2.4 Ground Studies of Pc 3–4 at High Latitudes ................. 39  
  2.4.1 Amplitude .................................................. 39  
  2.4.2 Coherence .................................................. 42  
  2.4.3 Cross-Phase ............................................... 42  
  2.4.4 Conjugate Point Studies .................................. 45
# CONTENTS

2.4.5 Nighttime Events ................................................. 47  
2.5 Location of Magnetospheric Boundaries: Magnetopause and Plasma-pause ................................................. 48  
2.5.1 The Magnetopause ............................................. 49  
2.6 The Plasmapause ................................................... 50  
2.7 Effects of the Ground and Ionosphere ......................... 52  
2.7.1 Earth Induction Effects ....................................... 52  
2.7.2 Ionospheric Effects ........................................... 55  

3 DATA COLLECTION AND ANALYSIS ................................. 59  
3.1 Data Sources ...................................................... 59  
3.1.1 The IMAGE Magnetometer Array ............................. 59  
3.1.2 Antarctic Induction Systems .................................. 62  
3.1.3 MACCS ......................................................... 63  
3.1.4 The WIND Satellite ............................................ 63  
3.1.5 DMSP ........................................................ 65  
3.1.6 The LANL-097A satellite ..................................... 66  
3.2 Data Preparation .................................................. 67  
3.2.1 IMAGE Magnetometer Array ................................ 67  
3.2.2 MACCS ......................................................... 67  
3.2.3 Antarctic Magnetometer Data ................................. 68  
3.2.4 WIND Spacecraft .............................................. 68  
3.2.5 DMSP ........................................................ 69  
3.3 Data Analysis Theory ............................................. 69  
3.3.1 The Fast Fourier Transform (FFT) Analysis and Filtering 69  
3.3.2 Artifacts of the FFT: Aliasing and Leakage ............... 70  
3.4 Bivariate Signal Analysis ...................................... 76  
3.4.1 Cross-Phase ................................................... 76  
3.4.2 Identification of reliable signals: Coherence ............. 77  
3.5 Polarization ....................................................... 78  
3.5.1 Definition of Polarization .................................... 79  
3.5.2 The Stokes Parameters ....................................... 82  
3.6 Statistical Relevance to Analysis Techniques ................. 85  
3.6.1 The Sinusoidal Model ....................................... 85  
3.6.2 Smoothing the Periodogram .................................. 88  
3.6.3 An Alternative Method for Coherence and Phase Determination 89  
3.6.4 Degrees of Freedom ........................................ 90
CONTENTS

3.6.5 Confidence Limits ........................................... 90
3.6.6 Confidence Limits for Coherence and Phase ............... 91

4 ANALYSIS OF OBSERVATIONAL DATA .............................. 97
4.1 Timing Error Correction ........................................ 97
4.2 Event Selection .................................................. 100
4.3 Dynamic Cross-Phase .......................................... 103
4.4 Amplitude, Phase and Coherence Profiles and Errors .... 103
  4.4.1 Amplitude Profile ......................................... 105
  4.4.2 Coherence Profile ........................................ 106
  4.4.3 Cross-Phase Profile ........................................ 108
4.5 Polarization Profiles ........................................... 116
4.6 Determination of Wave Properties From the Profiles .... 118
  4.6.1 Coherence Length .......................................... 118
  4.6.2 Azimuthal Wave Number, m ................................ 120
  4.6.3 Ground Phase Velocity (Speed and Direction) ........ 121
  4.6.4 Relationship with IMF .................................... 123
4.7 Magnetospheric Boundary Determination ..................... 128
  4.7.1 The Magnetopause ........................................ 128
  4.7.2 The Plasmapause .......................................... 131
4.8 Comparison With Other Ground Arrays ....................... 133
  4.8.1 Event Selection ........................................... 133
  4.8.2 Profiles and Values ....................................... 137

5 RESULTS .............................................................. 143
5.1 Diurnal Occurrence ............................................. 143
5.2 Spectral Appearance ............................................ 144
  5.2.1 Relationship with IMF .................................... 144
5.3 Profiles with Latitude and Longitude ....................... 149
  5.3.1 Amplitude .................................................... 149
  5.3.2 Coherence .................................................... 156
  5.3.3 Coherence Length ......................................... 156
  5.3.4 Cross-Phase ................................................ 159
  5.3.5 Azimuthal Wavenumber, m ................................ 161
  5.3.6 Ground Phase Velocity (Speed and Direction) ........ 161
5.4 Polarization Profiles ........................................... 165
5.5 Comparison with Other Ground Arrays ....................... 168
  5.5.1 Azimuthal Extent (MACCS) ................................ 168
CONTENTS

5.5.2 Conjugate Points (Davis-Longyearbyen) .......................... 168
5.6 Summary ................................................................. 173

6 GENERATION AND PROPAGATION MECHANISMS ........................................ 175
6.1 Generation ................................................................. 175
   6.1.1 Surface Waves (Kelvin-Helmholtz Instability) ....................... 175
   6.1.2 Ion Cyclotron Resonance ........................................... 178
6.2 Propagation ............................................................... 181
   6.2.1 Harmonics of FLR’s .................................................. 181
   6.2.2 Direct Fast Mode Propagation ..................................... 187
   6.2.3 Waveguide/Cavity Modes ......................................... 197
   6.2.4 The Transistor Model ............................................. 205
   6.2.5 Field-Guided Propagation ........................................ 212
   6.2.6 Geomagnetic Field Model Production ............................. 214
   6.2.7 Simulation Production .......................................... 217
6.3 Nighttime Events .......................................................... 226

7 SOURCES OF ERROR AND CONCLUSIONS ............................................ 227
7.1 Analytical and Result Limitations ...................................... 227
   7.1.1 Timing Error Correction ....................................... 227
   7.1.2 Event Selection .................................................. 228
   7.1.3 Ground Profiles .................................................. 229
   7.1.4 Spatial extent of signals, MACCS ............................... 231
   7.1.5 Conjugate Points, Davis:Longyearbyen ........................... 232
7.2 Conclusions ............................................................... 232

A PUBLICATIONS RESULTING FROM THE WORK PRESENTED IN THIS THESIS .......... 235
A.1 Conference Presentations ............................................. 235
A.2 Publications ............................................................. 236

B SUMMARY OF RESULTS .................................................... 237
B.1 Results for Each Event ............................................... 237
B.2 Events Used in the Mathematical Model ............................... 253

C INDICES AND PARAMETERS .................................................. 255
C.1 $K_p$ and Dst Indices ................................................ 255
C.2 Magnetic Declination (D) Angles .................................... 259
C.3 Values used For the Plasmapause Model ............................. 260
D MAGNETOMETER DATA CONVERSION PROCESSES 261
  D.1 The IMAGE Magnetometer Array Data Conversion ..................................... 261
  D.2 The Antarctic Magnetometer Array Data Conversion ..................................... 262

E DERIVATION OF MATHEMATICAL FORMULAE 265
  E.1 Frequency and \( B_{IMF} \) Relationship (§2.2.4) .............................................. 265
  E.2 The Polarization Equations (§3.5) ................................................................. 266
  E.3 The Stokes Parameters (§3.5.2) ................................................................. 269
  E.4 Statistical Equations (§3.6) ......................................................................... 271
  E.5 Time of Flight For A Fast Mode Wave (§6.2.2) ............................................. 272
CHAPTER 1
INTRODUCTION

1.1 The Magnetosphere

The geomagnetic field is generated by convection in the Earth’s metallic core and resembles in form the dipole field of a bar magnet. However, distortions are created by the interaction of the geomagnetic field with the solar wind and interplanetary magnetic field (IMF), leading to the formation of the magnetosphere. This chapter presents a brief description of the magnetosphere and short-period variations of the geomagnetic field. Near-Earth space is populated by energetic charged particles constituting a plasma, confined to a cavity around the Earth by the interaction of the streaming solar wind encountering the geomagnetic field (Gold, 1959; Parker, 1959). The solar wind interacting with the geomagnetic field achieves a pressure balance, given by equation (2.41) thus creating a cavity within which the Earth is enclosed. Solar wind plasma enters the high-altitude magnetosphere through the cusp region and convection from the night-side while the low-altitude magnetosphere is populated from plasma primarily from the ionosphere. A diagram of the magnetosphere and its major components is shown in figure 1.1. At high latitudes the geomagnetic field lines are swept into the tail and can reconnect to the interplanetary field. These field lines are termed open. In this thesis, the terms outlined in table 1.1 will be used to describe the main field components.

Figure 1.1: An illustration of the geomagnetic field on the noon-midnight meridian plane (Ness, 1967).
Table 1.1: Summary of important features of the geomagnetic field.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bow Shock</td>
<td>A shock formed when the supersonic solar wind encounters the geomagnetic field. It occurs on the dayside at an average distance of around 15 Earth radii ($R_E$). Its thickness ranges from around 100 km to 2 $R_E$ (Reyes, 2001). Sunward of the bow shock lies a turbulent region called the foreshock and contains reflected ions flowing upstream to the solar wind.</td>
</tr>
<tr>
<td>Magnetosheath</td>
<td>Region immediately Earthward of the bow shock where the solar wind particles are decelerated to subsonic speeds.</td>
</tr>
<tr>
<td>Magnetopause</td>
<td>The last closed magnetic field line on the dayside, moving outwards from the Earth. Location of the magnetopause current system is approximately 10–12 $R_E$ (refer to section 2.5.1).</td>
</tr>
<tr>
<td>Magnetotail</td>
<td>Region on the nightside of the magnetosphere, extending to $\sim 10^3 R_E$, containing both open and closed field lines and hot and cold plasma distributions (Kivelson and Russell, 1995).</td>
</tr>
<tr>
<td>Cusp</td>
<td>Funnel-shaped region between the front and rear lobes of the magnetosphere. Through this region solar wind particles gain direct access to the ionosphere.</td>
</tr>
<tr>
<td>Auroral Oval</td>
<td>Projection to the ground of the region where auroral activity maximises.</td>
</tr>
<tr>
<td>Polar Cap</td>
<td>Projection to the ground of the region of the magnetosphere containing open field lines. Immediately equatorward of the cap at magnetic noon lies the region mapping to the cusp.</td>
</tr>
<tr>
<td>Ring Current</td>
<td>High energy ($\sim 10^{15}$ J total) current located at around 2–5 $R_E$, flowing westward in the equatorial plane. During extended periods of strong southward IMF, charged particles from the low latitude plasma sheet are convected into the ring current. This initiates a magnetic storm, outlined in §1.2 (Reyes, 2001).</td>
</tr>
</tbody>
</table>
1.2. GLOBAL MAGNETOSPHERIC PHENOMENA

Plasma enters the magnetosphere via diffusion from the solar wind, ionospheric escape processes and magnetic reconnection (refer to §1.2). This plasma accumulates in specific regions governed by convection processes associated with the movement of the solar wind and the rotating Earth. The magnetosphere can therefore be subdivided into regions governed by the properties of these trapped plasmas. These regions (shown in figure 1.2) are described in table 1.2. As suggested by

Figure 1.2: Schematic diagram of plasma regions of the Earth’s magnetosphere in the noon-midnight meridian (Kivelson and Russell, 1995).

Dungey (1961), the behaviour of the geomagnetic field is directly related to that of the surrounding plasma, i.e. any perturbations produced in the geomagnetic field will also drive perturbations in the plasma, and vice versa. In this sense the geomagnetic field lines can be considered to be “frozen in” to the surrounding plasma. Consequently waves produced in the solar wind plasma or at the magnetosphere’s boundary later (e.g. from reconnection, §1.2) can generate fluctuations in the geomagnetic field which may propagate through the magnetosphere as hydromagnetic (hm) waves.

1.2 Global Magnetospheric Phenomena

Geomagnetic storms represent a global interaction between the solar wind, magnetosphere and ionosphere and can deposit power > 6 GW into the ionosphere (Hargreaves, 1992). They are initiated when enhanced energy transfer from the solar wind/IMF into the magnetosphere leads to intensification of the ring current (Gonzales et al., 1994). The largest storms are often related to large solar disturbances such as coronal mass ejections (CME’s) or coronal holes (Rasinkangas et
Table 1.2: The plasma regions within the Earth’s magnetosphere (Kivelson and Russell, 1995).

<table>
<thead>
<tr>
<th>Feature</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasmasphere</td>
<td>Cold dense corotating plasma forming a teardrop-shaped “sphere” around the Earth. Plasma accumulates along equipotential lines determined by a balance between the $\mathbf{E} \times \mathbf{B}$ drift plasma, the antisunward solar wind pressure, plasma exchange with the ionosphere, and corotation of the plasma with the Earth.</td>
</tr>
<tr>
<td>Plasmapause</td>
<td>Outer boundary of the plasmasphere indicated by a sharp drop in electron density (with increasing altitude) typically near 4 $R_E$. Empirically represented by a relationship involving local time and the level of magnetic activity (§2.6).</td>
</tr>
<tr>
<td>Plasma Sheet</td>
<td>Hot plasma typically on closed field lines in the nightside of the magnetosphere.</td>
</tr>
<tr>
<td>Tail Lobes</td>
<td>Low density cool plasma regions in the tail, also believed to be located on open field lines.</td>
</tr>
<tr>
<td>Plasma Sheet</td>
<td>Transition region between the tail lobes and the plasma sheet.</td>
</tr>
<tr>
<td>Boundary Layer</td>
<td>Also referred to as the Alfvén layer (Kivelson et al., 1979).</td>
</tr>
</tbody>
</table>

As storms are simultaneously observed across the entire globe, their appearance affects many magnetospheric processes, including ULF wave generation (Yumoto et al., 1997).

Like storms, magnetospheric substorms are also global phenomena, but the two are otherwise unrelated. Satellites at geosynchronous distance have shown that only a small portion of the plasma convected to the tail from the solar wind is instantaneously transported to the dayside via reconnection, the rest contributes to the tail lobes (Hargreaves, 1992). Magnetic flux intensity is increased by the cumulative build up of plasma until a sudden reconnection of the previous tail lobe field lines occurs, and the stored magnetic energy is released explosively. This has a dramatic effect on the magnetosphere and is called a magnetospheric substorm. Substorms occur almost every night for approximately two hours at a time. They result in broadband geomagnetic activity that, in this study, was one of the most prominent features in nightside ULF time series and power plots.
Magnetic reconnection refers to breaking and reconnecting of oppositely directed magnetic field lines, and is thought to be the main link in the solar wind - magnetosphere coupling process (Carter, 2001). Reconnection is believed to occur both at the nose of the magnetosphere between the interplanetary magnetic field and the geomagnetic field, and at the geomagnetic field lines in the magnetotail. The physical processes governing magnetic reconnection are still unclear and will not be discussed in this text. Useful references include Hones (1984) and Kivelson and Russell (1995). It should be noted that magnetic reconnection is not generally regarded as a global phenomenon. While it can occur across several geomagnetic field lines simultaneously its signature in the ionosphere is highly localised (Neudegg et al., 2000).

1.3 Terminology and Coordinate Systems

This section will outline the coordinate systems, magnetic field models and activity indices used in this study.

There are three magnetic field models which have been used in this study. The first is that of the dipole. The dipolar magnetic field can be written in vector form (Roederer, 1970) as

$$\mathbf{B} = \frac{1}{r^3} \left[ 3(\mathbf{k}_0 \cdot \hat{u})\hat{u} - k_0 \right],$$  \hspace{1cm} (1.1)

where $k_0$ is the Earth’s dipole magnetic moment of magnitude $8.02 \times 10^{15}$ Wb.m$^{-1}$ and direction southward from the Earth’s centre, $r$ is the radial distance from the centre of the Earth and $\hat{u}$ is the unit vector in the $r$ direction. As a function of magnetic latitude $\lambda$ the field along any given dipolar field line is

$$B(\lambda) = \frac{k_0}{L^3 R_E^3} \frac{\sqrt{4 - 3 \cos^2 \lambda}}{\cos^6 \lambda}.$$  \hspace{1cm} (1.2)

The dipole model is appropriate at low latitudes if the dipole centre is located close to the centre of the Earth and the axis inclined at $\sim 11.5^\circ$ to the Earth’s rotational axis (Knecht and Shumann, 1985).

The parameter $L$ in equation (1.2) is the magnetic shell parameter, or $L$ value (McIlwain, 1961), that describes the radius of a field line, measured in Earth radii, in the equatorial plane. $L$ values begin at 1 (the Earth’s surface) and increase with increasing latitude. The equation for a field line can be given (Kivelson and Russell, 1995) as

$$r = r_0 \sin^2 \theta,$$  \hspace{1cm} (1.3)

where $r_0$ is the distance to the equatorial crossing of the field line and $\theta$ is the colatitude angle in the spherical coordinate system. In terms of magnetic latitude
\lambda and \( L \) value, the equation can be written as

\[ r = L \cos^2 \lambda, \]  

(1.4)

where \( L \) is measured in \( R_E \). At high latitudes such as in the polar cap, the field lines are open or non-dipolar, and so the \( L \) value becomes meaningless (described in this report as \( L > 15 \)). The \( L \) value becomes unreliable from around \( L \geq 10 \) and also at very low values (i.e. near the equator), due to the influences of the ring current (Singer et al., 1981). Tables 3.1 and 3.2 show some \( L \) values for various geographic latitudes.

When we move to higher latitudes the distortion effects due to the solar wind dominate the dipole field structure. It is therefore necessary to use an alternative model when describing the geomagnetic field at around \( L > 4 \). There are two models which are popular in literature. The first is the Corrected Geomagnetic model, or CGM (Hakura, 1965). By its initial definition this involves tracing magnetic field lines using an internal magnetic field model such as IGRF (Gustafsson, 1970; 1984), and all the points along a single magnetic field line which have the same coordinates. A table is then produced which relates CGM to geographic coordinates and interpolation is performed to include any missing points (Baker, 1989). Each point on the Earth can therefore be defined in terms of this new coordinate system, hereafter referred to as CGM coordinates. In this thesis, the CGM coordinates we use are always for an epoch of 1998, unless otherwise stated.

The second model is that of Tsyganenko (Tsyganenko and Usmanov, 1982). This is a semi-empirical best-fit representation for the magnetic field, based on a large number of satellite observations (e.g. IMP, HEOS, ISEE). The model includes contributions from magnetospheric sources such as the ring current, magnetotail and magnetopause current systems, and a large number of field aligned current systems. Figure 1.3 shows a cross-sectional view of the geomagnetic field taken from the Tsyganenko model, demonstrating the annual effects on the geomagnetic field. Note that around equinox (March, September) the field is almost symmetrical about the equatorial plane. There have been several modifications made to this model since its development, first in 1987 and later in 1989 and 1995 (Tsyganenko, 1987; 1989; 1995). The most recent version of the Tsyganenko model was released in January 2001, which was a modified version of the model published in 1996 and includes, for example, the effects of IMF variations (Tsyganenko and Stern, 1996). In this study, we will use 1996 model, commonly referred to as T96.

\( K_p \) (Bartels et al., 1939) is a general index of magnetic activity. Worldwide magnetic activity is scaled every 3 hours to produce a semilogarithmic index called
1.3. TERMINOLOGY AND COORDINATE SYSTEMS

Figure 1.3: The geomagnetic field from the T89c model for $K_p = 2$ for different seasons (Rasinkangas et al., 1998).

$K_p$, which ranges from 0 (completely quiet) to 9 (very large magnetic storm). A correction factor is then applied to each station to account for the variation in magnetic activity amplitude with latitude, and the values from all the stations averaged. Each interval is divided evenly into three using positive and negative signs. For example, a $K_p$ value of 4+ represents a value of 4.33 and a value of 4− represents a $K_p$ of 3.67. Table 1.3 gives a physical description of $K_p$ values.

Table 1.3: $K_p$ values and their corresponding physical description.

<table>
<thead>
<tr>
<th>$K_p$</th>
<th>Physical Description of Geomagnetic Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;2</td>
<td>Quiet magnetic conditions.</td>
</tr>
<tr>
<td>2−4</td>
<td>Average magnetic disturbance.</td>
</tr>
<tr>
<td>4−6</td>
<td>Disturbed magnetic conditions.</td>
</tr>
<tr>
<td>&gt;7</td>
<td>Very disturbed conditions.</td>
</tr>
</tbody>
</table>

The Disturbance Storm Time, or Dst index (Sugiura, 1964) is an index of magnetic activity which monitors variations in the ring current. It is derived from a network of near-equatorial geomagnetic observatories and is an hourly average of
perturbations in the H (northward) component at these latitudes. Dst values are typically negative and large negative perturbations (<−100) are indicative of an intensity increase in the ring current, which is the signature of the main phase of a geomagnetic storm (Hargreaves, 1992). Table 1.4 gives some examples of some Dst values corresponding to various geomagnetic events.

Table 1.4: Some typical Dst values associated with storms and substorms (Rasinkangas et al., 1998).

<table>
<thead>
<tr>
<th>Dst (nT)</th>
<th>Storm Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;−100</td>
<td>Intense geomagnetic storm.</td>
</tr>
<tr>
<td>−50</td>
<td>Moderate geomagnetic storm.</td>
</tr>
<tr>
<td>−30</td>
<td>Small geomagnetic storm or typical magnetic substorm.</td>
</tr>
<tr>
<td>+30</td>
<td>Magnetospheric compression.</td>
</tr>
</tbody>
</table>

In this thesis a coordinate rotation has been performed on the data to align the axes with the invariant magnetic pole. This is based on the corrected geomagnetic (CGM) coordinate system, described above. This is based on two parameters, H and D, where H is the magnitude of the geomagnetic field for both components and D is the angle the H vector makes with geographic north, termed the declination angle. If we let \( X \) be geographic north and \( Y \) be geographic east then H and D are defined by:

\[
H = \sqrt{X^2 + Y^2} \\
D = \sin^{-1}(X/Y).
\]  

A list of D values for the stations used in this thesis is given in appendix C.2. In this thesis, we will adopt the terminology of H and D to describe the components in the geomagnetic north and east directions respectively, in accordance with several papers (e.g. Matthews et al., 1996). We stress that this should not be confused with the definitions of H and D described above.

1.4 Pulsations in the Geomagnetic Field

This project focuses on ultra-low frequency, or ULF waves. These have frequencies which range from 1 to 1000 mHz and wavelengths which may approach the size of the magnetosphere itself. Traditionally (e.g. Campbell, 1959; Bol’shakova, 1965; Troitskaya and Gul’elmi, 1967), these geomagnetic field variations have been termed
1.4. PULSATIONS IN THE GEOMAGNETIC FIELD

‘micropulsations’ because of their small magnitude relative to the main field. More recent texts (e.g. Orr, 1973; Plyasova-Bakounina et al., 1978; Odera and Stuart, 1985) have referred to them as ‘pulsations’ and most recently (e.g. Le et al., 1993; Neudegg et al., 1995; Ables et al., 1998) as simply ‘ULF waves’. The latter terminology has been adopted as the term ‘pulsations’ only describes the appearance of the signals on time series data records and not the physical mechanism by which the waves are generated and propagated.

The original classification for ULF waves was proposed by an international committee (Jacobs et al., 1964) based on the oscillation period range and appearance on time series data records. It had been previously found that there were two types of pulsation: those of continuous quasi-sinusoidal and those of irregular appearance. The former were termed “Pc” and the latter “Pi”. Pc pulsations exhibit essentially regular and sustained signals which last for several cycles and typically occur on the dayside of the magnetosphere. Pi pulsations appear more as transient signals and are mainly associated with the nightside.

A suffix indicates the frequency of the signal, ranging from 1–6 for Pc and 1–3 for Pi waves. Table 1.5 summarises these pulsation categories, including the additional Pc 6 and Pi 3 categories (Saito, 1973).

<table>
<thead>
<tr>
<th>Wave Type</th>
<th>Period Range</th>
<th>Frequency Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pc 1–2</td>
<td>0.2 – 10 s</td>
<td>0.1 – 5.0 Hz</td>
</tr>
<tr>
<td>Pc 3–4</td>
<td>7 – 150 s</td>
<td>10 – 150 mHz</td>
</tr>
<tr>
<td>Pc 5–6</td>
<td>150 – 1000 s</td>
<td>1.0 – 7 mHz</td>
</tr>
<tr>
<td>Pi 1</td>
<td>1.0 – 40 s</td>
<td>25 – 1000 mHz</td>
</tr>
<tr>
<td>Pi 2</td>
<td>40 – 150 s</td>
<td>7 – 25 mHz</td>
</tr>
<tr>
<td>Pi 3</td>
<td>&gt;150 s</td>
<td>&lt;7 mHz</td>
</tr>
</tbody>
</table>

While useful for lending order to data sets, the Pc/Pi classification does not describe the physical ULF wave generation and propagation mechanisms. In this thesis, the Pc/Pi classification is used for descriptive convenience rather than implying a categorisation of physical mechanisms.
1.5 Project Objectives

There are several possible mechanisms by which Pc 1–2 and Pc 5–6 ULF waves may be generated in the high latitude magnetosphere. Theories also exist which explain acceptably the production of Pc 3–4 waves at low latitudes. These are due to field line resonances excited by fast mode waves propagating earthward from an ion-cyclotron resonance mechanism in the upstream solar wind (Yumoto et al., 1985). This will be discussed in chapter 2. However, the mechanism by which Pc 3–4 signals are occurring at high latitudes remains unclear. The longer field line length here moves the eigenfrequency down into the Pc 5 range, making the field line resonance model a less likely description of wave generation. Resonance-related mechanisms such as higher harmonics have been suggested by some workers (e.g. Fukunishi and Landerotti, 1974; Tonegawa and Fukunishi, 1984; Ziesolleck et al., 1997) and this possibility has been investigated in the course of this work. The aim of this project is to determine the source and propagation mechanism of Pc 3–4 ULF waves at high latitudes. To this end an analysis of propagation, coherence, phase and polarization characteristics has been made for over 100 events observed in ground magnetometer array data over two months. Mathematical models of possible mechanisms have also been compared with the observational results of this study.

Several characteristics have been investigated in this project. The majority of the study was conducted with digital data from an array of ground-based magnetometers in the northern polar region in Scandinavia (3·3 < L < 15). Events were from January and March 1998 and were selected on the basis of high coherence across the majority of this array. The meridional and azimuthal profiles in amplitude, phase, coherence and polarization were determined following a statistical review of the relevant analytical techniques. From these plots the ground velocity, azimuthal wave number and coherence length were determined. Comparison with satellite data from the solar wind provided information on the relationship between signal frequency and the interplanetary magnetic field strength and cone angle. Possible evidence of standing field line oscillations was investigated using a conjugate station at Davis station, Antarctica, and the MACCS magnetometer array, which lies 7 LT hours from IMAGE provided information on the properties of these waves across large distances. A peak in the amplitude profile was found for events on the dayside and its position was compared to the location of the magnetopause using data from two satellites as well as three mathematical models. Finally, the observed properties of the signals were compared with five possible models of wave
generation and propagation.

This thesis consists of seven chapters. The next chapter will include a review on the history of ULF wave studies and the work done to date regarding them, chapter 3 will discuss the data collection and the techniques employed during the analysis of the digital signals, and chapters 4 and 5 will present the results, first individually (chapter 4) and then collectively (chapter 5). Chapter 6 will consider five mathematical models which have been suggested for Pc 3–4 wave propagation and their comparison with the results obtained from the data will be included with the final discussion chapter.
CHAPTER 2
ULF WAVES IN GEOSPACE

2.1 History

ULF waves have become an essential element in magneto-spheric physics and a useful tool for diagnostic and space weather studies. Long period pulsations were reported in 1741 by Anders Celsius who compared the compass measurements in Uppsala with pulsations of aurora (Kangas et al., 1998). They were later observed by Johan Nervander in the 1840's and again by Balfour Stewart in 1861, who is often credited with their discovery. The International Geophysical Year (IGY) in 1957–58 sparked a large increase in the research efforts in many field of space physics, including long-period pulsations, prompting Jacobs et al. to propose a classification for them in 1964. By the early 1970's well over 5000 related papers had been published (Orr, 1973). Since that time they have been identified in almost all areas of space physics and used as a tool in several areas, including the identification of geomagnetic storms (e.g. Heacock and Hessler, 1965; Olson and Lee, 1983; Kangas et al., 1986) and substorms (e.g. Saito, 1969; Rostoker and Olson, 1978; Bösinger et al., 1981), plasma density (Obayashi and Jacobs, 1958; Warner and Orr, 1979), magnetospheric topology (Fukunishi and Lanzerotti, 1974a; 1974b; Ables et al., 1998) and as a diagnostic tool for physical phenomena such as magnetic reconnection (Prikril et al., 1998; Neudegg et al., 1999) and magnetic field line resonances (Menk et al., 1994 and references therein). They have also been used as a natural probe of the solar wind and the outer magnetosphere (e.g. Russell and Fleming, 1976; Troitskaya, 1997).

The most recent work on ULF waves has been on magnetospheric field topology, such as the use of Pc 5 waves to determine the location of the plasmapause (Menk et al., 1999; Milling et al., 2001) and the open/closed magnetic field line transition (Ables et al., 1998; Mathie et al., 1999a). Other work includes the use of Pc 5 ULF waves to study waveguide modes (Mathie et al., 1999b; Mann and Wright, 1999; Mathie and Mann, 2000). There are currently discussions taking place regarding the development of a ULF wave index (Mann, 2001).

2.1.1 Wave Modes

Historically, ULF waves have been classified into wave modes, based on their propagation characteristics. If we assume a compressible fluid of infinite electrical con-
ductivity and introduce a uniform magnetic field then a disturbance in the system can excite several distinctly different wave modes (Orr, 1973). There are three basic types of low frequency hydromagnetic wave modes:

1. Shear Alfvén mode waves;

2. Fast magneto-acoustic mode waves;

3. Slow magneto-acoustic mode waves.

Shear Alfvén mode waves are sometimes referred to as resonant, toroidal or Alfvén waves, although these terms can be misleading as they are only appropriate when the surrounding plasma meets certain conditions. These waves can be described by a simple physical picture according to Alfvén and Fälthammar (1963). The following description refers to figure 2.1.

Figure 2.1: Alfvén mode waves (Alfvén and Fälthammar, 1963): a) Conducting fluid movement in y-direction as a result of a magnetic field perturbation in the z-direction which causes and electric field $E'$ in the x-direction. b) Qualitative illustration of the induced current system. c) The translation of the initial motion of ABCD to adjacent sections parallel to the magnetic field lines.

Consider an incompressible electrically conducting fluid which is immersed in a homogeneous magnetic field $\mathbf{B}$ in the $z$-direction (figure 2.1a). Also assume that the fluid is at rest everywhere except for a cross-section ABCD in the $xz$-plane, which is disturbed in the positive y-direction with velocity $\mathbf{v}$. The motion of the fluid in this direction induces an electric field $\mathbf{E} = \mathbf{v} \times \mathbf{B}$ in the x-direction. The current
2.2. GENERATION MECHANISMS OF ULF WAVES

The system produced is shown in figure 2.1b. In accordance with Lenz’s law, a force is induced to oppose the motion of ABCD. This is a volume force in this region only, and so a resulting acceleration is produced in section EFBA and DCGH which are adjacent to the original disturbed section. These two sections now exhibit the same motion as the original section ABCD. Consequently, these sections will induce the same motion in their adjacent section IJKL and MNOP (figure 2.1c). By repetition of this motion the disturbance will propagate along the z-axis (i.e. the direction of the magnetic field) with velocity known as the Alfvén velocity. The Alfvén velocity is defined by

\[ V_A = \frac{B_0}{\sqrt{\mu_0 \rho}}, \]

where \( B_0 \) is the surrounding magnetic field, \( \mu_0 \) is the permeability of free space and \( \rho \) is the mass density of the particles present. This is analogous to the speed of propagation of a travelling wave such as in a stretched string, where the velocity is governed by \( v = \sqrt{T/\mu} \). In this case, \( T \) is the restoring force (tension) and \( \mu \) is the mass density of the string. Thus, the Alfvén speed varies with plasma density \( \rho \) and radial distance (and hence \( B_0 \)) in the magnetosphere.

Fast Magneto-acoustic waves are often called poloidal or compressional mode waves but this is also misleading as it implies only one compressional mode exists. In a “cold” plasma (one in which the kinetic energy of the plasma due to thermal energy is neglected) this is indeed the case. However, a thermal contribution to the plasma (i.e. a “warm” plasma) introduces another compressional mode; the Slow Magneto-acoustic mode. The term compressional is, however, descriptive of the type of wave a fast mode represents. A shear Alfvén wave is transverse in nature and is restricted to motion along magnetic field lines, while fast (or slow) mode waves are compressional and are able to propagate perpendicularly to magnetic field lines. This gives them a strong penetrating ability into the Earth’s magnetosphere.

In nature it is rare for either mode to propagate independently of the other and Alfvén mode waves nearly always have a fast mode wave component.

2.2 Generation Mechanisms of ULF Waves

There have essentially been two generation mechanisms attributed to ULF waves, both originating in the solar wind. The first is that of surface waves, primarily produced by the Kelvin-Helmholtz Instability, or KHI (Kelvin, 1910). The KHI involves the creation of waves at the interface of two interacting, dynamic fluids, much in the same way as the production of water waves are created on interaction.
with the wind (Dungey, 1955). The second is the ion-cyclotron resonance mechanism, which is caused by the enhancement of waves by resonance with the cyclotron resonance frequency of ions in the surrounding plasma and magnetic field. When generated in the interplanetary magnetic field these produce waves in the Pc 3 frequency range while in the stronger geomagnetic field Pc 1 waves are produced. This section will discuss these two processes and their responsibility for the production of ULF waves.

2.2.1 Surface Waves on the Magnetosphere

For our discussion we may consider the magnetopause as the solar wind/magnetosphere interface. This assumption is valid for a first-order approximation as the magnetosheath, which lies just beyond the magnetopause, is dominated by solar wind processes. In general, there must be a velocity shear across the boundary for the KHI waves to form. The field lines associated with this shear map to high latitude dayside regions of the magnetosphere and are unable to couple to the open field lines extending into the magnetotail. As a result, the instability is, in general, not observed in the nightside magnetosphere. This condition supports the KHI theory for the production of Pc 5 ULF waves as these signals are mostly observed on the dayside.

![Schematic diagram of the equatorial plane of the magnetosphere, viewed from the north pole, showing the different sense of polarization for KHI generated waves on either side of the stagnation point P (Orr, 1973).](image)

Figure 2.2: Schematic diagram of the equatorial plane of the magnetosphere, viewed from the north pole, showing the different sense of polarization for KHI generated waves on either side of the stagnation point P (Orr, 1973).

If surface waves are generated at the magnetopause then the plasma will have
an approximate elliptical motion, with the rotation being in the opposite sense along the dawn and dusk meridians (Orr, 1973). As the field lines are frozen into the surrounding plasma they will also rotate, generating elliptically polarized hydromagnetic waves. These are then able to propagate to the Earth via the field lines.

Let the wavenumber of these waves on the boundary be $k_y$ (the $y$-component of $k$). Under appropriate conditions these waves can couple to sound waves in the magnetosheath and fast mode waves in the magnetosphere. If $\omega < k_y v_A^2$ then these waves decay exponentially in the radial direction, with amplitude varying as

$$e^{-\sqrt{k_y v_A^2 - \omega^2}}$$

on either side of the boundary. For this reason the KHI waves are called surface waves. They are, however, able to penetrate a few $R_e$ into the magnetosphere since the rate of decay is relatively gradual. They then take the form of evanescent fast mode waves which can couple to shear Alfvén waves already present in the magnetosphere, giving rise to the characteristic signature of a field line resonance (Walker et al., 1992). Walker (1981) has suggested that the KHI occurs in the boundary layer just inside the magnetopause and that the wavelength which grows most rapidly is given by $k_y d \approx 0.6$ where $d$ is the thickness of the boundary layer. This means that the resultant frequency for the waves is typically $\leq 10$ mHz, i.e. in the Pc 5 range. Some observations (e.g. Lanzerotti and Fukunishi, 1974a) have also suggested that surface waves may also be generated on the plasmapause but it remains uncertain as to whether these are produced by the KHI or some other mechanism.

### 2.2.2 Theory Associated with KHIs

To detect these waves at the Earth’s surface they must couple with the magnetic field lines, whose oscillations will drive currents in the ionosphere which will be detected by ground-based magnetometers. Here we will derive the differential equations associated with the perturbed magnetic field in dipole coordinates. This derivation will be in accordance with Chen and Hasegawa (1974a) with additional steps derived by the author to provide continuity and clarification.

We begin with the linearised MHD equations for a single component plasma neglecting interaction with electrostatic drift:

$$\rho_m \frac{\partial^2 \vec{\xi}}{\partial t^2} = \frac{1}{\mu_0} (\nabla \times \vec{b}) \times \vec{B} + \frac{1}{\mu_0} (\nabla \times \vec{B}) \times \vec{b} - \nabla p,$$

$$\vec{b} = \nabla \times (\vec{\xi} \times \vec{B}),$$

(2.3)
where $\mathbf{B}$ and $\mathbf{b}$ are the vectors representing the main and perturbed magnetic fields respectively, $\rho_m$ is the mass density, $p$ is the perturbed pressure and $\mu_0$ is the permittivity of free space, given as $4\pi \times 10^{-7}$ Wb/A·m. $\mathbf{\xi}$ is the displacement vector, defined using the perturbed fluid velocity $\mathbf{v}$ and the following relationship:

$$\frac{\partial \mathbf{\xi}}{\partial t} = \mathbf{v}. \tag{2.4}$$

We now consider variations in both the number density and plasma pressure. For number density, we have the conservation of mass law which requires

$$n + N \nabla \cdot \mathbf{\xi} + \mathbf{\xi} \cdot \nabla N = 0, \tag{2.5}$$

where $N$ and $n$ are the main and perturbed number densities respectively. For the plasma pressure we take equation (2.5) and introduce an adiabatic assumption. This yields

$$p = -NT(\Box_p \cdot \mathbf{\xi} + \gamma \nabla \cdot \mathbf{\xi}), \tag{2.6}$$

where $T$ is the temperature and $\Box_p$ is the adiabatic thermal conductivity, given by $\Box_p \approx \nabla (\ln NT)$. We now have a mathematical description of three basic types of MHD waves. Equations (2.3–2.6) describe the ion acoustic wave, the magnetosonic wave and the shear Alfvén wave.

Now, as the surface wave passes into the magnetosphere it couples with shear Alfvén modes already present. If we apply standard equalities of div and curl to equations (2.3) we reveal

$$\rho_m\mathbf{\xi} = -\nabla [p + (\mathbf{b} \cdot \mathbf{B}/\mu_0)] + \frac{1}{\mu_0} [(\mathbf{b} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{b}], \tag{2.7}$$

and

$$\mathbf{b} = -\mathbf{B}(\nabla \cdot \mathbf{\xi}) + (\mathbf{B} \cdot \nabla)\mathbf{\xi} - (\mathbf{\xi} \cdot \nabla)\mathbf{B} \tag{2.8}$$

respectively. Also, combining equations (2.3) and (2.4) reveals

$$\mu_0\rho_0 \frac{\partial^2 \mathbf{\xi}}{\partial t^2} - (\mathbf{B} \cdot \nabla)^2 \mathbf{\xi} = -\mu_0 \nabla [p + (\mathbf{B} \cdot \mathbf{b}/\mu_0)] + \mathbf{C}, \tag{2.9}$$

where

$$\mathbf{C} = (\mathbf{b} \cdot \nabla)\mathbf{B} - (\mathbf{B} \cdot \nabla)[(\mathbf{\xi} \cdot \nabla)\mathbf{B} + \mathbf{B}(\nabla \cdot \mathbf{\xi})] \tag{2.10}$$

and $\mathbf{b}$ is defined by equation (2.3). Equation (2.9) represents the coupling between the shear Alfvén and surface waves via the term $\mathbf{C}$, called the coupling coefficient, which is due to the non-uniformity of the perturbed magnetic field.
We now change to dipole coordinates \((\nu, \mu, \phi)\). These are shown in figure 2.3 and are related to spherical coordinates \((r, \theta, \phi)\) by the following equations:

\[
\begin{align*}
\nu &= \sin^2 \frac{\theta}{r}, & \mu &= \cos \frac{\theta}{r}, & \phi &= \phi, \\
h_\nu &= \frac{r^2}{\sin^2 \sqrt{1+3\cos^2 \theta}} & h_\mu &= h_\nu h_\phi = M/B, & h_\phi &= r \sin \theta,
\end{align*}
\]

(2.11)

where \(M\) is the Earth’s magnetic dipole moment, \(B\) is the field strength and \(h\) and \(r\) represent scale factors. Using these coordinates we can write an expression for the relationship between the perturbed magnetic field \(\mathbf{b}=(b_\nu, b_\mu, b_\phi)\) and \(\mathbf{\xi}=(\xi_\nu, \xi_\mu, \xi_\phi)\) using equation (2.10):

\[
\begin{align*}
\frac{b_\nu}{B} &= -\left[ \frac{im}{h_\phi} \xi_\phi + \frac{1}{h_\nu} \frac{\partial \xi_\nu}{\partial \nu} - \xi_\nu \left( \frac{1}{h_\phi} \frac{\partial h_\phi}{\partial \phi} \right) \right], \\
\frac{b_\mu}{B} &= \frac{1}{h_\mu} \frac{\partial \xi_\mu}{\partial \mu} - \left( \frac{1}{h_\nu h_\mu} \frac{\partial h_\mu}{\partial \mu} \right) \xi_\phi, \\
\frac{b_\phi}{B} &= \frac{1}{h_\mu} \frac{\partial \xi_\mu}{\partial \mu} - \left( \frac{1}{h_\nu h_\mu} \frac{\partial h_\mu}{\partial \mu} \right) \xi_\nu.
\end{align*}
\]

(2.12)

This set of equations is somewhat difficult to solve and is primarily treated only for special cases such as when the plasma is cold and incompressible (e.g. Orr, 1973).

### 2.2.3 Solar Wind Perturbations

During the past few years there has been an accumulation of evidence to suggest that geomagnetic pulsations are related to solar wind properties. A relationship between solar wind velocity and \(K_p\) value has been found by Snyder et al. (1963) and Wilcox et al. (1967) and a relationship between \(K_p\) index and pulsation activity has also been found (Campbell, 1959; Bol’shakova, 1965; Nagata and Fukunishi, 1968). Several studies have also found a relationship between Pc 3–4 activity and the solar wind velocity (Troitskaya and Gul’elmi, 1967; Verö, 1981; Wolfe et al., 1985). This can be described by a simple mathematical expression relating the IMF strength and the frequency of the Pc 3–4 pulsation on the ground and has
been determined by Troitskaya et al. (1971) and later by Verő and Holló (1978) and Odera and Stuart (1985). This relationship takes the form

\[ f \approx 6B_{IMF}, \]  

(2.13)

where \( f \) is the frequency of the pulsation in mHz and \( B_{IMF} \) is the interplanetary magnetic field strength in nT.

It has also been suggested (Bol’shakova and Troitskaya, 1968) that the orientation of the IMF relative to the magnetosphere has an influence on Pc 3–4 activity. The relative orientation is described by the angle between the IMF and the Earth-Sun line and is called the IMF cone angle. Plyasova-Bakounina (1972) has demonstrated a relationship between the cone angle and Pc 3–4 activity, with a minimum in activity occurring when the angle is 90° and 270° and a maximum at 0° and 180°. This study will consider two relationships between \( B_{IMF} \) to determine the possible influence of the solar wind on high-latitude Pc 3–4 activity.

All of the previous work mentioned above suggests that there may be a source of IMF waves present in the solar wind before reaching the magnetosphere. When the IMF cone angle is \(< 45°\), the upstream region of the bow shock is filled with ion beams which have been reflected by the magnetosphere back towards the Sun (Hoppe et al., 1981). These ions have two components to their propagation; parallel (\( v_\parallel \)) and perpendicular (\( v_\perp \)) to the IMF. The component which is perpendicular to the IMF will be forced in a circular direction according to the right hand rule and the parallel component will remain unaffected. The result is a spiral (cyclotron) motion with frequency governed by

\[ f_{ci} = \frac{qB_{IMF}}{2\pi m}. \]  

(2.14)

The backstreaming ions spiral sunward along the IMF lines with an ion-cyclotron frequency and rotation direction which depends on the mass and charge of the particle and the IMF strength. Incoming waves in the solar wind with frequencies matching the ion-cyclotron frequency will experience resonance, increasing in magnitude. These are fast mode waves which are then swept towards the Earth where they are able to propagate into the magnetosphere without any significant alteration to their spectra (Greenstadt et al., 1983). From equation (2.14) it can be shown that for protons in a typical \( B_{IMF} \) the upstream ion-cyclotron frequency lies in the Pc 3–4 range. As an example consider the following: A proton carries a charge of \( 1.60 \times 10^{-19} \) C and has a mass of \( 1.67 \times 10^{-27} \) kg. A typical value for the interplanetary magnetic field is 4 nT. Applying these values to equation (2.14) reveals a cyclotron frequency of \( \sim 60 \) mHz, which lies in the upper Pc 3 range.
This would suggest that fast mode Pc 3–4 waves are being produced in the solar wind then passing into the magnetosphere. Variations in the frequency are caused by varying values for $B_{IMF}$ as well as doppler shifting of the backstreaming ions with the incoming supersonic solar wind. Several studies using satellite data have confirmed the existence of these waves (e.g. Greenstadt et al. 1968; Fairfield, 1969; Plyasova-Bakounina et al. 1978). These waves may then couple to field line oscillations or propagate directly through the magnetospheric cavity to the ionosphere, resulting in Pc 3–4 pulsations detected on the ground.

Figure 2.4: The principle internal (I-1, I-2 and I-3) and external (E-1, E-2 and E-3) sources for exciting long period MHD waves in the magnetosphere. These sources are classified into two groups: monochromatic and broadband frequency (Yumoto, 1988).

2.2.4 $B_{IMF}$, Cone Angle and Pulsation Frequency Relationship

Here we will derive the relationships between the interplanetary magnetic field measured by spacecraft and pulsation frequency observed on the ground, which have been used in this study. This section will focus on two approaches, one theoretical, based on Takahashi et al. (1984), and one semi-empirical, from Le and Russell (1996). Preliminary analysis is made in accordance with Gary (1991) with additions and clarifications made by the author.

We begin with an expression of wave frequency as predicted by linear theory
(Gary, 1991) and apply the assumption that the waves in the solar wind are produced by some form of resonance, specifically, the ion-cyclotron resonance mechanism. Mathematically, an individual charged particle of species $j$ is resonant with a wave or field fluctuation if the component of velocity which is magnetic field-aligned ($k_z$) satisfies the cyclotron resonance condition. Namely,

$$\omega_r - k_z v_z \pm m \Omega_j = 0, \quad m = 1, 2, 3, ..., (2.15)$$

where $\omega_r$ is the wave frequency in the solar wind frame, $\Omega_j$ is the cyclotron frequency and $v_j$ is the statistical population of drift velocities of species $j$. These conditions occur when the particle experiences a constant electric field in its own frame of reference, allowing it to exchange a significant amount of energy with the wave. Let $v_{0j} = \hat{z}v_{0j}$ represent the drift velocity of the $j$th component along the magnetic field in the solar wind frame and assume that the $j$th component of the drift velocity distribution function has an average of $v_{0j}$. Also assume the plasma is cool, so that $v_j \ll v_{0j}$. This produces a distribution function which is relatively narrow (with variance, $\sigma^2 \ll 1$), allowing us to consider a single velocity $v_{0j}$, rather than a statistical estimation of the distribution function. Finally, we must set $m = \pm 1$ in equation (2.15) as only these resonances contribute at $k \times B_0 = 0$, which is the condition for propagation parallel or antiparallel to the background field $B_0$ (Gary, 1991). These two values for $m$ correspond to resonance modes of right-handed ($+$) and left-handed ($-$) circularly polarized waves. Observations have revealed that the low frequency waves which are typically enhanced are those in the right hand mode (Le et al., 1989), allowing $m = 1$. Under these assumptions, equation (2.15) becomes

$$\omega_r - \mathbf{k} \cdot \mathbf{v}_{0j} + \Omega_j = 0. \quad (2.16)$$

Using the definitions of vector dot product, wavenumber and phase velocity (refer to Appendix E.1), we arrive at

$$\omega_r = \frac{\Omega_p}{(v_{0j}/v_{ph}) \cos \theta_{kB} - 1}, \quad (2.17)$$

where $v_{ph}$ is the phase velocity of the reflected wave and $\theta_{kB}$ is the angle between $k$ and the IMF (Le and Russell, 1996). The wave frequency observed by the spacecraft, $\omega_{sc}$, is Doppler shifted since the coupled wave is flowing downstream, in the opposite direction to the wave described by equation (2.16). So we have

$$-\omega_{s/c} = \omega_r - \mathbf{k} \cdot \mathbf{v}_{sw}, \quad (2.18)$$

where $\omega_{s/c}$ is the frequency in the spacecraft frame and $v_{sw}$ is the solar wind velocity vector. Again using vector analysis and combining equations (2.17) and (2.18)
2.2. GENERATION MECHANISMS OF ULF WAVES

(Appendix E.1), we have

\[ \frac{\omega_s/c}{\Omega_p} = \left( \frac{v_{sw}}{v_{ph}} \right) \cos \theta_k - 1 \]

where \( \theta_k \) is the angle between \( k \) and the solar wind flow.

Now, we assume that the wave propagates along the IMF with speed equal to the Alfvén speed, i.e. \( v_{ph} = v_A \) and \( \theta_k = 0 \). Also \( k \) is parallel to \( B \), and so \( \theta_k = \theta_Bz \), which is the cone angle described previously. So equation (2.19) becomes

\[ \frac{\omega_s/c}{\Omega_p} = \left( \frac{v_{sw}}{v_A} \right) \cos \theta_Bz - 1 \]

Now we apply one further definition. The Alfvén Mach number is defined in the solar wind in the same way as the general Mach number was defined on the Earth, i.e. as the ratio between the speed of the wave and the speed of sound, or \( M_A = \frac{v_{sw}}{v_A} \). Equation (2.20) now becomes

\[ \omega_s/c = \Omega_p \frac{M_A \cos \theta_Bz - 1}{M_A(v_{0j}/v_{sw}) - 1} \]

where \( \cos \theta = B_{Bz}/B_{IMF} \), the solar wind cone angle. According to equation (2.21), the frequency observed is dependent on the Alfvén Mach number, the cone angle, solar wind speed and the bulk speed of the backstreaming ions. In order to determine a relationship between the frequency \( f \), cone angle \( \theta_Bz \) and IMF strength \( B_{IMF} \) approximations need to be applied to the other variables in equation (2.21). These can be made theoretically or empirically. We will construct a relationship using both approaches.

Several attempts have been made to obtain a theoretical relationship between \( B_{IMF} \) and \( f \) but until recently, only a few have included \( \theta_Bz \). The following has been made in accordance with Takahashi et al. (1984) and beginning with equation (2.21).

Using vector geometry shown in figure 2.5 and letting \( v_{0j} = v_r \) we have the following expression

\[ v_{0j} = v_r = (v_{||i} + v_{||r})\hat{B}. \]

We can now apply the following expression for \( v_{0j} \) (refer to Appendix E.1):

\[ v_{0j} = (1 + \delta) \left( \frac{\hat{n} \cdot \hat{x}}{\hat{n} \cdot \hat{B}} \right) v_{sw}\hat{B}, \]

where \( \delta \) is the ratio of guiding centre velocities along the magnetic field before and after reflection (\( \delta = v_{||r}/v_{||i} \)), and \( \hat{n}, \hat{x}, \hat{B} \) are the unit vectors along the shock normal, Earth-Sun line and IMF respectively. Assuming adiabatic reflection at the
Figure 2.5: (a) Vector diagram showing how the incident particle guiding-centre velocity vector, $v_i$, is decomposed into a component, $v_{∥i}$, along the IMF and a component, $v_t$, along the shock front. (b) Similarly the velocity vector of the reflected particle is decomposed into $v_{∥r}$ along the magnetic field and $v_t$ along the shock front. In this analysis we are neglecting and thermal contributions to velocity, and so angle $\gamma$ is zero and $v_{sw} \parallel v_i$. (Sonnerup, 1969).

The shock front ($\delta = 1$) and that the shock is perpendicular to the Earth-Sun line ($\hat{n} \parallel \hat{x}$), equation (2.23) becomes (Appendix E.1)

$$v_{0j} = 2v_{sw} \sec \theta B_x,$$  

(2.24)

and so we can write equation (2.21) as

$$\omega_{s/c} = \Omega_p \frac{M_A \cos \theta B_x - 1}{2M_A \sec \theta B_x - 1}.$$  

(2.25)

The Alfvén Mach number is typically of order 10 and so if we assume that $\theta B_x$ is not close to 90° we can neglect the $-1$ term at the end of the numerator and denominator. Equation (2.25) now becomes

$$\omega_{s/c} = \frac{1}{2}\Omega_p \cos^2 \theta B_x,$$  

(2.26)

or, in physical units, using the definition of $\Omega_p$,

$$f_{s/c}(\text{mHz}) = \frac{\omega_{s/c}}{2\pi} \sim 7.6B_{IMF}(\text{nT}) \cos^2 \theta B_x = 7.6B_{Bx}^2/B_{IMF} = 7.6B_{Bx} \cos \theta B_x,$$  

(2.27)
where $f_{s/c}$ is the frequency of the wave as observed by the spacecraft.

Empirical approaches are often preferred to theoretical attempts as they can take into consideration factors which are not considered in the theoretical model. They are limited to the observations made at the time which can sometimes lead to bias. This empirical derivation is in accordance with Le and Russell (1996) and also begins with equation (2.21), rewritten in the form

$$\omega_{s/c} = \Omega_p \cos \theta_B - 1/M_A. \quad (2.28)$$

As mentioned previously, $M_A \gg 1$ and we apply one further condition that $(v_{0j}/v_{sw}) > 1$. Equation (2.28) is now reduced to

$$\omega_{s/c} = \Omega_p \cos \theta_B - 1/M_A v_{0j}/v_{sw}. \quad (2.29)$$

This equation reveals a dependence of frequency on the IMF cone angle, Alfvén Mach number and $(v_{0j}/v_{sw})$. Le and Russell (1996) determined a relationship between $\theta_B$ and $B_{IMF}$ using measurements obtained from the ISEE spacecraft in 1978–79. They first produced a relationship between the spacecraft frequency and the cone angle,

$$f = (0.0408 + 0.307 \cos \theta_B) f_{ci}, \quad (2.30)$$

where $f_{ci}$ is the ion-cyclotron frequency. As shown in equation (2.14) this value is dependent on the value of $B_{IMF}$, via $f_{ci} \sim 15.2 B_{IMF}$. Applying this value to equation (2.30) reveals

$$f(mHz) = (K + 4.67 \cos \theta_B) B_{IMF}(nT), \quad (2.31)$$

where $K$ is a constant. The value of $K$ was determined by the production of the trend in a frequency against IMF strength plot produced from experimental observations. This plot was normalised by setting the cone angle to zero and allowing the trends to pass through the origin. The relationship was found to be $f(mHz) = 5.39 B_{IMF}(nT)$ and must satisfy equation (2.31). The value of $K$ must therefore be 0.72, producing the following equation:

$$f(mHz) = (0.72 + 4.67 \cos \theta_B) B_{IMF}(nT). \quad (2.32)$$

Le and Russell (1996) then attempted to produce a relationship between $(v_{0j}/v_{sw})$ and $M_A$. Upon linear regression of their measured results, they discovered a low correlation and concluded that the two parameters were not related.

If we assume that the frequency of the wave remains unaltered during its passage through the magnetosphere then the frequency predicted by equations (2.27) and
(2.32) also represents those measured by magnetometers on the ground. These are the two equations used to compare the pulsation frequency with solar wind parameters in this study.

2.3 Propagation Mechanisms

Once produced the ULF waves require a mechanism by which to propagate into the magnetosphere. They can assume one of two modes and can mode-convert upon interaction with a boundary such as the magnetopause or plasmapause. This section considers wave propagation for both shear Alfvén and fast mode propagation.

2.3.1 Shear Alfvén Modes: Travelling Waves and Field Line Resonances

Shear Alfvén mode waves are either standing or travelling, depending on the wavelength relative to the geomagnetic field line length.

Lower frequency waves have wavelengths which approximate the length of the field lines themselves, and are thus able to establish standing field line oscillations, or field line resonances (FLR’s). In the same way that standing waves are formed in stretched strings, closed magnetospheric field lines, which are “anchored” at conjugate ionospheres, are able to sustain resonances.

A field line oscillation can be described by the equation for forced, damped simple harmonic oscillations given as (Orr and Hanson, 1981):

\[
\ddot{b}_\phi + 2\gamma \dot{b}_\phi = \omega_R^2 b_z c (\sin \omega_D t),
\]  

(2.33)

where \( b_\phi \) represents the magnetic field of the shear Alfvén mode wave (i.e. a resonance), \( b_z \) is the magnetic field of the fast mode wave which drives the resonance (frequency \( \omega_D \)), and \( \omega_R \) is the natural resonant frequency of the field lines which are driven by the incoming fast mode wave. The incoming waves are coupled to field lines by the coupling constant \( c \) and \( \gamma \) is the damping term related to Joule dissipation in the ionosphere at each end of the field line.

The solution to (2.33) has two terms: the transient function where oscillations with frequencies near \( \omega_R \) have amplitudes which decay exponentially, and the steady state function at the driving frequency \( \omega_D \) (Menk et al.1994). If we assume a broadband, low dampened, uniform standing fast mode wave, and that the field lines are decoupled from their neighbours, then the steady state solution is indicative of the amplitude of the pulsations. According to texts such as Yavorsky and Detlaf...
2.3. PROPAGATION MECHANISMS

(1975):

\[
A(\omega) = \frac{\omega_R^2 b_z c}{\sqrt{(\omega_R^2 - \omega_D^2)^2 + 4\gamma^2 \omega_D^2}}.
\]  

(2.34)

This reaches a maximum near \(\omega_R = \omega_D\), i.e. at resonance. Similarly, an expression can be derived for phase:

\[
\phi(\omega) = \tan^{-1}\left[\frac{-2\gamma \omega_D}{\omega_R^2 - \omega_D^2}\right],
\]  

(2.35)

which undergoes a phase reversal when \(\omega_R = \omega_D\). In experimental measurements, the phase is compared with a reference point or origin. For this reason, all phase measurements in this study are made by measuring the phase difference between a pair of stations, one of which was always the same (Kilpisjärvi for IMAGE, Repulse Bay for MACCS). Figure 2.6 gives the plots of amplitude and phase across a resonant frequency for a pair of stations. For stations which are separated in latitude, the resonant frequency will be different for each. These are denoted \(\omega_P\) for the poleward station and \(\omega_E\) for the equatorward station.

Another indicator of a field line resonance appears in its polarization properties. Polarization (§3.5) is a property of wave-plane orientation and its properties can be divided into four, called the Stokes parameters (§3.5.2). Signals near a resonance are expected to display predominantly toroidal mode characteristics, i.e. mainly linear polarization in the longitudinal direction (Ziesolleck \textit{et al.}, 1993). As with the amplitude and phase characteristics a further resonance signature is a polarization reversal at the resonant latitude (Samson \textit{et al.}, 1971; Kivelson and Southwood, 1986), and at latitudes corresponding to each node and antinode of the global mode for higher harmonics (Kivelson and Southwood, 1986). Figure 2.7 shows the amplitude and polarization characteristics from a model proposed by Lanzerotti \textit{et al.} (1974). Note the polarization reversal at point (1) in panel (b), where the resonance frequency is away from the plasmapause, and the double reversal on the inner and outer regions of the plasmapause in panel (c).
Figure 2.6: Schematic plots for two damped resonant systems with slightly different eigenfrequencies. Top Panel: The amplitude response $A(\omega)$ in each case. Middle Panel: The meridional amplitude difference $A(\omega_P) - A(\omega_E)$ between the poleward and equatorial stations. Bottom Panel: The meridional cross-phase $\phi(\omega_P) - \phi(\omega_E)$. The resonant frequency is identified by the vertical dashed line (Waters et al., 1991b).
Figure 2.7: Model of the radial plasma density for two wave frequencies (a), where $\eta_r(y) = \omega^2 \mu_0 \rho_0$. Amplitude and polarization characteristics for the two wave types in the Pc 3 (b) and Pc 4 (c) frequency range (Lanzerotti et al., 1974)).
Several workers have identified field line resonances at all latitudes. Mathematical models for this type of wave were established early, from magnetohydrodynamic (MHD) investigations conducted by Alfven (1942), Dungey (1954) and Tamao (1966). First identified at high latitudes by Sugiura (1961) they were quickly linked with the Kelvin-Helmholtz mechanism, based on the evidence of a polarization reversal around local noon (Nagata et al., 1963; Samson et al., 1971). More recent work by Miura (1987) used a KHI mathematical model in the low-latitude boundary layer and produced wavelengths and frequencies in good agreement with those observed at the magnetopause. Current evidence of standing field line properties has been summarised by Kato et al. (1994), based on the works of previous researchers:

1. The latitude corresponding to the peak in amplitude increases with pulsation frequency, implying that the standing wave is a fundamental mode (Samson and Rostoker, 1972).

2. A Pc 5 event has the same period at separated stations at the same geomagnetic latitude (Ellis, 1960; Obertz and Raspopov, 1968).

3. Conjugate phase relations of H- and D-components resemble that of an odd mode standing wave (Kokubun et al., 1976).

4. Phase relationship between the Pc 5 waves and low-energy particle flux fluctuations suggest a fundamental mode standing oscillation (Kokubun et al., 1977; Cummings et al., 1978; Tonegawa and Fukunishi, 1984).

The detection of polarization reversals on ground latitude profiles have long been used to identify FLRs at both high (Samson et al., 1971; Lanzerotti et al., 1974; Ables et al., 1998) and low (Kuwashima et al., 1979) latitudes. More recently ground studies have found amplitude and phase signatures of FLR’s on the ground for Pc 5 at high latitudes (Ziesolleck and McDiarmid, 1995; Mathie et al., 1999a; Waters, 2000) and Pc 3 at low latitudes (Waters et al., 1991a; 1991b; Ziesolleck et al., 1993; Menk et al., 2000). In summary, a ground signature for a field line resonance is a peak in amplitude, an $\sim \pi$ cross-phase change and polarization reversal at the same latitude.

One application of Pc 5 FLR’s is that of magnetospheric topology. Because they are located at near-cusp latitudes and can only exist on closed field lines, then Pc 5 wave activity would be expected to cease for stations in the polar cap. This has indeed been observed by Ables et al. (1998) using coss-phase and Mathie et al. (1999a) using amplitude analysis. Furthermore, the dirunal variation of near-cusp
2.3. PROPAGATION MECHANISMS

FLR cross-phase has been monitored by McHarg et al. (1994), revealing a feature known as the ‘arch’, which was identified as a possible signature of the cusp (figure 2.8).

![Image of FLR 'arch' as observed in a dynamic cross-phase spectrum from Gillam and Back of the CANOPUS array on February 4, 1995 (Waters, 2000).](image)

For low-latitude studies, ground arrays are favoured over satellites because of the relatively fast spacecraft passage time, which causes spectral broadening and phase shear of signals (Anderson et al., 1989). Also the large distance between the solar wind and these regions makes production via the KHI at the magnetopause unlikely, prompting a different driving mechanism to be considered. The mid 1980’s saw research efforts turn to the role of fast mode waves in FLR excitation (Kivelson et al., 1984; Kivelson and Southwood, 1985; Allan et al., 1985), a proposal made a decade earlier by Troitskaya et al. (1971) and then by Chen and Hasegawa (1974a; 1974b) and Southwood (1974). The fast mode waves were believed to be generated in the solar wind, following the implication of a relationship between the waves on the ground and solar wind parameters (Greenstadt et al., 1979; Odera, 1986). The current theory (Orr, 1984; Allan and Poulter, 1992; Le and Russell, 1996) is that fast mode wave energy passes isotropically through the magnetosphere to the low-latitude region within the plasmasphere (Yumoto et al., 1985), where it undergoes mode conversion and excites FLR wave activity. For the field line lengths and plasma densities at $1.3 \leq L \leq 2.8$ the resonant frequency lies in the Pc 3 frequency range, which corresponds to the frequency of the fast mode waves produced by the ion-cyclotron resonance mechanism. The result is a high level of Pc 3 activity, seen almost daily and lasting the entire day (Waters et al., 1994; Menk et al., 2000). Menk et al. (2000) also showed that the resonant frequency increased with
decreasing latitude until $L \sim 1.6$, where it decreased again. This was believed to be due to mass loading due to ionospheric heavy ions, as the majority of the field line at these latitudes lies within the ionosphere (Sutcliffe et al., 1987; Poulter et al., 1988).

![Figure 2.9: H-component cross-phase between Gloucester (GLO) and Newcastle (NEW) in Australia ($L = 1.8$) (Waters et al., 1991).](image)

At mid-latitudes, the resonance frequency moves into the Pc 4 frequency range. Baker (1977) demonstrated the difficulty in determining the Pc 4 resonance region based on its proximity to the plasmapause/plasmatrough region. A recent paper by Howard and Menk (2001) noted that for the lower frequency Pc 4 events a peak in amplitude was noted at the low-latitude end of the IMAGE magnetometer array ($L \sim 3.4$). While outside the detection region of this array an increase in amplitude could be indicative of an FLR.

Some good reviews on FLR’s include Orr (1973) and Waters (2000). While they are now the generally accepted model for Pc 5 wave propagation at high-latitudes and Pc 3 (4) at low- (mid-) latitudes there remain a few workers who challenge the concept (e.g. Bellan, 1994; 1996).

### 2.3.2 FLR Harmonics

A natural corollary of the fundamental mode field line resonance is the implication of the existence of harmonics. This was first suggested with regard to Pc 3–4 pulsations by Fukunishi and Lanzerotti (1974a) and the results of Arthur et al. (1977), and Singer et al. (1982) suggested that harmonic oscillations were more common than previously thought. Evidence of their existence was produced by Takahashi and McPherron (1982) using magnetometer data from the ATS-6 spacecraft. They concluded that at least 10–30% of high latitude Pc 3 pulsations can be classified
as harmonic events. These findings were supported by observations on the ground (Tonegawa and Fukunishi, 1984), and with ground-satellite comparison (Tonegawa et al., 1984). Both papers showed the curious result that higher harmonics can be excited in the absence of fundamental mode activity. This was explained in terms of the frequency of the driving source for the standing waves, which was believed to be different for Pc 5 and Pc 3–4 waves.

Figure 2.10: *Dynamic power of the H-component at Syowa on August 18, 1977* (Tonegawa and Fukunishi, 1984). The lines are representative of possible harmonic structure as defined by conjugate phase properties.

Recently, Ziesolleck et al. (1997) conducted phase and polarization studies on 1 month of ground and satellite data, and concluded that some events, especially in the Pc 4 band, displayed clear characteristics of FLR’s. Of the ~30 azimuthally transverse Pc 3–4 events observed simultaneously by spacecraft and the ground they concluded that around 20% of them were consistent with FLR harmonics. It was
implied that this value could in fact be higher, with the suggestion that ionospheric screening effects and/or wave propagation effects play a role in spacecraft/ground comparison. Their percentage, however, was consistent with the findings of Takahashi and McPherron (1982) and with Howard and Menk (2001), who concluded that 2 of their 11 (22%) Pc 3–4 events observed on the ground displayed characteristics similar to those of FLR’s.

Figure 2.11: *Ground Pc 3–4 polarization and phase characteristics associated with azimuthally polarized transverse waves in March 1990 (Ziesolleck et al., 1997). Solar circles represent events with clear FLR characteristics in the ground data.*

### 2.3.3 Azimuthal Wavenumber

One important feature of these waves is the azimuthal wavenumber, $m$, which accounts for relative changes in phase with the longitude components. This is defined by $m = d\phi/d\lambda$, where $\phi$ is the ULF phase difference between longitudinally separated stations and $\lambda$ is the angular separation of stations in geomagnetic coordinates. A wavenumber $m = 0$ means the signals are in phase around the entire parallel of longitude, while a large $m$ indicates the adjacent field lines are largely decoupled and perform independent azimuthal oscillations. If we consider the shear mode, the phase difference leads to a compressional perturbation in the magnetic field, and the Alfvén and fast modes can no longer occur independently.

### 2.3.4 Fast Modes: Travelling and Cavity/Waveguide Resonances

#### Direct Fast Mode Propagation

The shear Alfvén consideration of ULF waves requires a mode-conversion at an appropriate boundary, such as the magnetopause. Another possibility for the passage
of Pc 3–4 wave energy into the magnetosphere is that of direct propagation from the solar wind. This can also assume either a standing or propagating form. One suggestion (Yumoto and Saito, 1985) is that the waves can penetrate the Earth’s bow shock, through the magnetosheath and into the magnetosphere without significant alteration to their spectra, and can be observed as fast mode waves.

According to Takahashi et al. (1994), if we regard the magnetopause as a tangential discontinuity then the energy of the upstream waves can be transmitted into the magnetosphere in wave modes which allow perturbation only in the total (plasma and magnetic field) pressure, i.e. as fast mode waves. Given the upstream ion-cyclotron generation mechanism for ULF compressional waves, one would expect to find such waves commonplace in the magnetosphere. Indeed, as discussed previously, it is now believed that these waves propagate far enough into the magnetosphere to drive Pc 3–4 FLR’s at low-latitudes. The passage of Pc 3–4 waves through the magnetosphere has been monitored by several workers with satellite data (e.g. Yumoto and Saito, 1983; Yumoto et al., 1985; Engebretson et al., 1987; Odera et al., 1991; Lin et al., 1991a) but the transmission of their signals to high-latitude ground magnetometers remains a mystery. To date, this continues to be the main conflict with the direct fast mode wave propagation model, which is otherwise very popular.

Fast mode wave propagation in the magnetosphere is governed by the plasma densities of each magnetospheric region. Changing density changes the Alfvén speed as well as the refractive index, encouraging wave refraction and even diffraction at plasma boundaries (Moore et al., 1987; Zhang et al, 1993; 1995). The model proposed by Moore et al. (1987) used empirical models of the geomagnetic field, plasma density and temperature to produce a distribution of Alfvén speeds within the magnetosphere (figure 2.12). Their model showed a deep minimum of the wave propagation speeds between 4–6 Rₑ, and using an analogy of acoustic waves in a gas with shallow water waves implied what was termed “optical effects”, such as refraction and diffraction. Furthermore they suggested that the magnetospheric “shoal” (low \( V_A \) region) would cause fast mode waves to “break” and form a shock front, much in the same way as water waves break on a beach. This breaking wave effect would allow wave dispersion around the plasmasphere, and possibly permit propagation to the ground.

A further model suggested by Zhang et al. in 1993 and followed up in 1995 used three-dimensional ray tracing of Pc 3 compressional waves from the magnetosheath. This implied a frequency-dependent barrier in the refractive index contours at the ion-ion cutoff for the \( \text{He}^+ \) and \( \text{O}^+ \) gyroresonances. As shown in figure 2.13a, the
O\textsuperscript{+} resonance contour encircles the Earth for a fast mode wave of 100 mHz, but becomes open at 60 mHz (figure 2.13b). This was defined as the critical frequency, i.e. the frequency for which the O\textsuperscript{+} resonance location just becomes tangent to the magnetopause at the equator. The result was a natural low-pass Pc 3 filter at this frequency as waves with periods above this contour will be blocked by the O\textsuperscript{+} resonance contour. This implies a method by which to test if Pc 3 at high latitudes could be fast mode wave. Any signal detected above 60 mHz could not be a fast mode wave as this model prevents any such signal from penetrating the magnetosphere.

Determination of the process by which the transmission of the signals is made to the ground is one of the objectives of this project. Fast mode propagation is one of our considerations and mathematical models describing possible magnetosphere-ground transmission will be discussed further in chapter 6.

**Waveguide/Cavity Modes**

Another possibility is that of cavity or waveguide modes. When considering an analysis of FLR’s it is often best to describe the magnetospheric system as a cavity...
2.3. PROPAGATION MECHANISMS

Figure 2.13: Log scale contours of the refractive index for fast mode waves of 100 mHz (panel a) and 60 mHz (panel b) in the noon-midnight plane (Zang et al., 1993).

bound on one side by the magnetopause and at the other by some low altitude turning point as shown in figure 2.14. In this figure a wave excited by a disturbance on the magnetopause is reflected at the turning point and the outer boundary. Some energy leaks past the turning point by evanescent barrier penetration to the radius where the wave frequency matches the resonant frequency for that field line.

First, we consider a simple model in a rectangular coordinate system as discussed by Samson et al. (1991; 1992a; 1992b). This system defines an origin located deep within the magnetosphere, where the $x$-axis is measured outward from its origin, $y$ is measured in the azimuthal eastward direction and $z$ is along the field lines themselves. In this sense, the azimuthal wavenumber is $k_y$ and the wavenumber along the field lines is $k_z$. The Alfvén velocity is a function of $x$ and increases as we move toward the origin. The turning point mentioned above occurs when the following condition is satisfied:

$$\frac{\omega^2}{V_A^2} - k_y^2 - k_z^2 = 0,$$

where $\omega$ is the frequency of the toroidal mode. The resonance occurs when

$$\frac{\omega^2}{V_A^2} - k_y^2 = 0$$

is satisfied. Under certain approximations the modes will occur between the magnetopause and the turning point, and there is evanescent barrier penetration of these modes to the resonance. Coupling is the driving force for resonance.
Figure 2.14: Schematic of waveguide mode magnetopause boundary conditions for (a) moderate solar wind speed and (b) high solar wind speed (Mann and Wright, 1999).

The general solution for the displacement in the $x$ direction, $\xi_x$ is given by

$$\xi_x(\omega, x) = A \exp \left( i \int x^x k_x dx \right) + B \exp \left( -i \int x^x k_x dx \right), \quad (2.38)$$

where $A$ and $B$ are arbitrary constants, and

$$k_x = k_x(\omega, x) = \sqrt{\frac{\omega^2}{V_A^2} - k_y^2 - k_z^2}. \quad (2.39)$$

The positive and negative exponents represent propagation in the positive and negative $x$ direction respectively. The boundary condition at the magnetopause is assumed to be a drop in Alfvén velocity large enough to allow a $\pi$ phase change. At the other boundary there is a phase change of $\pi/2$ since near the turning point the wave takes the form of an Airy function. This presents us with a complication as we must have a total phase of $2n\pi$ in order to produce a self-sustaining cavity mode. To accommodate for this we must include the following condition:

$$\int_{x_T}^{x_M} k_x(\omega, x) dx = \int_{x_T}^{x_M} \sqrt{\frac{\omega^2}{V_A^2} - k_y^2 - k_z^2} dx = (n - \frac{1}{4}) \pi, \quad (2.40)$$
2.4. GROUND STUDIES OF PC 3–4 AT HIGH LATITUDES

where $x_M$ and $x_T$ correspond to the $x$ position of the magnetopause and turning point respectively. The solutions of this equation for $\omega$ are the normal modes for the cavity.

The resonance structure shown in figure 2.14 gives rise to a waveguide mode with its frequency determined by $k_y$. For $m$ or $k_y$ to be provided as in the analysis above we must impose an appropriate boundary condition for $y$ or $\phi$. Assuming cylindrical symmetry we can allow $m$ to be integer. If $k_y$ is not fixed then equation (2.40) gives the dispersion relation for the waveguide in terms of $k_y$. If we assume the turning point to occur at the zero of the integrand in the equation then the resultant dispersion relation provides a frequency which is insensitive to changes in $k_y$ (Walker et al., 1992).

Observational evidence for cavity modes has been identified in works with Pi 2 pulsations in the plasmasphere (Lin et al., 1991; Sutcliffe and Yumoto, 1991; Itonaga et al., 1992; Allan et al., 1996) and with Pc 3 at low latitudes (Samson et al., 1995; Waters et al., 2000; Menk et al., 2000). Evidence for Pc 3–4 at high latitudes remains largely outstanding, however recent works (Man and Wright, 1999; Waters et al., 2002) have shed some light on this question. This will be discussed further in the discussion of modelling in chapter 6.

2.4 Ground Studies of Pc 3–4 at High Latitudes

This thesis includes an investigation of four properties of Pc 3–4 waves detected on the ground. These are amplitude, coherence and cross-phase, as well as polarization features. This section will present a review of previous studies involving these properties.

2.4.1 Amplitude

The amplitude (or power) distribution of Pc 3–4 waves has been investigated by several workers. Data from the Western Pacific Magnetometer Array and other mid-low-latitude arrays have produced several papers which present an amplitude profile with latitude across almost all $L$ values (e.g. Matsuoka et al., 1997 and references therein). These report a peak at $L \sim 2$ and an increasing amplitude with latitude. These magnetometer arrays did not achieve latitudes with $L > 6$ and so it was not possible to detect any high-latitude peaks in amplitude. Using four high-latitude stations ($58^\circ – 85^\circ$ latitude), Bol’shakova and Troitskaya (1984) produced a plot of amplitude against latitude and found the location of the peak was dependent on $K_p$ value (figure 2.15a). For a $K_p$ value of 2, they placed the
location of the peak at around 77° geographic latitude. Other papers (Takahashi, 1985; Plyasova-Bakounina et al., 1986; Morris and Cole, 1987; Engebretson et al., 1990; Matsuoka et al., 2002) also found an amplitude peak at high latitudes and suggested that it was related to the cusp/cleft region. Howard and Menk (2001) also revealed this amplitude peak at around 75° CGM ($\sim$ 78° GEO), although values for the H-component for the higher latitudes were not shown in their figure. Ziesolleck et al. (1997) produced plots of power with $L$ value (figure 2.16 and gave the peak location at $L = 7$–8 (Geographic latitude $\sim$ 72°). A further three plots were also given (not shown, their fig. 6) all for low-frequency Pc 4 events (frequencies 13.8, 16.3 and 11.9) and were attributed to FLR’s by comparison with cross-phase and polarization profiles. While station density was sparse, there was evidence in these plots of a second peak at around $L = 10$ (GEO $\lambda = \sim 75^\circ$).

Figure 2.15: Panel a): Amplitude profile with geographic latitude for two $K_p$ values; Panel b): The location of the amplitude peak with $K_p$ value (Bol’shakova and Troitskaya, 1984).
Figure 2.16: Pure state power and ellipticity with L value, and relative phase with CGM longitude as observed by the CANOPUS array (Ziesolleck et al., 1997). The event shown corresponds to an azimuthally polarized transverse Pc 4 event which occurred between 1720 and 1750 UT on March 8, 1990 in the 13.8±4.0 mHz frequency band.
2.4.2 Coherence

Coherence (and coherency) have long been accepted as a useful tool for the comparison of wave structure between two signals. Their application to Pc 3–4 at high latitudes dates back to Takahashi and McPherron (1984), who used this parameter to compare satellite data. Coherence on high-latitude Pc 3–4 pulsations has been conducted by few workers, (e.g. Takahashi et al., 1994; Matsuoka et al., 1997; Szuberla et al., 1998) and virtually none appear to provide the adequate statistical evaluation required for this parameter. One of the better analyses is from Olson and Fraser (1994), who computed coherence estimates using the Welch method (Jenkins and Watts, 1968). Using this technique they estimated the coherence of “noise” to have a variance of $0.14 \pm 0.37$, indicating a coherence lower limit of $\sim 0.5$. This means that according to this estimate, a signal with a coherence value $> 0.5$ could be regarded as being distinguished from the noise. While apparently more detailed than most coherence analyses on ULF waves this study was still incomplete, as no confidence limits or degrees of freedom were assigned to the noise distribution (refer to section 3.6). More recently attempts have been made to clarify the generally loosely adopted definition of coherence. Olson and Szuberla (1997) indicated a confidence limit of 90% with an upper coherence value of 0.65 and 10 degrees of freedom and used them to produce probability density functions. Unfortunately they then took an average across every frequency in the spectrum, thus giving coherence only for broadband noise. Howard and Menk (2001) produced coherence limits representing the 90% confidence intervals for upper and lower limits of 0.65 and 0.0, with 13 degrees of freedom. This meant that any value of coherence above 0.65 could be distinguished from the noise (of coherence 0.0) with 90% confidence. This plot given in figure 2.18.

2.4.3 Cross-Phase

To date, we are aware of only two papers which have used ground cross-phase measurements for the study of high latitude Pc 3–4 waves, and two of these are by the author and colleagues. The first (Ziesolleck et al., 1997) used phase with longitude profiles from the CANOPUS array to ascertain azimuthal wavenumbers and phase propagation directions for their Pc 3–4 events, and phase with latitude profiles used to imply resonance features in these waves (figure 2.17). Howard and Menk (2001) used the IMAGE array and produced cross-phase profiles with latitude and longitude for 11 events from March 1996. These were used to determine ground and magnetospheric speed and direction, and azimuthal wavenumber.
Figure 2.17: Latitudinal phase variation during three azimuthally polarized transverse Pc 4 events at geosynchronous orbit from March 8, 1990 (1720–1750 UT), March 8, 1990 (1930–2000 UT) and March 23, 1990 (1545–1615 UT). The frequencies ranged from 12.78–17.78 mHz (Ziesolleck et al., 1997).
Figure 2.18: Amplitude, coherence and cross-power with CGM latitude and longitude for H- and D-component for a Pc 3 (26.6 mHz) event on March 23, 1996 at 0730–0800 UT (Howard and Menk, 2001).
2.4.4 Conjugate Point Studies

Determination of possible FLR properties of waves at high latitudes is difficult using amplitude and cross-phase alone, as only too often the cusp is reached before a convincing profile can be achieved. Poleward of the cusp lies the polar cap containing open field lines which are unable to support standing waves. The first mathematical analysis of conjugate points was made by Sugiura and Wilson in 1964. They began with the assumption that ULF waves in a geomagnetic field line behave similarly to those in a stretched elastic string (Alfvén, 1950). They then assumed the model remains valid when the string is extended so it assumes the shape of a closed geomagnetic field line. Thus the pulsations which cause the fluctuations in the geomagnetic field lines are expected to be observed simultaneously at magnetically conjugate points of the Earth. Odd modes would occur if a parallel perturbation of H is detected at the conjugate points, while antiparallel oscillations indicate the presence of an even mode. Figure 2.19 demonstrates this analogy. An observer at one of these conjugate points would detect the end points of oscillating field lines as it is driven by the magnetohydrodynamic (MHD) wave. The polarization of the waves can be determined by analysing the trace of the end of this field line. According to this model, for an odd mode standing oscillation, observers at conjugate points would detect H-components which are in phase and D-components out of phase.

At high latitudes, the majority of conjugate point work has been with the Pc 5 FLR’s (e.g. REFERENCE; Kato et al., 1994), many used to confirm the FLR nature of this type of wave. Conjugate point work on Pc 3–4 was made early at around L = 4 by Lanzerotti et al. (1972) and Fukunishi and Lanzerotti (1974b), who suggested that Pc 3–4 waves here are predominantly odd mode standing field line oscillations. Further work by Barker (1977), also at L = 4 implied that Pc 3 wave frequency decreased throughout the day, and was dependent on $K_p$ value. Papers on Pc 3–4 at conjugate points include Wolfe et al. (1990) and Takahashi et al. (1994). Olson and Fraser (1994) conducted a study using Longyearbyen in Svalbard and Davis station, Antarctica and found good correlation between narrowband packets, which they interpreted as precipitating kilovolt electrons fluctuating in the upper ionosphere. Recent studies by Howard et al. (2001), using the same conjugate pair found no coherent interhemispheric Pc 3–4 activity.
Figure 2.19: The symmetry relations at magnetically conjugate points for the oscillation of the lines of magnetic force (Sugiura and Wilson, 1964). $H$ and $D$ represent the horizontal and easterly components respectively and the arrow indicates the direction of the magnetic perturbation itself.
2.4.5 Nighttime Events

Since their discovery Pc 3–4 have always been associated with the dayside magnetosphere (e.g. Orr 1973 and references therein). This report considers the rare occurrence of Pc 3–4 on the nightside, i.e. ULF waves in the 10–100 mHz range which meet the selection criteria for a legitimate event (section 4.2). These have not been investigated in the past as they have been dismissed as substorm-related Pi 2 signals. To check this belief, this section will outline some of the physical properties of nighttime Pi 2 waves, for comparison with the events we have identified in chapter 5.

High-latitude nighttime ULF wave activity is attributed to impulsive Pi waves and are related to substorm activity. Pi 2 waves are in the 7–25 mHz frequency range and are created by magnetic wave energy released during substorms (Saito, 1961). All Pi 2 are created during a disturbance in the plasma sheet which produces a field-aligned current. This disturbance launches a transverse polarized wave along the magnetic field to the polar ionosphere, where it causes a brightening of the auroral arc (Olson, 1999). The wave is then partially reflected, propagates back along the geomagnetic field lines, and interacts with the forming substorm current wedge. A westward-travelling current surge is created, which is registered at high latitudes as an impulsive Pi 2 event. A fast mode wave created at the same time in the plasma sheet propagates inward, creating surface waves along the plasmapause and field line resonances within the plasmasphere. If a substorm does not develop then the bright arc dims and Pi 2 activity ceases. This event is termed a pseudobreakup (Olson, 1999).

On the ground Pi 2 activity is known to maximise in the region mapping to the brightening auroral oval (Jacobs and Sinno, 1960; Olson and Rostoker, 1975), or more precisely, near the equatorward border of the electrojet (Rostoker and Samson, 1981; Samson, 1982). Latitude profiles have been produced by several workers (e.g. Lester, et al., 1983; Bradshaw and Lester, 1997) and reveal a peak in amplitude at around 66–67° CGM latitude. Longitudinal phase profiles show a westward motion west of the current wedge but eastward motion east if the wedge (Lester et al., 1984; Gelpi et al., 1985; Southwood and Hughes, 1985). Figure 2.20 show the results of Bradshaw and Lester (1997) for a single event on day 101, 1983. They note that the amplitude and phase becomes complicated about the amplitude peak and the longitudinal phase profile indicates westward propagation and an azimuthal wavenumber of ~17. Polarization profiles with latitude reveal a reversal in polarization across the electrojet (Pashin et al., 1982; Samson and Rostoker,
and the plasmapause (Fukunishi and Lanzerotti, 1974a; 1974b; Lanzerotti et al., 1974).

Figure 2.20: Amplitude and phase characteristics of Pc 3 with longitude (at latitude 66.6°E) and latitude (at longitude 5.5°E) observed by the SABRE radar system on day 101, 1983 (Bradshaw and Lester, 1997).

2.5 Location of Magnetospheric Boundaries: Magnetopause and Plasmapause

Previous workers have identified the magnetopause (Bol’shakova and Troitskaya, 1984) and the plasmapause (Lanzerotti et al., 1974; Barker, 1976) as two boundaries which influence Pc 3–4 propagation characteristics. To investigate these claims, the location of the two boundaries was estimated using three mathematical models for the magnetosphere and plasmasphere. This section will present a review of these mathematical models.
2.5. LOCATION OF MAGNETOSPHERIC BOUNDARIES: MAGNETOPAUSE AND PLASMA

Figure 2.21: Values of the $H$ and $D$ (a) amplitude and (b) phase and polarization ellipticity for a Pi 2 event observed by ISEE 1 on day 333, 1977 (Lester et al., 1983).

2.5.1 The Magnetopause

The first model for the magnetosphere calculations was using the geomagnetic field model of Tsyganenko (Tsyganenko and Stern, 1996). This has been discussed in chapter 1 and will be further discussed in section 4.7. The remaining three are empirical models, based on ground and satellite observations of the balance between the solar wind and magnetospheric pressures.

Farrugia et al. (1989) used a high-latitude ground magnetometer array and monitored the magnetopause motions by the magnetic footprint signatures of the cusp. These were correlated with solar wind data from the IMP 8 satellite and magnetopause signatures from the ISEE 1 and 2 spacecraft. They give the stand-off distance of the subsolar magnetopause by its relationship with the solar wind ram pressure, as according to Schield (1969). This relationship is given as

$$C = G/[(NV^2)_{SW}]^{1/6},$$

(2.41)
where $C$ is the magnetopause position in $R_E$ and $N_{SW}$ and $V_{SW}$ are the number density in parts per cubic centimetre ($\#/cc$) and speed of the local solar wind in kilometres per second (km/s). $G$ is a scaling factor which allows for the non-dipolar nature of the field. This parameter was determined for a particular time (1500 LT) using satellite data from ISEE 1 and 2, and IMP 8. This value was $129.4 \pm 2.6$.

The workers then produced a time history for the magnetopause at that local time. Their equation for the $L$ value of the magnetopause was therefore

$$L_{MP} = (129.4 \pm 2.6) \left( N_{SW} \times V_{SW}^2 \right)^{-1/6}, \quad (2.42)$$

Rodger (1998) described the location of the magnetopause as the region where the dynamic pressure of the shocked solar wind equals the magnetic pressure of the geomagnetic field. The following equation is given (their p117) for the magnetopause position:

$$L_{MP} = 107.4 \left( N_{SW} \times V_{SW}^2 \right)^{-1/6}. \quad (2.43)$$

where $L_{MP}$ is the magnetopause $L$ value, and and $N_{SW}$ and $V_{SW}$ defined as above. It was noted that the magnetopause is in fact extremely dynamic and can move at speeds comparable with that of the plasma in the magnetosheath.

### 2.6 The Plasmapause

This section will consider three models of the plasmasphere which estimate the location of the plasmapause. As with the magnetopause all three were empirical models, based on both satellite and ground data.

Rycroft and Burnell (1970) used data from the Alouette 1 satellite to conduct a statistical review of the plasmapause, based on 39 passes through this region. Using functions of the observed location of the trough with $K_p$ index and local time they produced a least-squares multidimensional plane to fit the data, based on previous works by Burnell and Rycroft (1969) and the theory of Weatherburn (1949). Their result was the following relationship:

$$\lambda_{pp} = (62.0 - 1.0K_p - 0.4t \pm 1.8)^\circ, \quad (2.44)$$

where $K_p$ is the value of the index for the time, $t$ is the number of hours from local midnight, and $\lambda_{pp}$ is the generalised invariant latitude at 1000 km altitude, given by

$$L \cos^2 \lambda_{pp} = 1 + \frac{1000}{6370} = 1.157. \quad (2.45)$$

To a first-order approximation the value of $\lambda_{pp}$ can be regarded as the CGM latitude for the plasmapause. Between local times 2100 ($t = -3$) and 0500 ($t = +5$) the
least-squares planes fit almost identically, but outside this time frame the uncertainty of the fit increased with increasing value of $|t|$. The location of the plasmapause was also estimated by Orr and Webb (1975). Following Chappell et al. (1971) they used the average $K_p$ value for the previous evening (2100–0600 LT) rather than that of the value at the time of the event. For those events which occurred before 0600 LT the average value from 2100 to the event was used. Applying this average $K_p$ to the results of Chappell et al. (1970, fig 6) they produced a plot of $L$ against $K_p$ and performed a second-order polynomial fit. The result was the following equation for the plasmapause $L$ value:

$$L_{pp} = 6.52 - 1.44K_p + 0.18K_p^2,$$  \hspace{1cm} (2.46)

which was only valid for local time 0200. The location of the plasmapause from other times of the day were given by Chappell et al. (1970) and are shown in figure 2.22. They then produced an estimate for the projection of the plasmapause to the ground using the following equation:

$$\left[ \frac{\text{The } L \text{ value of the plasmapause at } N \text{ hr LT}}{\text{The } L \text{ value of the plasmapause at 0200 hr LT}} \right]_{K_p=0} = \frac{L_0}{X},$$  \hspace{1cm} (2.47)

where $L_0$ is the $L$ value of a particular observatory on the ground and the left-hand side of the equation can be read from figure 2.22 (numerator) and equation (2.46) (denominator). The value of $X$ represents the 0200 LT $L$ value of the plasmapause for which, at $N$ hour LT, the plasmapause is overhead at the observatory at $L_0$. It is this value of $X$ which represents the location of the plasmapause on the ground. Orr and Webb (1975) confirmed this estimate using 12 years of data from 5 ground magnetometers from $3.1 \leq L \leq 6.6$.

An empirical model was proposed by Carpenter and Anderson (1992) using ISEE 1 satellite data and VLF whistler data. Given the definition of the plasmapause as “the last measured point prior to a steep plasmapause falloff” (Carpenter and Anderson, 1992, p1103) they identified over 120 crossings, mostly from 1977, 1982 and 1983, and produced a plot of plasmapause $L$ value against the maximum $K_p$ value for the previous 24 hours (figure 2.23). Applying a least squares first order polynomial fit to the data revealed the following relationship:

$$L_{pp} = 5.6 - 0.46K_{p\text{max}}.$$  \hspace{1cm} (2.48)

It was noted that this model was applicable mainly for 00-15 MLT, although a way to extend to other times was discussed in the paper. Section 4.7 discusses the application of these models to the events used in this study.
2.7 Effects of the Ground and Ionosphere

ULF wave detection on the ground is complicated further with the introduction of environmental factors. Among them, the ionosphere rotates the sense of polarization of the waves or even screen certain types of waves from the ground. Nearby ground effects can also distort ULF wave features, such as Earth induction effects from nearby coastlines (Chisham, 1991; Viljanen et al., 1995) or from local man-made interference (Viljanen et al., 1995). Here we will discuss these phenomena and their effects on ULF wave features observed on the ground.

2.7.1 Earth Induction Effects

It has been shown by several workers that anomalies in the Earth (Schmucker, 1970; Hughes, 1974; Chisham, 1991) and the proximity of a deep, saltwater ocean (Küppers, 1979; Jones, 1981) alter the conductivity profile of the Earth and influence signals detected on the ground. The following theory is based on Chisham (1991).

To a first-order approximation, the Earth can be regarded as horizontally flat homogeneous conducting surface. In practice the 1-dimensional assumption is unsufficient and conduction anomalies in the Earth’s crust can occur. These can be detected using the ‘transfer function’ technique (Schmucker, 1970) which produces the function in the form of induction vectors, which are frequency dependent. The technique assumes a linear relationship between the Fourier transforms (§3.3.1) of anomalous and normal internal field components. Let $F_0(f)$ be the observed fre-
2.7. EFFECTS OF THE GROUND AND IONOSPHERE

Figure 2.23: Plot of the L value of the plasmapause as determined from ISEE data with the maximum $K_p$ for the previous 24 hours. The linear fit is also included (Carpenter and Anderson, 1992).

quency spectrum of a variation in the total magnetic field, such that

$$F_0(f) = F_N(f) + F_A(f),$$

(2.49)

where $F_N(f)$ is the field expected without conductivity anomalies and $F_A(f)$ is the field induced by the local anomaly. The latter anomalous field can be related to the former normal field via the following, termed the transfer function:

$$h_A(f) = \begin{bmatrix} h_N(f) \\ d_N(f) \\ z_N(f) \end{bmatrix} + \begin{bmatrix} \delta_h(f) \\ \delta_d(f) \\ \delta_z(f) \end{bmatrix},$$

(2.50)

where $h_N(f)$, $d_N(f)$ and $z_N(f)$ are the H-, D- and Z-components of the normal field, and $h_A(f)$, $d_A(f)$ and $z_A(f)$ are the components for the anomalous field. More generally, this can be written as

$$F_A = T \cdot F_N + \Delta,$$

(2.51)

where $T$ represents the transfer function. Of course, for this function to be useful it requires comparison with a reference site, from which to obtain a normal definition of the field. This should be situated above a horizontally stratified conductivity structure, and spatially distant from anomalous discontinuities on the Earth’s surface (Banks, 1973). In the case where the achievement of such ideal conditions is difficult certain assumptions about an independent site can be determined in a process known as the single station transfer function (Schmucker, 1970).

The usual presentation of the single station transfer functions is in the form an induction vector pair. These are related to the orientation and intensity of the
underground conductivity anomaly, and are written as real (in phase) and imaginary (quadrature) vectors of magnitude

\[ A_R = \sqrt{Re\{t_{zh}\}^2 + Re\{t_{zd}\}^2} \]  

(2.52)

and

\[ A_I = \sqrt{Im\{t_{zh}\}^2 + Im\{t_{zd}\}^2}, \]  

(2.53)

where \( A_R \) and \( A_I \) are the real and imaginary components respectively, and are both functions of frequency. The angles between these vectors and local geomagnetic north are

\[ \theta_R = \tan^{-1}\left[\frac{Re\{t_{zd}\}}{Re\{t_{zh}\}}\right] \]  

(2.54)

and

\[ \theta_I = \tan^{-1}\left[\frac{Im\{t_{zd}\}}{Im\{t_{zh}\}}\right]. \]  

(2.55)

It has been shown by Hughes (1974) that even in the simple two-dimensional case the presence of magnetic anomalies can affect the polarization of many types of ULF waves. These mostly affected those with a shorter period (\(< 100s\)) but implications were made to the affects of the longer periods waves.

The primary source of data used in this study was the IMAGE magnetometer array (section 3.1.1). This is located in northern Scandinavia and several stations are located on either islands or near the coast. Viljanen et al. (1995) conducted a survey on the induction effects at the IMAGE array. They concluded that the induction influences of waves detected on the ground was fourfold:

1. An ocean effect, due to the proximity of the deep, saltwater Arctic ocean.
2. One strong inland anomaly, affecting the data at MAS.
3. Man-made disturbances, present from the STARE radar near HAN.
4. At other stations the anomalous effects are weaker and can generally be ignored.

However weak, the results revealed an ocean effect at MAS, KEV, KIL and SOR, and other anomalies at MAS, NUR, OUJ, PEL and SOD, with the main influence between frequencies 0.4 – 50 mHz in the \( z \) direction. Viljanen (1996) added to the anomaly contribution by revealing the effects of the overhead auroral electrojet. It was found that the induction vectors rotated further at stations in the proximity of the electrojet (MAS and SOR) and this effect was related to the intensity of the electrojet (up to 10’s of degrees during disturbed times). The effects of small-scale
induction effects on the large scale \(dB/dt\) in the auroral region were discussed in Viljanen (1997). He concludes that induction effects have the potential to cause disturbances to \(B\) in any direction, not just parallel to the electrojet.

In our study, induction effects were identified as a source of error in the measurements, but were neglected, as other errors were perceived to dominate further (chapters 4 and 5). It should be noted that the polarization results shown in 4.5 and 5.4 may also be affected.

### 2.7.2 Ionospheric Effects

Before hydromagnetic ULF waves can reach ground-based stations they must first propagate through the ionosphere. As it is a conducting layer, the ionosphere supports significant current systems and these can greatly modify the waves. The ionosphere can be represented as a slab of conductive medium, ranging from 70 to 2000 km altitude, of varying refractive index (Orr, 1973). The conditions for field line guided hydromagnetic waves in the ionosphere vary with latitude and the ionosphere can therefore be divided into three zones: the polar, mid latitude and equatorial zones. In the polar zone the hydromagnetic waves propagate along magnetic field lines which are perpendicular to the ionosphere, while in the equatorial region the field lines are parallel to the ionosphere. In the midlatitude region the field lines are at some angle in between. Mathematically the simplest case is the polar region, with the other cases analysed by considering extensions to this model (Hughes, 1983).

The ionosphere also can be divided into three regions according to altitude. Above 350 km is a region known as the Alfvén region in which collisions between nearby particles is negligible and the behaviour of MHD waves is similar to those discussed previously. Between the base of the ionosphere (~70 km) and 130 km lies the Hall region in which collisions occur much more frequently such that collision frequency is much higher than the ion-cyclotron frequency. A transition region between the Alfvén and Hall regions lies in between, in the region from ~130 to 350 km. Below the base of the ionosphere lies the atmosphere which is assumed to be a form of insulator.

We now consider the propagation of MHD waves through the ionosphere. The primary work on this subject is generally regarded as 4 papers published in the mid 1970’s (Hughes, 1974; Hughes and Southwood, 1974; 1976a; 1976b). These works proposed three effects of waves incident on the ionosphere from the magnetosphere;
1. Waves with a non-zero vertical current are screened out from the ground (Hughes, 1974; Hughes and Southwood, 1974a);

2. Waves with a zero vertical current have their polarization rotated in by 90° in the left-hand direction (Nishida, 1964; Hughes, 1974);

3. Signals are generally spread out over a circular region centred on the entry point of wave (Hughes and Southwood, 1976b).

**Screening and Rotation**

The following is a summary of the analysis presented by Hughes (1974). Defining the electric and magnetic disturbances in geographic coordinates as $\mathbf{E} = (E_x, E_y, E_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$ we consider two conditions, one in which the incident wave has a vertical component ($E_z \neq 0$) and the other in which none exists ($E_z = 0$). In both cases we have two of Maxwell equations;

$$\nabla \times \mathbf{E} = -\frac{i\omega}{c} \mathbf{B}$$  \hspace{1cm} (2.56)

$$\nabla \times \mathbf{B} = -\frac{1}{c} (i\omega \mathbf{E} + 4\pi \mathbf{J}),$$  \hspace{1cm} (2.57)

where $\mathbf{J}$ is the current density, $c$ is the speed of light and $\omega$ is defined such that $\partial/\partial t \equiv i\omega$.

First, we consider the first condition ($E_z \neq 0$). Here, the component of $E_z$ is derived from the $z$ component of equation (2.57)

$$\frac{1}{c} E_z (i\omega + 4\pi \sigma) = ik B_{xG},$$  \hspace{1cm} (2.58)

where $G$ refers to values on the ground and $\sigma$ is the conductivity. Next we apply the assumption that the ground is a perfectly conducting medium, so $E_{zG} = E_{yG} = B_{yG} = 0$. This leaves us only $B_x$ to consider. In a similar fashion the $x$-component of equation (2.56) gives us

$$\frac{\partial E_y}{\partial z} = ik E_z + \frac{i\omega}{c} B_{xG} = \left( \frac{k^2 c}{i\omega + 4\pi \sigma} + \frac{i\omega}{c} \right) B_{xG}.$$  \hspace{1cm} (2.59)

Neglecting the second term on the rhs and integrating equation (2.59) with the assumption that $\sigma(z) = \sigma_G e^{\alpha z}$ where $1/\alpha$ is the scale height reveals

$$E_y = -i k^2 c B_{xG} \ln \left( \frac{i\omega + 4\pi \sigma_G}{4\pi \sigma_G} \right),$$  \hspace{1cm} (2.60)

where $\sigma_G$ is the atmospheric conductivity at ground level. Now, the currents caused by this electric field perturbation causes changes in the magnetic field above the
ionosphere, given by
\[ \frac{\partial B_x}{\partial z} = \frac{4\pi}{c} \sigma_P E_y, \]
\[ \frac{\partial B_y}{\partial z} = -\frac{4\pi}{c} \sigma_H \cosec I E_y, \]
where \( \sigma_P \) and \( \sigma_H \) are the Pederson and Hall conductivities respectively and \( I \) is the angle between the ambient magnetic field and the horizontal. Integration of these reveals
\[ \Delta B_x = \frac{4\pi}{c} E_y \Sigma_P \]
\[ \Delta B_y = \frac{4\pi}{c} E_y \cosec I \Sigma_H, \]
where \( \Sigma \) is a height integrated conductivity. Hughes (1974) then applied typical values of \( \Sigma = 2 \times 10^{13} \text{ e.s.u}, k = 10^{-3} \text{ km}^{-1}, \alpha = 0.2 \text{ km}^{-1} \) and \( \omega = 0.1 \text{ sec}^{-1} \) and found that \( \Delta B_x \) and \( \Delta B_y \) are \( \sim 10^5 \) times larger than \( B_{xG} \) in equation (2.60). Thus this component is effectively screened by the ionosphere.

We now consider the second condition (\( E_z = 0 \)). This begins with the \( y \)-component of equation (2.56)
\[ \frac{\partial E_x}{\partial z} = -\frac{i\omega}{c} B_y, \]
and assuming a constant \( B_y \) we arrive at
\[ E_x = -\frac{i\omega z}{c} B_{yG}. \]

The resultant magnetic field components associated with the ground magnetic field here becomes
\[ \Delta B_x = \frac{4\pi i\omega}{c^2} B_{yG} \cosec^2 I \Sigma_P \]
\[ \Delta B_y = \frac{4\pi i\omega}{c} B_{yG} \cosec I \Sigma_H, \]
thus limiting the magnetic field observed on the ground to the \( y \) direction. So the currents in the ionosphere are such that they cancel the \( B_x \) in the magnetosphere and produce a comparable sized \( B_y \) (Hughes, 1974). The result is a rotation of the magnetic field vector in the ionosphere by 90\( ^\circ \) and hence the horizontal polarization of any wave incident on that field line will by rotated by the same amount (Hughes and Southwood, 1976b).

**Spatial Integration**

One aspect which has become important in multistation wave analysis is the field of view of the instruments used. Hughes and Southwood (1974b) demonstrated that the signal observed on the ground is a weighted average of that in the magnetosphere taken over a circular region of radius \( z \) and centred directly above the observation point. This implied that horizontal signals of region \( < z \) would be
smoothed out when observed on the ground. Here the value of \( z \) is the location of the ionospheric head or Hall layer (altitude where the Hall conductivity achieves maximum conductivity), approximately 120 km.

Hence a point source incident on the ionosphere would be spread out in a circle of radius \( \sim 100 \text{ km} \) when observed by a magnetometer on the ground. This defines the so-called field of view of a magnetometer. Two sites within \( \sim 200 \text{ km} \) of each other would in fact be observing the same signal in the ionosphere. This value is most important in studies related to noise and coherence as it represents the maximum region in which one is guaranteed to observe a coherent signal on the ground. This will be discussed further in later chapters.
CHAPTER 3
DATA COLLECTION AND ANALYSIS

This high latitude study utilised resources from three magnetometer arrays and two satellites. Each group of magnetometers sampled data in different formats and with different digital sampling rates and so a common format needed to be established. Satellite data required time correlation with the ground and use of to appropriate mathematical models for magnetopause location and solar wind relationship with frequency on the ground. Analytical techniques such as coherence required a detailed statistical assessment to ensure result reliability. This chapter presents a review of the instruments and analytical techniques employed.

3.1 Data Sources

The main body of data for this project was provided by the International Monitor for Auroral Geomagnetic Effects (IMAGE) ground magnetometer array. The array currently comprises of 27 stations maintained by a joint effort of 10 institutions from Estonia, Finland, Germany, Norway, Poland, Russia and Sweden (Viljanen, 2001). Figure 3.1 gives a map of the array while details about the stations are outlined in table 3.1.

3.1.1 The IMAGE Magnetometer Array

The IMAGE array (Lühr et al., 1998; Viljanen, 2001) evolved from the EISCAT magnetometer cross, which was operational from October 1982 until October 1991. Originally, the EISCAT cross consisted of five sites at Sørøya, Muonio, Alta, Pello and Kautokeino, with two more sites at Kevo and Kilpisjärvi added in June 1983. All of these magnetometers had a sampling interval of 20 seconds and a resolution of 1 nT. In 1990, the fluxgates at Alta and Kautokeino were removed and in October 1991, a site at Masi was include. At this time managements of the EISCAT cross was reorganised, and the array became IMAGE. By the end of October 1993, IMAGE consisted of 15 stations and 4 more had been added by January 1996. By the end of 2000 the number had reached 25, with Abisko and Uppsala added in 1998 (the time for the data provided for this study), Lycksele and Rørvik in 1999, and Dombås and Leknes in January 2000. The most recent additions to IMAGE were at Ivalo and Tartu in February and September, 2001. All stations now have a sampling rate of 10 seconds, a resolution of 0.1 nT and are aligned in geographic
coordinates. The chief objective of IMAGE is to study auroral electrojets and moving two-dimensional current systems. It should be noted that the data used for this project were obtained in 1998 when only 21 of the IMAGE stations were operational. These are listed in table 3.1.

Figure 3.1: The IMAGE array and its 27 stations (Viljanen, 2001).
### 3.1. DATA SOURCES

Table 3.1: Geographical and CGM coordinates and $L$ values of the IMAGE stations used in this study (Viljanen, 2001), and the additional two Antarctic stations added for conjugate point analysis. CGM coordinates correspond to an epoch of 1998 and altitude of 100 km.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Station name</th>
<th>Geographic latitude</th>
<th>Geographic longitude</th>
<th>CGM latitude</th>
<th>CGM longitude</th>
<th>$L$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IMAGE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NAL</td>
<td>Ny Ålesund</td>
<td>78.92°</td>
<td>11.95°</td>
<td>76.08°</td>
<td>112.44°</td>
<td>&gt;15</td>
</tr>
<tr>
<td>LYR</td>
<td>Longyearbyen</td>
<td>78.20°</td>
<td>15.82°</td>
<td>75.13°</td>
<td>113.17°</td>
<td>&gt;15</td>
</tr>
<tr>
<td>HOR</td>
<td>Hornsund</td>
<td>77.00°</td>
<td>15.60°</td>
<td>74.04°</td>
<td>110.63°</td>
<td>13.4</td>
</tr>
<tr>
<td>HOP</td>
<td>Hopen Island</td>
<td>76.51°</td>
<td>25.01°</td>
<td>72.94°</td>
<td>116.03°</td>
<td>11.8</td>
</tr>
<tr>
<td>BJN</td>
<td>Bear Island</td>
<td>74.50°</td>
<td>19.20°</td>
<td>71.35°</td>
<td>108.82°</td>
<td>9.9</td>
</tr>
<tr>
<td>SOR</td>
<td>Sørøya</td>
<td>70.54°</td>
<td>22.22°</td>
<td>67.27°</td>
<td>106.74°</td>
<td>6.8</td>
</tr>
<tr>
<td>KEV</td>
<td>Kevo</td>
<td>69.76°</td>
<td>27.01°</td>
<td>66.23°</td>
<td>109.75°</td>
<td>6.2</td>
</tr>
<tr>
<td>TRO</td>
<td>Tromsø</td>
<td>69.66°</td>
<td>18.94°</td>
<td>66.57°</td>
<td>103.47°</td>
<td>6.4</td>
</tr>
<tr>
<td>MAS</td>
<td>Masi</td>
<td>69.46°</td>
<td>23.70°</td>
<td>66.10°</td>
<td>106.94°</td>
<td>6.2</td>
</tr>
<tr>
<td>AND</td>
<td>Andenes</td>
<td>69.30°</td>
<td>16.03°</td>
<td>66.40°</td>
<td>100.94°</td>
<td>6.3</td>
</tr>
<tr>
<td>KIL</td>
<td>Kilpisjärvi</td>
<td>69.02°</td>
<td>20.79°</td>
<td>65.81°</td>
<td>104.32°</td>
<td>6.0</td>
</tr>
<tr>
<td>ABK</td>
<td>Abisko</td>
<td>68.35°</td>
<td>18.82°</td>
<td>65.21°</td>
<td>102.27°</td>
<td>5.8</td>
</tr>
<tr>
<td>MUO</td>
<td>Muonio</td>
<td>68.02°</td>
<td>25.53°</td>
<td>64.65°</td>
<td>105.70°</td>
<td>5.5</td>
</tr>
<tr>
<td>KIR</td>
<td>Kiruna</td>
<td>67.84°</td>
<td>20.42°</td>
<td>64.63°</td>
<td>103.14°</td>
<td>5.5</td>
</tr>
<tr>
<td>SOD</td>
<td>Sodankylä</td>
<td>67.37°</td>
<td>26.63°</td>
<td>63.85°</td>
<td>107.71°</td>
<td>5.2</td>
</tr>
<tr>
<td>PEL</td>
<td>Pello</td>
<td>66.90°</td>
<td>24.08°</td>
<td>63.49°</td>
<td>105.30°</td>
<td>5.1</td>
</tr>
<tr>
<td>OUJ</td>
<td>Oulujärvi</td>
<td>64.52°</td>
<td>27.23°</td>
<td>60.92°</td>
<td>106.51°</td>
<td>4.3</td>
</tr>
<tr>
<td>HAN</td>
<td>Hankasalmi</td>
<td>62.30°</td>
<td>26.65°</td>
<td>58.66°</td>
<td>104.94°</td>
<td>3.7</td>
</tr>
<tr>
<td>NUR</td>
<td>Nurmijärvi</td>
<td>60.50°</td>
<td>24.65°</td>
<td>56.84°</td>
<td>102.48°</td>
<td>3.4</td>
</tr>
<tr>
<td>UPS</td>
<td>Uppsalà</td>
<td>59.90°</td>
<td>17.35°</td>
<td>56.45°</td>
<td>96.22°</td>
<td>3.3</td>
</tr>
<tr>
<td><strong>Antarctic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DAV</td>
<td>Davis</td>
<td>−68.60°</td>
<td>78.00°</td>
<td>−76.76°</td>
<td>125.15°</td>
<td>14.4</td>
</tr>
</tbody>
</table>
3.1.2 Antarctic Induction Systems

Also included in Table 3.1 are two induction magnetometer sites at Davis station in Antarctica. These were managed by the Australian Antarctic Division and the University of Newcastle, Australia. The induction systems were installed at Davis during the summer of 1990-91 and comprise of two sensor coils of 250K turns, each aligned to local magnetic north (H) and east (D). A secondary coil wound around each primary allows calibration using a test signal at 1630 UT each day (Howard, 2000). Data were sampled digitally at 10 Hz after a low pass filter at 13 Hz and removal of any the DC bias from the signal (Neudegg, 1997). The induction sensors have peak sensitivity at 1 Hz. Data were stored on disk in whole day files daily commencing at 0000 UT. Figure 3.2 shows the amplitude and frequency response for the Davis magnetometer.

![Figure 3.2: The a) amplitude and b) frequency response with amplitude of the Davis induction magnetometer (Howard, 2000).](image)

By using the Tsyganenko (T96) and the Corrected Geomagnetic (CGM) models (§1.3), it can be shown that the magnetic conjugate point of Davis lands just west of the Svalbard archipelago, which contains the IMAGE sites Ny Ålesund, Longyearbyen and Hornsund (§5.5.2). Figure 3.3 shows the location of these stations.
3.1. DATA SOURCES

Figure 3.3: Mawson and Davis stations in the Antarctic (Ponomarenko, 2000).

3.1.3 MACCS

The Magnetometer Array for Cusp and Cleft Studies (MACCS) (Engebretson, 2001) is a joint project between the Physics department at Augsburg College, Minnesota, and the Astronomy Department at Boston University, Massachusetts, with the assistance of the University of Alberta. The first four stations of the array were installed in arctic Canada in 1992 and another 4 in 1993. A further four were included in August 1995. At each station a ringcore fluxgate magnetometer samples the geomagnetic field at 8 Hz with resolution of 0.01 nT (Hughes and Engebretson, 1997). Like the IMAGE array they are aligned in geographic coordinates and calibrated to nT. Data are stored at 0.5 Hz and made available from Augsburg College and the Geological Survey of Canada as 5 or 10 second averages. Figure 3.4 shows the location of the MACCS stations and table 3.2 gives details on their location and L value.

3.1.4 The WIND Satellite

Data on the interplanetary magnetic field (IMF) and solar wind were obtained from the WIND satellite public domain database(http://web.mit.edu/space/www/wind.html), which is managed by the National Space Science Data Center (NSSDC). WIND was launched on November 1, 1994 and was the first of two NASA spacecraft in the Global Geospace Science Initiative, part of the International Solar-Terrestrial Physics (ISTP) program.
The sensor system of the satellite consists of three electric antenna systems and a triaxial magnetic search coil (magnetometer), which is mounted on the end of a 12 meter boom. The antenna system comprises of two coplanar, orthogonal wire antennas in the spin-plane and a rigid spin-axis dipole. There are five main receiver systems, all managed by a Digital Processing Unit (DPU) which is also housed within the spacecraft body.

The instruments used in this study include the magnetometer, which is used for the Magnetic Fields Investigation (MFI), and a Vector Electron Ion spectrometer, two Faraday cup ion detectors and a Strahl Detector for the Solar Wind Experiment (SWE). The MFI magnetometer system is actually a pair of magnetometers, the first mounted at the end of a 12 meter boom and the other midway along the boom. This dual system provides real-time magnetic field estimation and elimination of magnetic noise generated by the spacecraft (Taggart, 2001). The magnetometers have a resolution of 0.001 nT and data are logged digitally via a 12-bit analogue-digital converter system and the DPU. The SWE instrument includes two Faraday cup detectors which measure solar wind protons and atomic particles at energies up to 8 keV (Lazarus and Kasper, 2000) and an array of sensors for characterising the solar wind electrons. Data from the MFI and SWE were made available courtesy of R.P. Lepping and K.W. Ogilvie at Goddard Space Flight Center (NASA), and

Figure 3.4: The 12 stations of the MACCS array (Engebretson, 2001).
Table 3.2: The MACCS stations used in this study and their geographical and CGM coordinates, and \( L \) values for epoch 1998 and altitude 100 km (Engebretson, 2001).

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Station name</th>
<th>Geographic latitude</th>
<th>Geographic longitude</th>
<th>CGM latitude</th>
<th>CGM longitude</th>
<th>( L ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RE</td>
<td>Resolute Bay</td>
<td>74.7°</td>
<td>265.0°</td>
<td>83.5°</td>
<td>315.8°</td>
<td>&gt;15</td>
</tr>
<tr>
<td>CY</td>
<td>Clyde River</td>
<td>70.5°</td>
<td>291.4°</td>
<td>79.6°</td>
<td>18.5°</td>
<td>&gt;15</td>
</tr>
<tr>
<td>CB</td>
<td>Cambridge Bay</td>
<td>69.1°</td>
<td>255.0°</td>
<td>77.5°</td>
<td>306.9°</td>
<td>&gt;15</td>
</tr>
<tr>
<td>GH</td>
<td>Gjoa Haven</td>
<td>68.6°</td>
<td>264.1°</td>
<td>78.2°</td>
<td>323.4°</td>
<td>&gt;15</td>
</tr>
<tr>
<td>PB</td>
<td>Pelly Bay</td>
<td>68.5°</td>
<td>270.3°</td>
<td>78.6°</td>
<td>335.5°</td>
<td>&gt;15</td>
</tr>
<tr>
<td>RB</td>
<td>Repulse Bay</td>
<td>66.5°</td>
<td>273.8°</td>
<td>76.9°</td>
<td>343.2°</td>
<td>&gt;15</td>
</tr>
<tr>
<td>BK</td>
<td>Baker Lake</td>
<td>64.3°</td>
<td>264.0°</td>
<td>74.3°</td>
<td>326.1°</td>
<td>13.4</td>
</tr>
<tr>
<td>CD</td>
<td>Cape Dorset</td>
<td>64.2°</td>
<td>283.4°</td>
<td>74.6°</td>
<td>1.2°</td>
<td>13.6</td>
</tr>
<tr>
<td>IQ</td>
<td>Iqaluit</td>
<td>63.8°</td>
<td>291.5°</td>
<td>73.4°</td>
<td>14.7°</td>
<td>11.9</td>
</tr>
</tbody>
</table>

CDAWeb (http://cdaweb.gsfc.nasa.gov/).

The orbit of WIND ranges from within the magnetospheric cavity to far upstream of the solar wind. It has a halo orbit at the L-1 Lagrangian point between the Earth and the sun, with a perigee of \( 7.72 \, R_E \sim 4.8 \times 10^4 \, \text{km} \), apogee of \( 250 \, R_E \sim 1.6 \times 10^6 \, \text{km} \), and a 221 day orbit. The satellite is ideal for upstream solar wind measurements and has been used for all of the IMF and solar wind velocity and ion density measurements in this project.

### 3.1.5 DMSP

The Defence Meteorological Satellite Program (DMSP) is a Department of Defence program run by the U.S. Air Force Space and Missile Systems Center. The first group of satellites were launched in 1965 and there have been a total 38 launches to date, most recently in April 1997, although several have either failed at launch or been placed into an ‘unusable’ orbit (Wade, 2001). The satellite network has been divided into two blocks (4 and 5) and orbit at \( \sim 850 \, \text{km} \) altitude. Along with atmospheric and oceanographic data, the DMSP satellites record the densities, velocities, composition and drifts of plasma particles in the Earth’s magnetosphere, and the magnetic field.

Data used in this project data were collected by the SSJ/4 particle detectors on board the three satellites F11, F12 and F13. The instrument consists of four electrostatic analyzers and curved plate detectors designed for recording electrons...
and ions between 30 eV and 30 keV (Clark, 2001). Data were processed onboard the satellite and at the National Geophysical Data Center (NGDC). These data were used to characterise the magnetospheric topology during the intervals of interest. Section 3.2.5 provides more details on the analysis of the DMSP data.

### 3.1.6 The LANL-097A satellite

Magnetospheric density values required for the mathematical modelling part of the study was obtained from the Los Alamos National Laboratory LANL-097A satellite (e.g. Reeves et al., 1996 and references therein; Belian, 2000). This is part of a

Table 3.3: The DMSP satellites used in this study; Block, launch dates and orbit details (Wade, 2001).

<table>
<thead>
<tr>
<th>Block</th>
<th>Name</th>
<th>Launch date</th>
<th>Mass (kg)</th>
<th>Perigee (km)</th>
<th>Apogee (km)</th>
<th>Inclination</th>
</tr>
</thead>
<tbody>
<tr>
<td>4A</td>
<td>F11</td>
<td>May 1968</td>
<td>130</td>
<td>806</td>
<td>883</td>
<td>98.9°</td>
</tr>
<tr>
<td>4A</td>
<td>F12</td>
<td>Oct 1968</td>
<td>130</td>
<td>783</td>
<td>828</td>
<td>98.7°</td>
</tr>
<tr>
<td>5D-2</td>
<td>F14</td>
<td>Apr 1997</td>
<td>770</td>
<td>842</td>
<td>855</td>
<td>98.9°</td>
</tr>
</tbody>
</table>
network of 13 satellites, the first of which was launched in 1976. Each occupies a geosynchronous orbit at \( \sim 6.6 \, R_E \) at the geomagnetic equator and a fixed longitude.

The instrument on board the LANL-097A satellite of relevance to this project was the Synchronous Orbit Particle Analyser, or SOPA (Belián et al., 1992; Belian, 2000).

### 3.2 Data Preparation

As the data for this project were recorded by several different organisations worldwide it was necessary to convert each into a single standard for use by the analysis software at the University of Newcastle.

The analysis software at Newcastle was written in the IDL code and it was necessary for the ground magnetometer data to be converted into a standard data format used by the Newcastle group for codes written with this language. This format is an ASCII file, devised by C.L. Waters in 1994 (Waters et al., 1994), and consists of a header containing the station name, component, date, starting time and number of points, followed by the data. Figure D.1 gives an example of part of one of these files. This will hereafter be referred to as “Newcastle format”.

#### 3.2.1 IMAGE Magnetometer Array

The data from the IMAGE stations were obtained by request to the administrators of the network at the IMAGE web site (Viljanen, 2001). These data are stored in IAGA (International Association for Geomagnetism and Aeronomy) format on that database maintained by the Finnish Meteorological Institute (FMI) in Finland. The data are divided into whole day files, with the header and all data for the three geographic components (X,Y,Z) recorded as a single line. These were sent to the University of York where they were transferred into Newcastle format via an intermediate step into York format. The conversion process involved a coordinate rotation to geomagnetic coordinates to a format specific to the University of York. Details on this conversion process are given in the appendix section D.1. The final form of the data in Newcastle format is an ASCII file for each component of each day of approximately 140 KB.

#### 3.2.2 MACCS

Data were obtained from M.J. Engebretson at Augsburg College via FTP but are generally available on the MACCS website. The files were in binary format sampled every 5 seconds, and an IDL program `sec_conv.pro` (also available on the MACCS
webpage) allowed conversion to ASCII. A program `MACCSSconv.pro` was written by the author which included this subroutine to convert the data to the same format used by analysis programs developed for IMAGE data. Finally, a coordinate rotation was performed into geomagnetic H and D components in the same manner as that of IMAGE (refer to Appendix C.2 for declination angles). In this format, a whole-day 2-component data file is around 490 KB in size.

### 3.2.3 Antarctic Magnetometer Data

The Davis data were stored in an Antarctic Division format called ADAS (Analogue Data Acquisition System) (Symons, 1996) in dip magnetic coordinates (i.e. aligned with a compass) and were made available by request from the Antarctic Division Web Page (http://www.antdiv.gov.au). The data were transferred to the computers at Newcastle by FTP. A format conversion from ADAS to Newcastle was necessary as well as a calibration into nT. Both of these were performed with two IDL programs, `trueind.pro`, developed by P.V. Ponomarenko, and `adas2idl.pro`, written by the author. Appendix D.2 provides details on this conversion process.

Comparison with IMAGE data also required a change in sample rate. This was included with each analysis program developed by the author. As with the other data a coordinate rotation to CGM coordinates was required. This involved a determination of the angle between the location of magnetic north as determined by the compass (bearing is +111° for DAV, +122° for MAW) and the declination angle D (Appendix C.2) of the field. This required a rotation of \((-78.06+111.00 = +32.94)° \) for Davis. Once in its final form a whole-day single-component data file occupies 1.5 MB.

### 3.2.4 WIND Spacecraft

The WIND satellite data were provided on the WIND satellite webpage (Lazarus and Kasper, 2000) and includes information from both the Magnetic Fields Investigation (MFI) and the Solar Wind Experiment (SWE) The data were plotted as 90 minute intervals with a resolution of 1 minute, the above parameters as the $y$-axis and UT time as $x$. The plots were saved as *.gif files and measurements were made with close-up views of the plots using the graphics analysis software Paintshop Pro. An example of these plots are shown in figures 4.11 and 4.12.
3.3 DATA ANALYSIS THEORY

3.2.5 DMSP

The DMSP data were used to aid in the identification of the magnetopause and auroral oval, and were obtained from the Auroral Particles and Imagery Group at the Johns Hopkins University Applied Physics Laboratory (http://sd-www.jhuapl.edu/Aurora/). This uses an automated algorithm designed to identify various boundary layers in the magnetosphere through predetermined features observed in the data. The cusp, for example, is identified as the region “where the ion and electron fluxes begin to approximate magnetosheath values, which generally occurs where the ion cutoff drops to at least 1-3 keV and below.” (Newell, 2000). Further identification was also made by the author by examining the location of these boundaries in the online spectrograms, also provided on the webpage.

3.3 Data Analysis Theory

3.3.1 The Fast Fourier Transform (FFT) Analysis and Filtering

In analysing digital signals there are several problems to overcome, mainly because analysis techniques try to achieve an analogue approximate from the digital signal. A visual analysis of the raw data is easily obtained via an amplitude-time plot, where each point represents the magnitude at an integral multiple of the sample rate. Particular signals, however, are identified by their frequency components and so a transformation is performed on the data from the time to the frequency domain.

If we begin with a discrete signal in the time domain $s(t)$ then its counterpart in the frequency domain $S(f)$ can be determined with the Fourier Transform:

$$S(f) = \int_{-\infty}^{\infty} s(t)e^{-2\pi ift}dt. \quad (3.1)$$

Mathematically, the Fourier Transform decomposes any signal into a sum of sinuosoids and transforms each into the frequency domain (Brigham, 1988). Typically, the function in the time domain does not geometrically resemble the function in the frequency domain, e.g. a boxcar function is transformed into a sinc function (i.e. $\sin(x)/x$). The useful aspect of the Fourier Transform is it transforms a sinusoidal function into a delta function at the frequency of the signal. It also resolves signals containing different frequencies into spikes representing the relevant frequency of each component.

Digitising a signal changes an indefinite integral into a definite sum of $N$ points.
and equation (3.1) becomes
\[
S(f_k) = \sum_{i=0}^{N-1} s(t_i)e^{-2\pi i f_k(t_{i+1} - t_i)} \quad k = 0, 1, ..., (N - 1). \tag{3.2}
\]

The problem with using equation (3.2) is that if there are \(N\) data points and \(N\) separate sinusoids, then the computation time is proportional to \(N^2\), the number of multiplications. Considering the fact that a whole day data set sampling at 1 second produces 86400 points and as many separate sinusoids, the computation time is excessive, even for the faster computers. To overcome this problem, the Fast Fourier Transform, or FFT was developed (Cooley and Tukey, 1965). The FFT reduces the computational time to \(N\log_2 N\).

The function \(S(f_k)\) described in equation (3.2) is a digitised complex vector of the same dimension as \(s(t_i)\). Each value of \(f_k\) represents an integral multiple of the frequency resolution, which is determined from the sample rate and dimension of the original signal, namely:
\[
\Delta f = \frac{1}{N\Delta t}, \tag{3.3}
\]
where \(\Delta f\) is the frequency resolution, \(N\) is the number of points in the sample window (called the FFT length) and \(\Delta t\) is the time interval between points. From this equation it can be seen that the larger the value of \(N\) and the higher the sample rate, the better the resolution. However, increasing the window length also increases the contribution from noise signals, and increasing the sample rate introduces problems with data storage and computation time. The key is to find a balance where the value of \(N\) and \(\Delta t\) are low enough to allow a suitably sized time domain and storage capacity without an excessive reduction to frequency resolution.

The total power of \(s(t)\) is determined by summing the magnitudes of \(S^2(f_k)\) across all of the frequencies, i.e:
\[
Pow = \sum_{i=0}^{N-1} |\text{FFT}\{s(t_i)\}|^2 = \sum_{i=0}^{N-1} |S(f_i)|^2, \tag{3.4}
\]
where \(S(f) \in \mathbb{C}\). Again, care must be taken to choose the value of \(N\) to remove noise contributions. This is discussed later.

### 3.3.2 Artifacts of the FFT: Aliasing and Leakage

The mathematics surrounding the FFT creates two physical problems in the range of the transformation. These are aliasing of signals above the Nyquist frequency and leakage. Both of these must be addressed before any experimentally viable measurements can be made, as discussed below.
Aliasing is a problem associated with frequency resolution. The data are collected at a specific rate with a distinct time between each interval. There is a corresponding frequency, called the Nyquist frequency, which requires the sampling frequency to be high enough to sample each frequency component at least twice in a cycle (Cooley et al., 1969). This frequency is:

\[ f_N = \frac{1}{2\Delta t} \]  

(3.5)

For example, the IMAGE magnetometer stations sample at 10 second intervals, resulting in a Nyquist frequency of 50 mHz.

As mentioned in §3.3.1, for the Fourier Transformation, the FFT will produce a value for any frequency up to \( N \) times the resolution. The FFT is mathematically symmetrical about zero, and a mirror-image of the positive frequency spectrum will be produced for the negative frequencies. Digital analysis programs such as IDL represent the negative frequency spectrum by reproducing the signals in the positive, thus introducing false power signatures in the spectrum. This problem is easily overcome by the application of a low pass filter at the Nyquist frequency to the data before analysis.

The sample rate of the IMAGE magnetometers is 0.1 Hz, producing a Nyquist frequency of 50 mHz. The signals examined in this thesis therefore cannot exceed this frequency which is the reason why we have only focused upon activity up to 50 mHz, rather than the maximum value of Pc 3 at 100 mHz.

Another effect of aliasing is the bisection of the power distribution across the Nyquist frequency axis of symmetry. Filtering the signal at the Nyquist frequency removes the negative frequency spectral peak, effectively dividing the power of the input signal by 2. When restoring the amplitude from the power it is therefore necessary to multiply the power by 2.

Leakage arises from the sampling of data across a particular window. An unaltered data set in the time domain can be considered as being windowed with a boxcar function the width of the set. It was noted in section 3.3.1 that a boxcar function is transformed into a \( \text{sinc} \) function, which when digitised creates side bands symmetrically about the signal’s frequency. Each of these carry power which must be removed from the central frequency in order to preserve equation (3.4). This is shown in figure 3.6. Another method to overcome leakage is to choose a band of frequencies on either side of the central value and apply equation (3.4). One must be careful in band selection as a large band may include unwanted signals but a narrow band may not include all the power from the signal. In this project a bandwidth of \( \pm 2.5 \) mHz was chosen.
It has been shown (e.g. Ramirez, 1985) that certain functions in the time domain can reduce power leakage into the side lobes when transformed into the frequency domain. It is therefore commonplace to modulate the time series with such a function before performing the transform. This technique is called *windowing*. Figure 3.7 demonstrates a selection of different window geometries and their normalised Fourier Transformed counterparts. In this study, a Hanning window, or inverted cosine, was used to modulate the ground data.

The modulation of the time series by a window creates a modification in amplitude which can reduce the power at the given frequency considerably. In the case of the Hanning window the power is effectively reduced by almost 70%. To restore to units of amplitude after windowing it is therefore necessary to apply a correction factor. This is discussed further in section 4.4.1.
Figure 3.6: A selection of functions in the time domain and their Fourier Transformed counterparts in the frequency domain (Ramirez, 1985).
### Figure 3.7: Some common data windows and their frequency domain parameters (Ramirez, 1985).

<table>
<thead>
<tr>
<th>Window</th>
<th>Shape Equation</th>
<th>Frequency Domain Magnitude</th>
<th>Highest Side Lobe (dB)</th>
<th>Bandwidth (at 3 dB)</th>
<th>Theoretical Roll-Off (dB/0.1 octave)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$A = 1$ for $t=0$ to $T$</td>
<td>$-T$</td>
<td>$-12.2$</td>
<td>$0.86T$</td>
<td>6</td>
</tr>
<tr>
<td>Extended Cosine Bell</td>
<td>$A = 0.5 (1 - \cos 2\pi t/T)$ for $t=0$ to $T/10$ and $t=9T/10$ to $T$</td>
<td>$0.9 T$</td>
<td>$-13.5$</td>
<td>$0.95T$</td>
<td>18 (beyond 5%)</td>
</tr>
<tr>
<td>Hanning</td>
<td>$A = 1$ for $t=T/10$ to $9T/10$</td>
<td>$0.5 T$</td>
<td>$-22.4$</td>
<td>$1.15T$</td>
<td>12</td>
</tr>
<tr>
<td>Triangle</td>
<td>$A = 2/T$ for $t=0$ to $T/2$</td>
<td>$0.5 T$</td>
<td>$-26.7$</td>
<td>$1.27T$</td>
<td>12</td>
</tr>
<tr>
<td>Coaxial $^1$ (Hanning)</td>
<td>$A = 0.5 (1 - \cos 2\pi t/T)$ for $t=0$ to $T$</td>
<td>$0.5 T$</td>
<td>$-31.6$</td>
<td>$1.39T$</td>
<td>18</td>
</tr>
<tr>
<td>Hanning</td>
<td>$A = \cos^2 t + 0.5/t$ for $t=0$</td>
<td>$0.4 T$</td>
<td>$-39.5$</td>
<td>$1.61T$</td>
<td>24</td>
</tr>
<tr>
<td>Hamming</td>
<td>$A = 0.5 \left[ 0.45 \left( 1 - \cos 2\pi t/T \right) \right]$ for $t=0$ to $T$</td>
<td>$0.5 T$</td>
<td>$-41.9$</td>
<td>$1.26T$</td>
<td>6 (beyond 3%)</td>
</tr>
<tr>
<td>Coaxial $^2$</td>
<td>$A = 0.5 \left[ 1 - \cos 2\pi t/T \right] ^2$ for $t=0$ to $T$</td>
<td>$0.3 T$</td>
<td>$-45.9$</td>
<td>$1.73T$</td>
<td>39</td>
</tr>
<tr>
<td>Parzen</td>
<td>$A = \left( 1 - \cos (2\pi T/T - 1) \right)^2$ and $A = 2(1 - \cos (2\pi T/T - 1/2) \right)$ for $t=0$ to $T/4$ and $t=3T/4$ to $T$</td>
<td>$0.37 T$</td>
<td>$-53.2$</td>
<td>$1.84T$</td>
<td>24</td>
</tr>
</tbody>
</table>
3.3. DATA ANALYSIS THEORY

Filtering is the removal of signals of particular frequencies from the data set. This is easily done once the data have been transformed into the frequency domain. However, there are several problems in the selection of filter as it is often desired to return to the time domain after filtering has been performed. In a similar nature to the problems posed by windowing, filtering edges can become distorted upon transformation into the time domain. To overcome this problem, various filters have been designed by several workers (e.g. REFERENCE; Bourke, 1999). These involve creating a form of step-function in the frequency domain, performing a reverse FFT to produce the function in the time domain then optionally widowing the data to reduce possible artifacts (the Gibbs phenomenon). We have used two filters in this thesis. The first was made in the time domain using non-recursive digital filter for evenly spaced points included with the IDL software package (following Walraven, 1985). The second was the application of a smoothing technique as discussed in section 3.6.2.

The development of a power spectrum involves the production of a power against frequency plot from the FFT of the input time series. In order to relate the output power in the frequency domain to the input in the time domain it is necessary to take several things into consideration. The list below shows the steps taken during this study to produce the amplitude profiles, shown in the next chapter.

Power spectrum development can be taken one step further to produce the dynamic power spectrum. This is a three-dimensional plot of power, frequency and time. The daytime time series is divided into a series of overlapping windows. Each window contributes a value of power for each frequency thus producing a series of power values with frequency across the day. In our case the size of the window was 30 minutes, overlapping every 15 minutes and windowed with a Hanning function. The frequency resolution was 0.56 mHz and power was plotted in the same sense as contours are plotted on a map but colour coded. An example of a dynamic power spectrum is provided in panel b) of figure 4.2.

In such studies which involve an average power across a window one must consider the time stationarity of the signal. If the selection of window is too large then the average power can be affected by signals in the same frequency band but unrelated to that of interest. It is therefore important to select a window size which is sufficiently small to minimise the possibility of this interference. Window reduction, however, reduces the number of overlapping windows which can be used for a given frequency resolution, which affects the statistical reliability of the results. This will be discussed further in section 3.6.4.
3.4 Bivariate Signal Analysis

The analytical techniques of this project focus mainly on the study of multivariate signals, i.e. multiple signals being processed simultaneously and compared. Analysis of such signals can prove very useful in revealing the spatial characteristics of waves. Mathematically, multivariate signal analysis removes several of the problems indicated in the previous section such as power loss from windowing, which cancels in cross-spectra study. The measurements in this thesis will focus on three aspects of bivariate analysis, cross-power, cross-phase and coherence.

3.4.1 Cross-Phase

In this project, two cross-phase techniques were used. The first, the “static” cross-phase, involves the production of a cross-phase against frequency spectrum for any pair of stations for a specific time interval. Using this technique, cross-phase values at specific frequencies can be determined by identifying the value in the spectrum corresponding to the desired frequency. The second method is the dynamic cross-phase method, which is simply an extension of the static cross-phase. It involves the production of a three-dimensional colour coded plot of phase ($z$), frequency ($y$) and time ($x$). It is produced in a similar manner to the dynamic power spectrum and can be regarded as a sequential application of static cross-phase spectra.

The Static Cross-Phase Spectrum. This technique is well known and several authors discuss its application (e.g. Kanasewich, 1973; Bath, 1974; Fraser, 1979). The following discussion is based on Fraser (1979).

We begin with two time series function $f_1(t)$ and $f_2(t)$ which for our purposes represent two sets of data from different stations. Let the FFT of $f_m$ be $F_m(\omega)$ ($m = 1, 2$). The power spectrum for $f_m$ is given by

$$S_{mm}(\omega) = |F_m(\omega)|^2 = F_m^*(\omega)F_m(\omega). \quad (3.6)$$

When the squared modulus is taken it will always be real and positive.

The cross-power spectrum can be found from the bivariate form of equation (3.6), i.e.

$$S_{12}(\omega) = F_1^*(\omega)F_2(\omega). \quad (3.7)$$

The cross-power spectrum is a complex quantity and can be written in the following form:

$$S_{12}(\omega) = Re\{S_{12}\} + Im\{S_{12}\}i. \quad (3.8)$$
The magnitude of $S_{12}$ represents the cross-power spectrum and the phase $\varphi_{12}$ can be determined from

$$
\varphi_{12} = \tan^{-1} \left[ \frac{|\text{Im}\{S_{12}\}|}{|\text{Re}\{S_{12}\}|} \right].
$$

The cross-power indicates the association of frequency components with amplitudes at the same frequency at both series. This is a useful tool in the identification of FLRs (§2.3.1). The phase spectrum shows the lead or lag of components at the same frequency for one series when compared with another. When performed over a selection of frequencies a phase difference against frequency plot can be produced. This plot allows the measurement of phase difference of a signal between any two stations at a particular frequency for a given time interval.

**Dynamic Cross-Phase.** Since its development in 1991 by Waters *et al.*, this technique has been utilised in many publications involving the identification of FLRs and the study of ULF waves (e.g. Waters *et al.*, 1991a, b, 1994; Menk *et al.*, 1994; Chi and Russell, 1998). It was originally devised as a method to aid in the detection and measurement of FLRs. Previous studies have revealed that the cross-phase spectrum reveals information not available in the power spectrum (Lefeuvre *et al.*, 1984; Waters *et al.*, 1991a).

The dynamic cross-phase can be calculated by FFT using the technique outlined for the static cross-phase and its development with time can be monitored for any time sequence. This produces a plot of phase, frequency and time, called the dynamic cross-phase spectrum. Again the choice of window sizes can alter cross-phase measurements due to the time stationarity of the signal, discussed above.

In this study dynamic cross-phase was used as a visual guide to aid in the detection of signals, and to detect timing errors between stations. In the dynamic spectrum, a timing error between stations will reveal an increasing or decreasing trend in phase as we move up the frequency axis. As the phase is colour coded, this will be indicated by a colour sweep across the spectrum as the frequency increases. This is often easier to detect than a trend slope in the two dimensional static phase spectrum. Actual measurements of phase were made using the static cross-phase technique.

### 3.4.2 Identification of reliable signals: Coherence

Before we can begin analysis of phase or amplitude, we must first confirm the reliability of the signal. A reliable signal at two stations should not only have relatively high power at the same frequency but also be correlated. As indicated by equation (3.7) the cross-power is the magnitude of $S_{12}$. Thus a relatively high cross-
power at a given frequency indicates a high level of activity at that frequency, but not necessarily the same signal. One indicator of the reliability of wave structure is coherency.

**Coherency** is a complex value which relates the cross-power of a signal between two stations with the power at each of the individual stations (Fraser, 1979). It is normalised by definition, such that a value of unity indicates a signal of high interrelation and zero for noise signals indigenous to a single station only. Coherency is defined by

\[
\gamma(\omega) = \frac{S_{12}(\omega)}{\sqrt{S_{11}(\omega)S_{22}(\omega)}},
\]

and its complex nature indicates both a magnitude and phase. It is important not to confuse coherency with coherence; coherence is the magnitude of coherency $|\gamma(\omega)|$, and has no imaginary component (Fraser, 1979).

For a signal to be considered experimentally viable across two stations it must have significantly high cross-power and a value of coherence as close as possible to unity. For statistical purposes which will be discussed later the chosen cutoff for coherence in this study was 0.65, such that any signal with a coherence value <0.65 was considered unreliable.

A closer inspection of equation (3.10) reveals a fundamental problem with coherency if performed across an entire FFT length. Combining equations (3.6) and (3.7 with 3.10) we have

\[
|\gamma(\omega)| = \frac{|S_{12}(\omega)|}{\sqrt{S_{11}(\omega)S_{22}(\omega)}} = \frac{|F_1^*(\omega)F_2(\omega)|}{\sqrt{F_1^*(\omega)F_1(\omega)F_2^*(\omega)F_2(\omega)}} = \frac{|F_1F_2|}{\sqrt{|F_1^2F_2^2|}} = 1,
\]

rendering the value of coherency meaningless. This property, which we will call this the unity paradox is a demonstration of the fact that coherency is a statistical quantity. Thus, to find coherence one must determine the average values of $S_{12}$, $S_{11}$ and $S_{22}$ across a series of smaller windows within the selected time period or apply some other statistically valid method to the frequency domain such as smoothing. These techniques will be discussed in section 3.6.

### 3.5 Polarization

The final feature in our wave analysis is the comparison of H and D components of the wave to determine wave orientation. This is called polarization and requires data from a single station only.
3.5. POLARIZATION

3.5.1 Definition of Polarization

The following is based on Smith and Thomson (1971) with additions and corrections made by the author. Consider two waves travelling in the same direction, each plane polarized, but with their planes at right angles to each other. They produce a resultant electric field which can be found by vector addition of the individual field vectors. This addition must be done at all times, i.e. across all data points in a discrete time series. In an ideal case, the frequency remains constant and the addition need only be performed once. If the two waves are in phase with each other the vector addition will produce a resultant which, while it may differ in magnitude, is always directed with the same angle, as shown in panel (a) of figure 3.8. If the two are $\pi/2$ out of phase then the resultant traces out a circle as shown in panel (b) of figure 3.8. If the magnitudes of the two signals are unequal then the result is an ellipse.

![Figure 3.8: Vector addition of two oscillating electric fields at successive moments through one half cycle. The two fields are of equal magnitude and are mutually perpendicular to each other. Panel (a): The waves are in phase; Panel (b): The waves are $\pi/2$ out of phase. The resultant polarization is (a) linear and (b) circular (Smith and Thomson, 1971).](image)

Convention dictates that a left handed polarized magnetic wave produces a trace which rotates anticlockwise when viewed in the direction of the positive magnetic
field. A clockwise rotation indicates a wave which is right hand polarized in nature.

From a mathematical perspective consider two waves, both propagating in the z direction, of respective amplitudes \( a \) and \( b \), which are oriented perpendicularly to each other and the direction of propagation (i.e. the x and y axes). These components are two plane polarized waves, given by

\[
E_x = ae^{i(\omega t - kz)} \hat{x}, \\
E_y = be^{i(\omega t - kz + \phi)} \hat{y},
\]

(3.12)

where \( \phi \) is the phase difference between the waves and \( \hat{x} \) and \( \hat{y} \) are unit vectors.

The value of \( k \) is the wavenumber defined by \( k = 2\pi/\lambda \). Now consider the following cases.

1. \( \phi = 0 \). In this case the two vectors combine to form a resultant of magnitude \( \sqrt{a^2 + b^2} \) and an angle to the x-axis of

\[
\tan \psi = \frac{b}{a}.
\]

(3.13)

2. \( \phi = \pm \pi/2 \). In the special case where \( a = b \) the wave is circularly polarized and the resultant vector traces out a circle. When \( a \neq b \) the resultant is an ellipse with a major axis oriented along the axis of the largest amplitude. The case where \( \phi = -\pi/2 \) is identical except the direction of oscillation is reversed.

3. \( -\pi/2 < \phi < \pi/2 \). This is the more general case where the major axis of the ellipse does not lie along either of the axes and is discussed below.

We will now derive the equation of the trace of the resultant vector as it develops with time. Equating the real parts of the waves in equation (3.12) we have

\[
\frac{E_x}{a} = \cos \omega t, \\
\frac{E_y}{b} = \cos(\omega t + \phi) = \cos \omega t \cos \phi - \sin \omega t \sin \phi,
\]

(3.14)

where \( Re\{E_x\} = E_x \) and \( Re\{E_y\} = E_y \). Combining these equations and eliminating the time factor (derivation is in Appendix E.2) we have

\[
\frac{E_y}{b} = \frac{E_x}{a} \cos \phi - \left[ \sqrt{1 - \cos^2 \omega t} \right] \sin \phi.
\]

(3.15)

Rearranging this equation (again refer to Appendix E.2) we have

\[
\frac{E_y^2}{b^2} + \frac{E_x^2}{a^2} - \frac{2E_xE_y}{ab} \cos \phi \sin^2 \phi = 0
\]

(3.16)

which gives the equation for an ellipse, shown in figure 3.9. At any time the tip of the resultant vector will reach a point on this curve. The ellipse is contained within
3.5. POLARIZATION

Figure 3.9: Elliptically polarized oscillation, combining linearly polarized oscillations on the x and y axes, with amplitudes a and b, and an arbitrary phase difference (Smith and Thomson, 1971).

A rectangle with sides 2a and 2b. The major axis is oriented at some angle $\psi$ to the x-axis. This angle can be found as follows (complete derivation is included in Appendix E.2).

Using the principles of multivariable calculus we begin with the general equation for the complete derivative of any function of two variables, $F(E_x, E_y)$:

$$dF = \frac{\partial F}{\partial E_x} dE_x + \frac{\partial F}{\partial E_y} dE_y.$$  \hfill (3.17)

The amplitude of the resultant vector, $F = \sqrt{E_x^2 + E_y^2}$ is maximum on the major axis, i.e. $dF = 0$. Applying this to equation (3.17) we have

$$E_x dE_x + E_y dE_y = 0.$$ \hfill (3.18)

Now let $F$ be the equation of the ellipse described in equation (3.16). Applying equation (3.17) reveals

$$\left( \frac{E_x}{a^2} - \frac{\cos \phi}{ab} E_y \right) dE_x + \left( \frac{E_y}{b^2} - \frac{\cos \phi}{ab} E_x \right) dE_y = 0.$$ \hfill (3.19)

Along the major axis at angle $\psi$ with the x-axis we have $\tan \psi = E_y/E_x$, so combining equations (3.18) and (3.19),

$$\frac{1}{a^2} - \frac{\cos \phi}{ab} \tan \psi = \frac{1}{b^2} - \frac{\cos \phi}{ab} \cot \psi.$$ \hfill (3.20)
Finally, using the trigonometric identity \( \tan \psi - \cot \psi = -2 \cot 2\psi \) we find

\[
\tan 2\psi = \frac{2ab \cos \phi}{a^2 - b^2}.
\] (3.21)

This shows that given the values of \( a \) and \( b \) and the phase difference between the signals \( \phi \) we can determine the angle of the major ellipse axis. The values of \( a \) and \( b \) can be read directly from the data as they represent the maximum values of each component, and \( \phi \) can be determined from cross-phase analysis.

### 3.5.2 The Stokes Parameters

The most common way to express the polarization characteristics of a wave is in terms of the Stokes parameters. These are values which describe trace power, polarized contribution to power and orientation of the polarization ellipse. Some of these have been discussed in previous sections but the equations governing their determination for this project were used according to the following. Derivation of the Stokes parameters requires a consideration of the coherency and time-dependance on signal amplitude and phase. The following is from Fowler et al. (1967).

First, we take the mean frequency across a sum of subwindows \( \overline{f} \) (section 3.6.3) and express the quasi-monochromatic signal \( H = H(t) \) in terms of the time-dependent amplitude \( A = A(t) \) and phase \( \varphi = \varphi(t) \) of the \( x \) and \( y \) components:

\[
\begin{align*}
H_x &= A_x e^{[\omega t + \varphi_x]i} \\
H_y &= A_y e^{[\omega t + \varphi_y]i},
\end{align*}
\] (3.22)

with \( \omega = 2\pi \overline{f} \). The polarization of the wave field can be characterised in terms of the coherency matrix, given as

\[
J = \begin{bmatrix}
<H_x H_x^*> & <H_x H_y^*> \\
<H_y H_x^*> & <H_y H_y^*>
\end{bmatrix}.
\] (3.23)

As shown by Born and Wolf (1964), any monochromatic signal can be written as a sum of a totally polarized and completely unpolarized signal. In the former case, time-dependent variables \( A \) and \( \varphi \) in equation (3.22) are constant. Its coherency matrix, which we will call \( P \), can be expressed as

\[
P = \begin{bmatrix}
P_{xx} & P_{xy} \\
P_{yx} & P_{yy}
\end{bmatrix} = \begin{bmatrix}
A & B \\
B^* & C
\end{bmatrix},
\] (3.24)

with \( A \geq 0 \) and \( C \geq 0 \). By putting equation (3.22) with constant \( A \) and \( \varphi \) into equation (3.23) it can be shown that \( \det J = 0 \) (appendix E.3) when the signal is completely polarized. Thus, from equation (3.24),

\[
AC - BB^* = 0.
\] (3.25)
3.5. POLARIZATION

Also, let the unpolarized component be $U$, such that the total coherency matrix is

$$J = P + U.$$  \hspace{1cm} (3.26)

For a completely unpolarized signal there is no coherency between components and so from equation (3.10),

$$\gamma_{xy} = \frac{J_{xy}}{\sqrt{J_{xx}J_{yy}}} = 0.$$ \hspace{1cm} (3.27)

Now, given that $J$ is a Hermitian Matrix (i.e. $J_{ik} = J_{ki}^*$), we have $J_{xy} = J_{yx} = 0$. From the definition of $J$ given in equation (3.23) we also have $J_{xx} = J_{yy}$. So we can write the coherency matrix for an unpolarized signal in the following form:

$$U = \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix},$$ \hspace{1cm} (3.28)

where $D \geq 0$. Substituting equations (3.24) and (3.28) into (3.26) and (3.23) we arrive at

$$\begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = \begin{bmatrix} A + D & B \\ B^* & C + D \end{bmatrix}.$$ \hspace{1cm} (3.29)

Expanding this matrix form into a series of simultaneous equations and solving for $D$ (appendix E.3) we have

$$D = \frac{1}{2}(J_{xx} + J_{yy}) \pm \frac{1}{2}\sqrt{(J_{xx} + J_{yy})^2 - 4|J|}. \hspace{1cm} (3.30)$$

As shown in appendix E.3 the root with the positive sign requires negative values for $A$ and $C$ which is in violation of their initial definition, so this root is omitted. Rearranging equation (3.26) we can now express a matrix for the polarized component of the signal,

$$P = \begin{bmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{yy} \end{bmatrix} = \begin{bmatrix} J_{xx} - D & J_{xy} \\ J_{yx} & J_{yy} - D \end{bmatrix}.$$ \hspace{1cm} (3.31)

We now consider the shape and orientation of the polarization ellipse, described in section ???. The polarization properties of the wave are completely determined by the matrix $P$, and the equation of the polarization ellipse in coordinates $(x, y)$ arises from the expansion of the following equation. Let

$$\begin{bmatrix} y \\ x \end{bmatrix} \begin{bmatrix} P_{xx} & P_{xy} \\ P_{xy}^* & P_{yy} \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = P_{xx}P_{yy}$$

and expand (appendix E.3) to reveal

$$\frac{x^2}{P_{xx}} + \frac{2\text{Re}\{P_{xy}\}}{P_{xx}P_{yy}}xy + \frac{y^2}{P_{yy}} = 1.$$ \hspace{1cm} (3.32)
Our objective here is to diagonalise $P$ through a coordinate rotation from the $(x, y)$ coordinate system to one which corresponds to the major and minor axes of the ellipse $(x', y')$. To diagonalise any matrix (under the assumption that the matrix is diagonalisable) we find the transformation matrix $T$ such that $T^TPT = Q$, where $T^T$ is the transpose of $T$ and $Q$ is the new diagonal matrix. In this case $T$ is the rotation matrix

$$T = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},$$

and the value of $\theta$ which diagonalises $P$ is given (Fowler et al., 1967) by

$$\tan 2\theta = \frac{2\text{Re}P_{xy}}{P_{xx} - P_{yy}}.$$ (3.34)

This angle therefore represents the angle between the $x$-axis and the major axis of the polarization ellipse. The ellipticity can be derived in a similar manner in terms of the angle $\beta$, defined (Born and Wolf, 1964) by

$$\sin 2\beta = \frac{(P_{yx} - P_{xy})i}{\sqrt{(P_{xx} - P_{yy})^2 + 4P_{yx}P_{xy}}}.$$ (3.35)

The ellipticity is then defined by $\tan \beta$ and the sense of polarization by the sign of $\beta$. By convention, a right- (left-) handed polarized wave corresponds to a value of $\beta < (> )0$, when looking into the propagating wave.

We are now in a position to define the four Stokes Parameters. The first is the Trace Power, derived from the trace of the coherency matrix for the total signal $J$. When applied to our original signal the trace power is

$$\text{trace} = Tr[J] = J_{xx} + J_{yy} = S_{xx} + S_{yy},$$ (3.36)

where $S_{xx}$ and $S_{yy}$ are the averaged power spectra for the $x$ and $y$ components.

The second Stokes parameter is the Polarized Power, which is the intensity of the polarized component of the trace power. This is obtained from the trace of the matrix for the polarized power $P$, with equation (3.30) representing $D$,

$$\text{pol} = Tr[P] = P_{xx} + P_{yy} = \sqrt{(J_{xx} + J_{yy})^2 - 4|J|}.$$ (3.30)

Now, as with the previous case we substitute $J$ with $S$ and apply the definition of the modulus of a complex parameter,

$$\text{pol} = \sqrt{(S_{xx} - S_{yy})^2 + 4[\text{Re}(S_{xy})^2 + \text{Im}(S_{xy})^2]}.$$ (3.37)

where $\text{Re}$ and $\text{Im}$ denote the real and imaginary components. Thirdly, the Ellipticity is defined as $\tan \beta$ with $\beta$ given in equation (3.35). In terms of our power
3.6 Statistical Relevance to Analysis Techniques

No data analysis project can be truly complete without statistics. The most important question a researcher can ask about a project is how well can we trust the results? This answer cannot be adequately addressed without a proper consideration of statistical significance. Few books have been published on statistics with digital signal analysis, but the fundamental text appears to be from Jenkins and Watts (1968). Other texts include Bendat and Piersol (1966) and Chatfield (1989). This section will outline the importance of statistics in this project along with experimental relevance to values such as cross-phase and coherence.

3.6.1 The Sinusoidal Model

The following discussion is based on Chatfield (1989).

Consider a digitised time series $X$ of $N$ elements which contains a sinusoidal component of frequency $\omega$ and a random error term $Z$. Then for each element $t$ of the series,

$$X_t = \mu + \alpha \cos \omega t + \beta \sin \omega t + Z_t$$

where $\mu$, $\alpha$ and $\beta$ are parameters to be estimated from the data and $N$ is the FFT length. This can be represented by a matrix as

$$\mathbf{E}(X) = A\theta$$
CHAPTER 3. DATA COLLECTION AND ANALYSIS

where

\[ X^T = (X_1, \ldots, X_N) \]
\[ \theta^T = (\mu, \alpha, \beta) \]
\[ A = \begin{bmatrix}
1 & \cos \omega & \sin \omega \\
1 & \cos 2\omega & \sin 2\omega \\
\vdots & \vdots & \vdots \\
1 & \cos N\omega & \sin N\omega
\end{bmatrix}. \]

To remove the error term we apply the least squares estimate technique. The least squares estimate of \( \theta \) can then be applied to approximate

\[ \min \left\{ \sum_{t=1}^{N} (x_t - \mu - \alpha \cos \omega t - \beta \sin \omega t)^2 \right\}, \]

where \( x_t \) is the \( t \)'th observation of the digital signal. The estimate is denoted by \( \hat{\theta} \) and is given by

\[ \hat{\theta} = (A^T A)^{-1} A^T x, \]

where \( x \) is defined as \( x^T = (x_1, \ldots, x_N) \). The values of \( \omega \) are digitised and a vector \( \omega \) can thus be determined, where each element represents an integral multiple of the frequency resolution. In terms of \( \omega \) the frequency resolution is the lowest frequency which can occupy \( N \) points in a single cycle, i.e. \( \omega_{\text{min}} = 2\pi/N \). Hence, \( \omega \) becomes restricted to one of the values \( \omega_p = \frac{2\pi}{N} \) \( p = 1, \ldots, N/2 \), where the \( N/2 \) limitation on \( p \) is governed by the Nyquist condition. Each value of \( \omega_p \) is called the \( p \)th harmonic of \( X \). Now, using the ‘well-known’ trigonometric results (equations E.36–E.39 in Appendix E.4) we can find \( \hat{\theta} \) given below.

\[
\hat{\theta} = \begin{bmatrix}
\hat{\mu} \\
\hat{\alpha} \\
\hat{\beta}
\end{bmatrix} = \begin{bmatrix}
\frac{\bar{x}}{\sum_{t=1}^{N} x_t \cos(\omega p t)/N} \\
\frac{2\sum_{t=1}^{N} x_t \sin(\omega p t)/N}{\sum_{t=1}^{N} x_t (1-t^2)/N} \\
\frac{\bar{x}}{\sum_{t=1}^{N} x_t (1-t^2)/N}
\end{bmatrix}, p \neq N/2, \text{ or } \begin{bmatrix}
\frac{\bar{x}}{\sum_{t=1}^{N} x_t (1-t^2)/N}
\end{bmatrix}, p = N/2.
\] (3.41)

We can now rewrite equation (3.40) in the digitised form, with the error removed by applying the least square approximations for \( \mu, \alpha \) and \( \beta \), i.e.

\[ x_t = \hat{\mu} + \hat{\alpha} \cos (\omega p t) + \hat{\beta} \sin (\omega p t). \] (3.42)

Substituting equation (3.41) into 3.42 and applying some algebra (derived in Appendix E.4) we arrive at the Fourier series representation of \( x \),

\[ x_t = a_0 + \sum_{p=1}^{N/2-1} \left[ a_p \cos(2\pi pt/N) + b_p \sin(2\pi pt/N) \right] + a_{N/2} \cos \pi t \quad t = 1, \ldots, N, \] (3.43)
3.6. STATISTICAL RELEVANCE TO ANALYSIS TECHNIQUES

where

\[
\begin{align*}
    a_0 &= \bar{x} \\
    a_{N/2} &= \sum_{t=1}^{N} (-1)^t x_t / N
\end{align*}
\]

\[
\begin{align*}
    a_p &= 2 \left[ \sum_{t=1}^{N} x_t \cos(2\pi pt/N) \right] / N \\
    b_p &= 2 \left[ \sum_{t=1}^{N} x_t \sin(2\pi pt/N) \right] / N
\end{align*}
\]

\[p = 1, \ldots, (N/2-1).\]  

(3.44)

When \( p \neq N/2 \) we can write the \( p \)th harmonic in the following form:

\[
[a_p \cos \omega_p t + b_p \sin \omega_p t] = R_p \cos(\omega_p t + \phi_p),
\]

\[N \sigma^2 = \sum_{t=1}^{N} \frac{(x_t - \bar{x})^2}{N}.\]  

(3.46)

When using the sum of squares this can be partitioned into two components; one being the sum of squares estimate “explained” by the periodic component at a particular frequency. This is given by

\[
\begin{align*}
    \sigma^2 &= \sum_{t=1}^{N} \frac{(\hat{\alpha} \cos \omega_p t + \hat{\beta} \sin \omega_p t)^2}{N} \\
    N \sigma^2 &= \begin{cases} 
        (\hat{\alpha}^2 + \hat{\beta}^2)N/2, & p \neq N/2 \\
        \hat{\alpha}^2 N, & p = N/2.
    \end{cases}
\end{align*}
\]

(3.47)

Using the trigonometric identities shown in Appendix E.4 with the derivation also in E.4 we arrive at

\[
N \sigma^2 = \begin{cases} 
        (\hat{\alpha}^2 + \hat{\beta}^2)N/2, & p \neq N/2 \\
        \hat{\alpha}^2 N, & p = N/2.
    \end{cases}
\]

(3.48)

Applying the condition \( p \neq N/2 \) and equation (3.44) we have the contribution of the \( p \)th harmonic to the sum of squares;

\[
(a_p^2 + b_p^2)N/2 = R_p^2 N/2
\]

from equation (3.45). Using the trigonometric identities once again and applying them to equation (3.43) we have

\[
\sum_{t=1}^{N} \frac{(x_t - \bar{x})^2}{N} = \sum_{p=1}^{N/2-1} R_p^2 / 2 + a_{N/2}^2.
\]

(3.49)

which is known as Parseval’s Theorem. The derivation of this equation can be found in Appendix E.4. Plotting \( R_p^2 / 2 \) against \( \omega_p = 2\pi p/N \) we get a line spectrum which can be considered a statistical version of the power spectrum. \( R_p^2 / 2 \) can be regarded as the contribution of the \( p \)th harmonic to the variance in the range \( \omega_p \pm \pi/N \). With this information we can plot a histogram with height such that \( R_p^2 \)
is equal to the area of the $p$th histogram rectangle. The rectangle width is given by the range limits of $\omega_p$, i.e. $2\pi/N$. Entering this information we have an expression for the height of each square in the histogram,

$$\text{Height} = I(\omega_p) = \begin{cases} \frac{NR_p^2}{4\pi}, & p = 1, \ldots, (N/2 - 1) \\ \frac{Na_{N/2}^2}{\pi}, & p = N/2. \end{cases}$$ (3.50)

A plot of $I(\omega)$ against $\omega$ can now be produced. This is called the periodogram. The name can often be misleading as it is a function of frequency, not period.

### 3.6.2 Smoothing the Periodogram

One process of statistically validating the data is to perform a process known as smoothing. This is essentially an average of each point in the periodogram with its neighbours (Chatfield, 1989). As each point is being scaled with respect to its consecutive points, any sharp peaks tend to be smoothed out, hence the name. Smoothing is done by grouping the periodogram ordinates in sets of size $m$ and finding their average value. Beginning with the periodogram $I(\omega)$ defined in equation (3.50) we can produce the smoothed periodogram $\hat{f}(\omega)$ using

$$\hat{f}(\omega) = \frac{1}{m} \sum_{j=1}^{m} I(\omega_j),$$ (3.51)

where $\omega_j = 2\pi j/N$ and $j$ varies over $m$ consecutive integers so that the $\omega_j$ values are symmetric about $\omega$. This requires $j$ to be an odd integer. At the extreme points, $\omega = 0$ and $\pi$, we must also assume the $\omega_j$ are symmetric about these values. So

$$\hat{f}(0) = 2 \frac{\sum_{j=1}^{(m-1)/2} I(2\pi j/N)}{m},$$

and

$$\hat{f}(\pi) = \frac{1}{m} \left[ I(\pi) + 2 \sum_{j=1}^{(m-1)/2} I(\pi - 2\pi j/N) \right].$$

The variance of $\hat{f}$ is of order $1/m$.

The value of $m$ is selected by the user and care must be taken to balance resolution against variance. Large values of $m$ reduce the variance but increase the bias on the results, while small values increase the variance, creating problems for statistical efforts. Also, large values of $m$ tend to smooth out features such as peaks, risking the loss of potentially valuable data. A rough value to select for $m$ is around $N/40$ but will vary for each data set. In this project we have chosen a sample window of 30 minutes which corresponds to a re-binned value of $N = 360$. Our chosen value of $m$ was 7 which is approximately $N/25$. 
Statistics are relevant to analysis techniques

3.6. SMoothing the data in the frequency domain is one way to overcome the unity paradox for coherence demonstrated in equation (3.11) and allows statistical validation for cross-phase and power. Smoothing in the frequency domain divides the spectrum into intervals which have width \( m \) and are windowed by a boxcar function. As outlined in section 3.3.2, a boxcar function is Fourier transformed into a sinc function of central lobe width \( 1/(m \Delta t) \), where \( \Delta t \) is the sample rate.

3.6.3 An Alternative Method for Coherence and Phase Determination

In order to preserve the high frequency components of a signal and to maintain statistical reliability an alternative method to smoothing must be used to determine coherence and phase. The technique for coherence determination is to take a series of smaller overlapping windows (hereafter referred to as subwindows) across a given time window, determine the average of the values for \( S_{12}(\omega) \), \( S_{11}(\omega) \) and \( S_{22}(\omega) \) for each subwindow and apply to them equation (3.10). The same technique can be adopted for cross-phase. This technique preserves frequency information as no averaging is performed in the time domain and maintains statistical relevance as performing the average creates parameters which can be analysed statistically. It also resolves the unity paradox from equation (3.11).

The subwindows are of equal size in the time domain, each windowed by a Hanning function and oriented so that they half overlap each other. This creates a “flattened out” Hanning window across the entire data set. The size of each window must allow an integral number of them fit within the FFT length, i.e. \( N \mod \Delta W = 0 \), where \( \Delta W \) is the length of each subwindow. For the windows to half overlap there must be \((2\Delta W - 1)\) windows within the FFT length. This value is therefore the number of parameters which are averaged in the frequency domain as each subwindow contributes one element to the frequency set. It is therefore statistically favourable to use as many windows as possible. However, as the subwindows are all averaged to produce a single value for each frequency counterpart, the resultant FFT length becomes \( \Delta W \). This means that increasing the number of windows reduces the size of \( \Delta W \), thereby reducing the frequency resolution. Care must therefore be taken in determining the size of \( \Delta W \) to optimise statistical favourability and frequency resolution, as well as maintaining the time stationarity of the signal. A general equation for this technique is given below.

\[
S_{12}(\omega_j) = \frac{\sum_{i=1}^{2\Delta W-1} \left[ FFT\{Window_j \times [s_1(t_{j,i})]\} \times FFT\{Window_j \times [s_2^*(t_{j,i})]\} \right]}{2\Delta W - 1},
\]

(3.52)

where \( j = 1, \ldots, \Delta W \); Window is the selected window function and \( s_1, s_2 \) are the
original data sets. Similar formulae can be used in determining $S_{11}$ and $S_{22}$. The frequency resolution becomes

$$\Delta f = \frac{1}{\Delta W \Delta t}.$$ 

Once in this form we can now apply them to equation (3.10) to find coherence. A similar process can be used to confirm previous cross-phase measurements by applying them to equation (3.9).

### 3.6.4 Degrees of Freedom

An important parameter in statistical analysis is the *degrees of freedom* of the data set, denoted $\nu$. This is defined as the number of independent measurements in a population (Jenkins and Watts, 1969). As an example consider a data set $X$ of $n$ points arranged in a normal (Gaussian) distribution with the mean as the centre. Assuming the mean is included in the set there will be $(n - 1)$ independent deviations $(X_t - \bar{X})$ since their sum is zero from the definition of the mean. Therefore, for a Gaussian distribution of $n$ points the number of degrees of freedom is $(n - 1)$.

Various statistical techniques influence the value of $\nu$ in different ways. The smoothing technique outlined in section 3.6.2 assumes a rectangular window of size $m$. The number of degrees of freedom will therefore be $N/m$ where $N$ is the FFT length. The averaged window technique employed in the previous section has a number of degrees of freedom equal to the number of independent windows in the FFT length. This is not equal to the number of windows as these are overlapping and not independent. Thus, for this procedure the value of $\nu$ is $N/\Delta W$.

Windowing a time series also influences the number of degrees of freedom. The spectral windows act as a kind of smoothing agent which decreases variance and increases the value of $\nu$. This is shown in table 3.4. The value of $M$ is the truncation point, which for our purposes represents the size of each window $\Delta W$. Statistically, the Hanning and Tukey windows are almost identical (Chatfield, 1989) and their influence on variance and $\nu$ can also be considered identical.

### 3.6.5 Confidence Limits

We can now assign a value to the statistical significance of our measurements. This value is called the *confidence* and represents a range of values within which we have a percentage confidence that the results are statistically viable (Bendat and Piersol, 1966). For example, in a normal (Gaussian), mean-centred distribution we can say with 95% confidence that an element in the population chosen at random lies within two standard deviations ($\sigma$’s) of the mean. The confidence percentage comes from
Table 3.4: Properties of spectral windows (Jenkins and Watts, 1969)

<table>
<thead>
<tr>
<th>Description</th>
<th>Spectral Window</th>
<th>Variance ratio</th>
<th>Degrees of freedom</th>
<th>Standardized bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>$2M \frac{\sin 2\pi f M}{2\pi f M}$</td>
<td>$\frac{2M}{N}$</td>
<td>$\frac{N}{M}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Bartlett</td>
<td>$M \left( \frac{\sin \pi f M}{\pi f M} \right)^2$</td>
<td>$0.667 \frac{M}{N}$</td>
<td>$3 \frac{N}{M}$</td>
<td>1.5</td>
</tr>
<tr>
<td>Tukey (Hanning)</td>
<td>$M \left( \frac{\sin 2\pi f M}{2\pi f M} \times \frac{1}{1-(2fM)^2} \right)$</td>
<td>$0.75 \frac{M}{N}$</td>
<td>$2.667 \frac{N}{M}$</td>
<td>1.333</td>
</tr>
<tr>
<td>Parzen</td>
<td>$\frac{3M}{4} \left( \frac{\sin(\pi f M/2)}{\pi f M/2} \right)^4$</td>
<td>$0.539 \frac{M}{N}$</td>
<td>$3.71 \frac{N}{M}$</td>
<td>1.86</td>
</tr>
</tbody>
</table>

the area under a normalised Gaussian distribution, the total under which is 1. Each percentage corresponds to a value along the $x$-axis, which is in units of standard deviation $\sigma$, henceforth referred to as $\eta_{1-\alpha/2}$, where the confidence percentage is given by $100(1 - \alpha)\%$.

To clarify this, refer to figure 3.10. This shows a confidence limit of 95%, indicating that the area under the distribution should be 0.95, for a Gaussian set (i.e. a white noise assumption). The limits on the “$x$” axis producing this area must be symmetric about $\tau$, and in this example correspond to the region $[\tau - 2\sigma, \tau + 2\sigma]$. Thus, the value of $\eta$ for this value is 2, or $\eta_{0.525} = 2$. Other values of $\eta$ are listed in table 3.5.

Table 3.5: Some $\eta$ values for various values of $\alpha$ (Bendat and Piersol, 1966).

<table>
<thead>
<tr>
<th>Confidence</th>
<th>$\alpha$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>0.30</td>
<td>1.04</td>
</tr>
<tr>
<td>75%</td>
<td>0.25</td>
<td>1.15</td>
</tr>
<tr>
<td>80%</td>
<td>0.20</td>
<td>1.28</td>
</tr>
<tr>
<td>85%</td>
<td>0.15</td>
<td>1.44</td>
</tr>
<tr>
<td>90%</td>
<td>0.10</td>
<td>1.65</td>
</tr>
<tr>
<td>95%</td>
<td>0.05</td>
<td>1.96</td>
</tr>
</tbody>
</table>

3.6.6 Confidence Limits for Coherence and Phase

Here we derive an expression for the confidence limits for coherence and phase.
Figure 3.10: Determination of the confidence limit for a normalised Gaussian. The example used here is for a confidence limit of 95% which corresponds to a value of $2\sigma$ in the $x$-axis.

Coherence

In order for coherence values to be significant it is necessary to be able to resolve the coherence of “good” signals from those of noise. Thus a confidence limit for coherence must be established so that we can say with a particular confidence that the signals are genuine and not the result of some random process. Consider figure 3.11.

Let $\gamma_a$ and $\gamma_b$ be coherences of noise and a “good” signal respectively. Jenkins and Watts (1969) have indicated that coherence noise values form a Gaussian distribution when the inverse hyperbolic tangent operation is performed on coherence.

Figure 3.11: Two values for coherence. $\gamma_a$ represents the coherence of noise and $\gamma_b$ represents a “good” signal.
So, if we let
\[ Y_a = \tanh^{-1}(\gamma_a) \]
\[ Y_b = \tanh^{-1}(\gamma_b) \]
then the \( Y \)'s will form a Gaussian distribution. Now, let the errors for each value of coherence be \( \Delta \gamma_a \) and \( \Delta \gamma_b \). From Jenkins and Watts (1969), we have
\[
\tanh^{-1}(\Delta \gamma) = \pm \frac{\eta_{1-\alpha/2}}{\sqrt{\nu}}.
\] (3.53)

For a signal to be declared "good" it must be clearly resolved from the noise, so condition 3.54 must be met as indicated in figure 3.11;
\[
Y_b - \frac{\eta_{1-\alpha/2}}{\sqrt{\nu}} \geq Y_a + \frac{\eta_{1-\alpha/2}}{\sqrt{\nu}}.
\] (3.54)

Equating the above we arrive at
\[
Y_b - Y_a = \Delta Y_{ab} \geq \frac{2\eta_{1-\alpha/2}}{\sqrt{\nu}},
\] (3.55)
where \( \Delta Y_{ab} = \tanh^{-1}(\gamma_b) - \tanh^{-1}(\gamma_a) \), \( \eta_{1-\alpha/2} \) is a value related to the confidence area under normal distribution curve and \( \nu \) is the number of degrees of freedom.

We can now produce an algorithm for coherence confidence limit determination. This is the technique used in this project to determine the errors in coherence and is given below. Assume a coherence profile has already been produced with the technique outline in §3.6.3.

1. Determine the appropriate value for \( \eta \) from the desired confidence value (e.g. table 3.5).

2. Determine the number of degrees of freedom from the chosen number of sub-windows. Remember the number of windows should be selected to optimise frequency resolution and degrees of freedom. Remember also that the choice of windowing affects the degrees of freedom, e.g. for a Hanning window the number of degrees of freedom should be multiplied by 2.667 (Jenkins and Watts, 1969). Finally, remember to remove 1 from the final value of \( \nu \) as a result of the exclusion of the mean from the Gaussian noise distribution (section 3.6.4).

3. Determine the value of \( \Delta Y_{ab} \) using equation (3.55).

4. Produce a table with each column as follows:
   - Coherence values \( \gamma \) ranging from 0 to 1 (stepping in intervals of 0.05 or whatever the chosen lower limit for coherence measurements will be).
CHAPTER 3. DATA COLLECTION AND ANALYSIS

- The inverse hyperbolic tangent of each of the coherence values, \( \tanh^{-1}(\gamma) \).
- The limits of \( \tanh^{-1} \) given as an upper and lower limit, determined from \( Y \pm \Delta Y_{ab} \).
- These same limits converted back into limits of \( \gamma \) by performing a \( \tanh \) function on each of the upper and lower limits.

5. Match the appropriate values for \( \Delta \gamma \) with each value of coherence to complete the coherence profile. These will appear as error flags on the plot.

Note that the error flags due to the confidence limits need not necessarily be symmetric about any value of \( \gamma \). Also note that the higher the coherence the smaller the error bars, which is to be expected. Refer to §4.4.2 for an example of this technique applied to actual data.

Phase

A similar procedure can be adopted for the confidence interval of phase, however the equation for determining the phase confidence is complicated and will only be summarised here. From Jenkins and Watts (1969) we have the variance of the windowed phase estimator

\[
Var[\hat{\phi}_{12}] \approx \frac{I}{2N} \left( \frac{1}{\gamma^2 - 1} \right),
\]

where \( I/N \) is the variance ratio defined in table 3.4 and \( \gamma \) is the coherence. For phase, a normal distribution is obtained from the tan of the phase estimator, and we have

\[
Var[\tan \hat{\phi}_{12}] \approx \sec^4 \varphi \frac{I}{2N} \left( \frac{1}{\gamma^2 - 1} \right).
\]

By approximating the distribution of \( \tan \phi_{12} \) by a normal distribution, appropriate confidence intervals can be obtained with confidence coefficient \( 100(1 - \alpha)\% \) from

\[
\tan \hat{\phi}_{12}(f) \pm \eta_{1-\alpha/2} \sqrt{\sec^4 \varphi \frac{I}{\nu} \left( \frac{1}{\gamma^2 - 1} \right)}.
\]  

(3.56)

An algorithm for determining the confidence limits for phase is shown below.

1. Determine the phase \( \phi_{12} \) along with values of \( \eta \) and \( \nu \).

2. Take the tangent of the phase, \( \tan(\phi_{12}) \).

3. Add and subtract the confidence limits described in 3.56 to get the upper and lower bounds for \( \tan(\phi_{12}) \). Note that the value of coherence should be entered here also.
4. Take the inverse tangent of these limits to get the real limits for phase.

5. Represent these on the phase profile as error bars.
CHAPTER 4

ANALYSIS OF OBSERVATIONAL DATA

The analytical techniques discussed in the previous chapter were applied to 170 events recorded during January and March, 1998. In order to illustrate the techniques employed, a single representative event has been selected and is presented in this chapter with details on each step taken throughout the analysis procedure. Table 4.1 gives a summary of the programs used in this analysis and their functions.

The event chosen here occurred on January 10, 1998 and is number 8 of the January events listed in Appendix B.1. It is a typical Pc 3–4 event, with a frequency of $20.9 \pm 1.67 \text{ mHz}$ and occurred between 0550–0605 UT, or 0720–0735 LT at IMAGE (local time at IMAGE is UT +90 minutes).

4.1 Timing Error Correction

The IMAGE magnetometers record data at 10 s and are generally used for studies of substorm current systems. In the last couple of years the timing mechanisms for the IMAGE magnetometers have been upgraded to GPS timing. In 1998, however, the clocks used were not as accurate, and a few stations were offset by several seconds. The magnetometer at Davis station introduce a calibration signal to the data at 1630 UT each day to assist in problems such as timing errors between stations. No such artificial signal was recorded with the IMAGE stations, so we must estimate timing errors using an estimate of the baseline noise for each station.

Timing errors appear in the frequency domain as a sloping trend in phase. This can be easily corrected by offsetting the starting point of the offending time series. In the case where this offset is not a multiple of the 10 second sampling rate, an appropriate interpolation of the data is employed. The difficulty lies in determining the amount of the offset required since it is often difficult to distinguish between a genuine timing error and a natural property within the data.

In this project timing errors were determined using cross-phase spectra before event selection took place. In the whole-day dynamic cross-phase spectra, timing errors were identified by a shift in colour (phase) with frequency. This is shown in figure 4.1a, which compares the offending station with one without a known timing error. For the static spectra, FFT windows of three hours were used to identify timing errors from the slope in phase. Figure 4.1b shows an example for the same station pair as was used for the dynamic phase.
Table 4.1: IDL programs used and their input variables and products. Each was produced by the author, excepting Pltphx.pro, originally developed by F.W. Menk, but altered by the author. Input files constitute 24 hours of data.

<table>
<thead>
<tr>
<th>Program</th>
<th>Inputs</th>
<th>Intermediate steps</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stakts.pro</td>
<td>Data filenames, event start &amp; end times, filter bandwidth.</td>
<td>Running mean removal, bandpass filter, amplitude scaling.</td>
<td>Stacked time series for any group of stations.</td>
</tr>
<tr>
<td>DPow.pro</td>
<td>Data filenames, date, power limits.</td>
<td>Spike and 2nd order polynomial removal, Hanning window, spectral weighting.</td>
<td>Whole-day Dynamic power (in dB).</td>
</tr>
<tr>
<td>DPhase.pro</td>
<td>Data filenames, date, phase limits.</td>
<td>Spike removal, Hanning window.</td>
<td>Whole-day Dynamic cross-phase spectrum.</td>
</tr>
<tr>
<td>Pltphx.pro</td>
<td>Data filename, event start &amp; end time, start offset, maximum freq.</td>
<td>Bandpass filter: 5–50 mHz, data rebin, data spike and running mean removal, Hanning window, spectral weighting, smoothing.</td>
<td>Plots of amplitude, with time; cross-power, coherence, cross-phase with frequency.</td>
</tr>
<tr>
<td>GUP.pro</td>
<td>Data filenames, start &amp; end time, timing corrections, frequency, frequency resolution.</td>
<td>Bandpass filter: 5–50 mHz, data spike and running mean removal, Hanning window, spectral weighting, averaged window power.</td>
<td>Plots of amplitude, cross-phase and coherence with latitude and longitude for entire IMAGE array.</td>
</tr>
<tr>
<td>GUPPol.pro</td>
<td>Same as for GUP.pro.</td>
<td>Same as for GUP.pro</td>
<td>Stokes parameters with latitude and longitude for entire IMAGE array.</td>
</tr>
<tr>
<td>Velocity.pro</td>
<td>File created in GUP containing eq’ns of lines of fit through cross-phase plots.</td>
<td>Gradient and y-intercept extraction.</td>
<td>Ground velocity and m-number for H- and D-components.</td>
</tr>
</tbody>
</table>
4.1. TIMING ERROR CORRECTION

Figure 4.1: Timing errors in the data as shown in the cross-phase spectrum for the KIL:TRO pair on January 10, 1998. a) Dynamic cross-phase with an FFT length of 300 points and overlapping windows stepping every 20 points. The cross-power low cutoff is -20 dB and the cross-phase limits are ±180°. Frequency resolution is 0.33 mHz. b) Static phase with an FFT window of 180 minutes, from 0300 to 0600 UT for the KIL:TRO pair (left) without any change to the timing, and (right) after the TRO time series has been advanced by 17 seconds.

To correct these errors linear interpolation was performed on the data, allowing for corrections by any fraction of the sample rate. The data start-point offsets were increased by increments of 1 second until no slope was visible in the static cross-phase spectrum. This was then confirmed using the same correction in the dynamic spectrum. The timing corrections required for the IMAGE stations are summarised in table 4.2. Even with these corrections the phase at AND was still considerably different compared to other IMAGE stations. It is unclear whether this was due to
a naturally occurring phenomenon such as ground induction effects (§2.7) or timing. While the coherence value was used for the longitude plots later the phase for AND was omitted in this study.

Table 4.2: Timing corrections to the IMAGE stations. The time indicated is the amount by which the start point of each station’s time series was adjusted.

<table>
<thead>
<tr>
<th>Station</th>
<th>Timing correction</th>
<th>Jan</th>
<th>Mar</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOR</td>
<td>+5 s</td>
<td>+10 s</td>
<td></td>
</tr>
<tr>
<td>TRO</td>
<td>+17 s</td>
<td>+6 s</td>
<td></td>
</tr>
<tr>
<td>AND</td>
<td>+5 s</td>
<td>+5 s</td>
<td></td>
</tr>
<tr>
<td>MUO</td>
<td>—2 s</td>
<td>—2 s</td>
<td></td>
</tr>
<tr>
<td>ABK</td>
<td>+5 s</td>
<td>+5 s</td>
<td></td>
</tr>
<tr>
<td>UPS</td>
<td>+5 s</td>
<td>+5 s</td>
<td></td>
</tr>
</tbody>
</table>

4.2 Event Selection

Event selection was based on amplitude, power, cross-power and coherence properties. This project focused specifically on pulsation signals exhibiting high coherence over great distance since these are likely signatures of ULF waves propagating through the magnetosphere. In order to be classified as an event suitable for further analysis it was required for the signal to meet all of the following conditions for the same frequency and time interval:

1. A sinusoidal signal lasting for at least 4 cycles must be observed on most of the bandpass filtered IMAGE stations’ time series.

2. A clear peak in power must be observed at a particular frequency on the whole-day dynamic power spectrum.

3. A peak in coherence (> 0.65, after Olson and Szuberla, 1997 and §3.4.2) at the same frequency as the dynamic power peak must be observed across at least three pairs of stations. The station pairs should be at high, middle and low latitudes and span the entire IMAGE array.

4. A corresponding peak in cross-power must be observed across the same three pairs of stations.
4.2. EVENT SELECTION

Figure 4.2 illustrates the selected example event. Panel a) shows half hour time series filtered between 15 and 30 mHz and was produced with the program Stakts.pro (table 4.1). The ULF wave event is the series of regular sinusoids roughly between 0550 and 0605 UT. Note that the series tends to move forward in time as we move poleward (indicated by the arrow). This indicates that the signal arrives on the ground at the equatorial stations first and moves poleward. The time delay is best represented on a phase with latitude profile, shown later.

Panel b) shows the whole-day dynamic power spectrum at KIL for January 10 as produced by DPow.pro (table 4.1). There are three points to note in this plot. First, the regions of high intensity, broadband signals from 2000–2200 UT correspond to substorm activity and in the local night sector. Second, the low-frequency (∼ 8 mHz), high-power signal spanning 0200–1300 identifies the field line resonance (FLR) at this latitude. This is more clearly represented in the dynamic cross-phase spectrum, shown later. Finally, the packet of activity at 0550–0605 UT and 20 mHz is the chosen event (indicated by a circle).

Panels c) show time series for three pairs of IMAGE stations along with plots of static cross-power, coherence and cross-phase with frequency, as produced in the program Pltxph.pro (table 4.1). In this case, coherence was determined by first smoothing the data with a \( m \) value of 7 (refer to §3.6.2). For event selection we examine the coherence and cross-power spectra. The left hand panel represents the high-latitude region (LYR:NAL), and shows coherence \( \gamma \geq 0.65 \) across the frequency band 10–24 mHz. A peak in coherence and cross-power occur near at 21 mHz. For the mid latitudes (KIR:KIL, the centre panel), the frequency band for \( \gamma \geq 0.65 \) is 13–26 mHz and the peaks in coherence and cross-power are at 20–21 mHz. For the low-latitude pair (OUJ:PEL, the right hand panel) \( \gamma \geq 0.65 \) for the frequency band 17–25 mHz and the peaks in coherence and cross-power occur at 21 mHz. There seems to be a stable, coherent signal across all three regions of the array at an average of 20.9 ± 1.67 mHz. This therefore meets the selection criteria for an event and is set aside for further analysis.

Dynamic power activity in the Pc 3–4 frequency range was identified during each day of both months January and March, 1998. We began by locating signals which met the above criteria across any two pairs of IMAGE stations in either of the three latitude regions (e.g. high-mid, mid-low latitudes), of which 1359 events were identified. We then restricted our selection to only those signals which were observed in all three latitude regions, from which 170 events were extracted. These 170 events are shown in appendix B.1 and formed the event database for this study.
Figure 4.2: Plots produced for the event selection process. 
a) Stacked time series of the January 10 interval from 0545—0615 UT bandpass filtered at 15–30 mHz. Each plot is labeled with the IMAGE station with its CGM latitude. The selected event is indicated by vertical dashed lines and the arrow indicates the direction of motion of the wave. 
b) Dynamic power spectrum for Kilpisjärvi, produced using overlapping windows stepping every 15 minutes and with a 2nd-order polynomial fit removed from the whole-day plot. The circle indicates the selected event. 
c) Amplitude against time (top), bandpass filtered at 5–50 mHz; and cross-power (2nd), coherence (3rd) and cross-phase (bottom) with frequency. Plots are given for three station pairs representing high (NAL:LYR, left), mid (KIR:KIL, middle) and low (OUJ:PEL, right) latitudes. Vertical dashed lines identify the example signal.
Those events selected have been classified into three categories, depending on their appearance on dynamic power spectra. The term ‘narrow’ of which the example event in this chapter belongs, refers to an event which stands alone, i.e. a relatively high intensity power packet with a low-power background. A ‘packet’ event is one which is part of a larger group of power packets, and ‘broad’ events are embedded in a region of broad-band noise. The latter mostly occurred during the local night and were classed as Pc 3–4 events because of their appearance in time series records, but we remain sceptical of this classification. They will be discussed later.

4.3 Dynamic Cross-Phase

Another technique to identify stable cross-phase features in the spectrum is the dynamic cross-phase (section 3.4.1). While only used as an aid for timing and an additional guide for event selection in this project, dynamic cross-phase is useful in identifying FLR’s and their harmonics (e.g. Waters, 2000). Figure 4.3 shows the dynamic cross-phase spectrum for the mid-latitude pair KIR:KIL for the example event, created using the program DPhase.pro. When compared to the dynamic power spectrum in figure 4.2 a few features become clear. The first is the band of activity at ~ 20°, beginning at 0000 and continuing until 1300 UT. The shape resembles an arch which peaks at ~ 11 mHz. This is the signature of the high-latitude FLR which has been discussed in chapter 2. The second is the patch at 0600 UT and 20 mHz (indicated by the circle). It has a phase of ~ 20° and corresponds to the time and frequency of the example event. Finally, the highly variable phase distribution across time 1900–2100 UT are in accordance with the substorm activity, also indicated in the dynamic power spectrum.

4.4 Amplitude, Phase and Coherence Profiles and Errors

The program GUP.pro was used to produce plots of amplitude, phase and coherence for the H- and D-components. This program used the input parameters described in table 4.1 and performed the analysis techniques outlined in chapter 3 to produce the profiles. Two techniques were used to achieve this. “Technique 1” employed the averaged window technique outlined in section 3.6.3. For all of the 170 events a 30 minute window was chosen, with a frequency resolution of 3.33 mHz, creating a series of 11 overlapping windows, each of 30 data points (5 minutes). For example, for the January 10 example event the window spanned the times 0545 to 0615 UT.
CHAPTER 4. ANALYSIS OF OBSERVATIONAL DATA

Figure 4.3: Dynamic cross-phase for the KIR:KIL pair for January 10, 1998. The FFT length was 300 points and windows step every 20 points. The cross-power low cutoff is $-20$ dB and the cross-phase limits are $\pm 90^\circ$. Frequency resolution is 0.33 mHz.

With a resolution of 3.33 mHz, the closest frequency to that of the event was $20.0 \pm 1.7$ mHz. “Technique 2” produced a single spectrum of the entire 30 minute window event and was used for amplitude and cross-phase, the latter only as a test. This technique significantly improved the frequency resolution (0.56 mHz) but coherence calculations required a smoothing with a window length of 7 points (70 s). Also cross-phase calculations were made across the entire window, thus resulting in only 1 degree of freedom. Section 3.6.2 explains the problems associated with using such a technique for statistical quantities. Amplitude, coherence and cross-phase were determined using both techniques and the most precise estimate of the spectral parameters was made. Technique 1 was used for coherence and cross-phase and technique 2 for amplitude.

In both cases a high-pass filter was applied at 1 mHz to remove any dc contribution and data spikes were removed by normalising any point which was significantly different to its adjacent neighbours in the time series. A low-pass filter at the Nyquist frequency (50 mHz) prevented aliasing, and timing error corrections were made by altering the start point of the data set for the ‘problem’ stations as shown in table 4.2. A Hanning window was then applied before performing the FFT. With technique 1 this occurred for each subwindow, thus creating 11 values for each frequency once the FFT was applied. These were then averaged to form a
single value for each frequency. For technique 2 the window was applied across the entire 30 minute interval.

The following describe the production of the five latitude and longitude profiles, shown in figures 4.6 and 4.7.

4.4.1 Amplitude Profile

Amplitude profiles were obtained from the power spectrum using the technique outlined in section ???. In the example event power was obtained at the point in the frequency domain corresponding to 20.9 mHz and the following actions performed.

1. ‘Total power’ for the event was determined by adding power from the frequencies in the band ±2.5 mHz centred at 20.9 mHz.

2. A mathematical consequence of the FFT is the creation of multiple peaks centred at zero (§3.3.2). The application of a low-pass filter at the Nyquist frequency removes one of these peaks and hence divides the power of the signal at $f$ by 2. This power must therefore be restored by multiplying the power obtained in our analysis by $2^2$. The index of 2 is required because of the squared nature of power.

3. Power loss due to the application of the Hanning window was returned by dividing by 0.374629. This value was obtained after several tests of the effects of Hanning windows with test signals of varying amplitudes and frequencies.

4. The square root was taken to put the units back into nT.

For the example event Masi (MAS) was ignored as data from this station were missing for this day. The errors in amplitude determination are given by the resolution and calibration of the magnetometers used for measurement. In 1998 the IMAGE stations had a quoted resolution of 0.1 nT, giving an error of ±0.05 nT.

The amplitude with latitude profile shown in figure 4.6a is for the example event on January 10. Amplitudes for the H- and D-components are shown, plotted as a function of CGM (epoch = 1998) latitude with the error bars representing the magnetometer limit of reading as discussed previously. Two features are of note here. First, is the peak in the H-component amplitude at the point corresponding to Hopen Island (HOP). This peak also appears for the D-component, but it is not as sharp as that of H. The second feature is the dip in amplitude for both components between $64^\circ$ and $67^\circ$. This will be discussed later.
4.4.2 Coherence Profile

Production of the coherence latitude and longitude profiles involved simply plotting the coherence values determined for each station compared with Kilpisjärvi (KIL), as produced by technique 1. The indicated errors represented the statistical confidence limits described in §3.6.6. These were determined using equation (3.56) and the procedure described in section 3.6.6, which involved the following:

1. The chosen confidence limit is 87% for which \( \eta_{0.93} = 1.5 \) (Rajgopal, 2002);

2. Choosing our upper (\( \gamma_b \)) and lower (\( \gamma_a \)) limits of coherence as 0.65 and 0 (3.6.6) respectively, we can rearrange equation (3.55) to find the degrees of freedom \( \nu \), such that

\[
\nu = \left[ \frac{2 \times \eta_{1-\alpha/2}}{\tanh^{-1}(\gamma_b) - \tanh^{-1}(\gamma_a)} \right]^2 = \left[ \frac{2 \times 1.5}{\tanh^{-1}(0.65)} \right]^2 = 15, \tag{4.1}
\]

thus requiring 15 degrees of freedom. As mentioned before, the chosen frequency resolution of 3.33 mHz and a 30 minute window produces 11 overlapping windows. The degrees of freedom here are dependent on the number of independent windows, in this case 6. The Hanning function applied to each subwindow affects the number of degrees of freedom as discussed in §3.6.4 and this is represented by multiplying by 2.667, bringing the value of \( \nu \) to 16. By removing the mean from the Gaussian noise distribution as demonstrated in section 3.6.4 we arrive at a value of \( \nu = 15 \). This satisfies the number of degrees of freedom representing with 87% confidence that a signal with coherence above 0.65 is distinguished from the noise (to which we have designated a coherence value of 0);

3. From equation (3.55) the value of \( \Delta Y_{ab} = \tanh^{-1}(0.65) - \tanh^{-1}(0) \sim 0.775 \), or \( Y \pm 0.388 \);

4. The last two columns of table 4.3 represent the upper and lower limits of coherence given the above value of \( \Delta Y_{ab} \);

5. These values must now be assigned to the profile. For example, the first two values of H-component coherence for the coherence with latitude plot for the example event are 0.87 (UPS) and 0.89 (NUR) (figure 4.6b). These are approximately equal to 0.85 and 0.90 and thus the values will be represented by the following:

UPS: \( 0.87^{+0.06}_{-0.15} \); NUR: \( 0.89^{+0.05}_{-0.11} \).
These uncertainties are represented by the error bars in each case in figure 4.6.

Table 4.3: Upper and lower limits of coherence in increments of 0.05. These represent the 87% confidence intervals for coherence limits of 0 to 0.65, with 15 degrees of freedom (Jenkins and Watts, 1968).

<table>
<thead>
<tr>
<th>Coherence</th>
<th>tanh(^{-1}) of Coherence</th>
<th>tanh(^{-1}) Limits</th>
<th>Coherence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper</td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.39</td>
<td>-0.39</td>
</tr>
<tr>
<td>0.05</td>
<td>0.05</td>
<td>0.44</td>
<td>-0.34</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01</td>
<td>0.49</td>
<td>-0.29</td>
</tr>
<tr>
<td>0.15</td>
<td>0.15</td>
<td>0.54</td>
<td>-0.24</td>
</tr>
<tr>
<td>0.20</td>
<td>0.20</td>
<td>0.59</td>
<td>-0.18</td>
</tr>
<tr>
<td>0.25</td>
<td>0.26</td>
<td>0.64</td>
<td>-0.13</td>
</tr>
<tr>
<td>0.30</td>
<td>0.31</td>
<td>0.70</td>
<td>-0.08</td>
</tr>
<tr>
<td>0.35</td>
<td>0.37</td>
<td>0.75</td>
<td>-0.02</td>
</tr>
<tr>
<td>0.40</td>
<td>0.42</td>
<td>0.81</td>
<td>0.04</td>
</tr>
<tr>
<td>0.45</td>
<td>0.48</td>
<td>0.87</td>
<td>0.10</td>
</tr>
<tr>
<td>0.50</td>
<td>0.55</td>
<td>0.94</td>
<td>0.16</td>
</tr>
<tr>
<td>0.55</td>
<td>0.62</td>
<td>1.01</td>
<td>0.23</td>
</tr>
<tr>
<td>0.60</td>
<td>0.69</td>
<td>1.08</td>
<td>0.31</td>
</tr>
<tr>
<td>0.65</td>
<td>0.78</td>
<td>1.16</td>
<td>0.39</td>
</tr>
<tr>
<td>0.70</td>
<td>0.87</td>
<td>1.25</td>
<td>0.48</td>
</tr>
<tr>
<td>0.75</td>
<td>0.97</td>
<td>1.36</td>
<td>0.59</td>
</tr>
<tr>
<td>0.80</td>
<td>1.10</td>
<td>1.49</td>
<td>0.71</td>
</tr>
<tr>
<td>0.85</td>
<td>1.26</td>
<td>1.64</td>
<td>0.87</td>
</tr>
<tr>
<td>0.90</td>
<td>1.47</td>
<td>1.86</td>
<td>1.08</td>
</tr>
<tr>
<td>0.95</td>
<td>1.83</td>
<td>2.22</td>
<td>1.44</td>
</tr>
<tr>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
Figures 4.6b and 4.7a show the coherence profile with latitude and longitude for the H- and D-components. In each case the coherence values were determined with a common reference station, in this case Kilpisjärvi (KIL). The point corresponding with this station (latitude 65.8°, longitude 104.3°, shown in the plots) therefore has a coherence of 1. As the points move away from this central station in both directions the coherence decreases, and we have assigned a 2nd-order polynomial fit to the plots for both components. The horizontal dashed lines represents the lower frequency cutoff at $\gamma = 0.65$. Note that in both the latitude and longitude plots each point is above this cutoff limit, with the polynomial fit for the latitude profile crossing somewhere $< 51^\circ$ and $\sim 76^\circ$. Also note that the y-axis for the longitude plot does not begin at 0, but rather the 0.65 cutoff. The absence of error bars for this plot are because the coherence values all lie between 0.95 and 1, and, as shown in table 4.3, errors for those are negligible.

Finally, it is important to note that coherence is a localised parameter and the profiles were found to change if a different central station was selected. An example of this is given in figure 4.4. Note the width of the polynomial fit decreases for the high latitude-stations. This is because the increase in the noise level near the cusp decreases the signal-to-noise ratio of the signal, and thus the coherence. Also, we have found that the spacing between each station can also influence the coherence values. To overcome these problems our coherence length calculations were made using station pairs of comparable interstation distance and a mathematical assumption for the coherence profiles. This will be discussed further in section 4.6.1.

### 4.4.3 Cross-Phase Profile

Cross-phase was determined using techniques 1 and 2 but only the values from technique 1 were plotted on the profiles. Those obtained using technique 2 were used as a test for consistency of phase trends and to help resolve $2\pi$ ambiguities. The latter posed the largest problem. For the data analysis software package we used (IDL), the domain for trigonometric functions is $[-\pi, \pi]$ and so phase values outside this domain will be wrapped to a value within. For example, a phase value of $3\pi/2$ will be registered by IDL as $-\pi/2$. Two problems arise. The first is the unwrapping of phase steps in the program output and the second is the resolution of cross-phase values between a pair of stations. It is sometimes unclear as to whether an interstation cross-phase value is $\varphi$ or $\varphi \pm 2\pi$. To overcome this problem, a technique known as phase closure was employed.
Figure 4.4: Coherence profiles with a different central station at a) LYR, b) KIL and c) OUJ for the H-component of the event on January 10.
Phase closure involves the measurement of phase between two closely-spaced stations, stepping through adjacent stations and then comparing the total phase with that of the phase between two stations which are more distant. With the IMAGE array for example, another method of measuring the phase between Kiruna (KIR) and Kilpisjärvi (KIL) would be to measure the phase between KIR and MUO, MUO and ABK, and ABK and KIL, adding the phase values and comparing these to the KIR-KIL cross-phase. Alternatively, one could produce multiple phase profiles for the same event, each beginning with a different central station. The blue-diamond plots in figure 4.5 show this technique for the January 10 event.

Phase results in this thesis are quoted relative to a reference station at KIL. In the case of the January 10 example event however, there is a discrepancy in the technique 1 phase between Hornsund and Hopen Island (the 3rd and 4th points from the right). There is clearly a 2π ambiguity here but it is unclear which stations should be corrected. To aid in this we consider the profiles centred at LYR (figure 4.5a, the second point from the right) and OUJ (figure 4.5c, at around latitude 64°). Since the coherence of the signal decreases with distance only the stations nearest the reference point should be regarded. Figure 4.5a suggests that the high-latitude phase values should be grouped together and figure 4.5c suggests the same for the low-latitude values. The phase at BJN and HOP (5th and 4th points from the right) relative to KIL (figure 4.5b) are both lower than the main body. Also, the high-latitude group of points has a trend which decreases with increasing latitude. This implies that the entire high-latitude group should be affected by the 2π ambiguity, requiring a downward shift of the points.

Another way to check for phase structure and 2π ambiguities is to use the phase profile produced using technique 2, shown as the red circles in figure 4.5. In both panels b) and c) the high-latitude points have a lower phase than the main body of points, with the exception of NAL (the 1st point on the right). This point represents the largest distance from both central stations and so it is logical to assume that this point has been wrapped into the $[-\pi, \pi]$ domain. In figure 4.5a, where the high-latitude station is the reference there are four points which deviate from the main body in the low latitudes. Again these points are some distance from LYR and do not correspond to the distribution in the other two points. Thus these points should have 2π added to restore them to their true values. Also, the similarity between the phase values derived with both techniques validates the values we have used to produce the plot.
Figure 4.5: Cross-Phase profiles with latitude for the H-component January 10 event. The three profiles use reference stations at Longyearbyen (panel a), Kilpisjärvi (panel b) and Oulujärvi (panel c). Phase is zero at these reference stations. The phase values derived from techniques 1 and 2 are shown.
Once the corrections have been made the three profiles would look almost identical in shape, as shown in figure 4.6b. The above procedure was then also applied to the D-component for each event.

When determining the phase with longitude profiles it was necessary to account for the variation in phase due to differences in latitude of the stations. As the longitude stations are all located on the Scandinavian mainland, a linear fit was made through the points corresponding to the stations here (i.e. equatorward of SOR). Once made a phase value for any latitude was achieved by simply entering the latitude value into the equation describing the linear fit with phase. This value was then subtracted from the longitude phase values to extract the latitude phase from the longitude measurement. The same procedure was also applied to the D-components. Table 4.4 shows the corrections made with the example event on January 10.

Table 4.4: Longitude profile cross-phase corrections to account for phase differences in latitude. Here the H-component only is considered. The last three columns represent the phase determined a) from the measurement with the analysis program, b) from the linear equation of phase with latitude, c) the new phase once corrected. The values in the last column were the ones plotted as the phase profile with longitude.

<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude</th>
<th>Measured</th>
<th>Equation</th>
<th>Corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABK</td>
<td>65.21°</td>
<td>-39°</td>
<td>+4.54°</td>
<td>-43.54°</td>
</tr>
<tr>
<td>KIL</td>
<td>65.81°</td>
<td>0°</td>
<td>+3.11°</td>
<td>-3.11°</td>
</tr>
<tr>
<td>KEV</td>
<td>66.23°</td>
<td>8°</td>
<td>+2.11°</td>
<td>+5.89°</td>
</tr>
</tbody>
</table>

Uncertainty in the cross-phase determination arises from two factors. The first relates to the confidence limits for the given degrees of freedom, as for coherence. The determine this, values of the number of degrees of freedom \( \nu \), the confidence limit and its corresponding value of \( \eta_{1-\alpha/2} \), and the coherence value \( \gamma \) for each phase measurement were required. As mentioned in the previous section, for the example event the values of \( \nu \) and \( \eta_{1-\alpha/2} \) were 15 and 1.5 respectively. Table 4.5 shows the cross-phase confidence uncertainties for some stations for the H-component for the example event.

The second source of uncertainty is due to the accuracy of the timing of the signal itself. It was difficult to correct timing errors with precision better than a
few seconds. We have assumed a timing error of $\pm 2.5$ seconds, which for the example event at frequency 20.9 mHz corresponds to a phase error of $\pm 18.6^\circ$. As shown in table 4.5, the timing errors were usually higher than the statistical uncertainty, and so in almost all cases the error bars shown on phase plots represent the former value.

Table 4.5: Cross-Phase error determination for the first 4 points of the H-component latitude profile for the Pc 3–4 Event on January 10, 1998. The two columns labelled “$\varphi$ limits” refer to the confidence limits for phase. The last two columns represent the limits from the timing error of $\varphi \pm 18.6^\circ$.

<table>
<thead>
<tr>
<th>Station</th>
<th>$\gamma$</th>
<th>$\varphi$</th>
<th>tan $\varphi$</th>
<th>tan $\varphi$ limits (eq’n 3.56)</th>
<th>$\varphi$ limits</th>
<th>Timing limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Upper</td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td>UPS</td>
<td>0.87</td>
<td>9.21</td>
<td>0.16</td>
<td>0.39</td>
<td>-0.06</td>
<td>21°</td>
</tr>
<tr>
<td>NUR</td>
<td>0.89</td>
<td>32.2°</td>
<td>0.63</td>
<td>0.91</td>
<td>0.35</td>
<td>42°</td>
</tr>
<tr>
<td>HAN</td>
<td>0.89</td>
<td>33.3°</td>
<td>0.66</td>
<td>0.95</td>
<td>0.37</td>
<td>43°</td>
</tr>
<tr>
<td>OUJ</td>
<td>0.96</td>
<td>11.0°</td>
<td>0.19</td>
<td>0.31</td>
<td>0.08</td>
<td>17°</td>
</tr>
</tbody>
</table>
Figure 4.6: a) Amplitude, b) coherence and c) cross-phase profiles with CGM latitude for the Pc 3–4 event on January 10, 1998 at 0545–0615 UT. The frequency resolution was 3.33 mHz and the measured frequency of the signal was at 20.9 mHz. The curves of fit through the coherence and cross-phase are 2nd-order and linear polynomial fits.
Figure 4.7: a) Coherence and b) cross-phase with CGM longitude for the Pc 3–4 event on January 10, 1998 at 0545–0615 UT. The frequency resolution was 3.33 mHz and the measured frequency of the signal was at 20.9 mHz. The curves of fit through the coherence and cross-phase are second-order and linear polynomial fits. Note that the ordinate values for both differ from those in the latitude profiles in figure 4.6.

To summarise, the main features from the plots shown in figures 4.6 and 4.7 are listed below.

1. The peak in amplitude at latitude $\sim 73^\circ$. This is to be compared later with models and satellite data which estimate the location of the cusp region ($\S$4.7).

2. The values of coherence shown are used to calculate coherence length. The intersection of the polynomial fit through the coherence plots with the $\gamma = 0.65$ cutoff was used as a first estimate ($\S$4.6.1).

3. The linear fit through the cross-phase plots were used to calculated the ground
speed and angle. This required both the phase from the latitude and longitude plots (§4.6.3) and was only used for the points on the Scandinavian mainland.

4. The gradient of the phase-longitude plot is the m-number (§4.6.2)

4.5 Polarization Profiles

As for the amplitude, coherence and cross-phase, the polarization parameters were also determined for the latitude and longitude ranges spanned by the IMAGE array. This involved plotting the four Stokes parameters, outlined in section 3.5.2 and the degree of polarization. These parameters were calculated by a version of GUP.pro designed for polarization; GUPPol.pro (table 4.1), in the following manner (after Fowler et al., 1967):

1. **Trace Power** is the sum of the orthogonal power spectra, given by
   \[ \text{trace} = S_{xx} + S_{yy} \] (equation 3.36), i.e. the sum of the averaged power spectra (equation 3.52) for the H- and D-components for each station.

2. **Polarized Power** is the regular polarized contribution to the trace power, defined by
   \[ \text{pol} = \sqrt{(S_{xx} - S_{yy})^2 + 4|\text{Re}\{S_{xy}\}|^2 + 4|\text{Im}\{S_{xy}\}|^2} \] (equation 3.37).

3. **Ellipticity** is the ratio of the major and minor axes of the polarization ellipse,
   \[ \text{ell} = \tan \left[ 0.5 \sin^{-1} \left( \frac{-2\text{Im}\{S_{xy}\}}{\text{pol}} \right) \right] \] (equation 3.38).

4. **Azimuth** is the orientation of the major axis of the ellipse,
   \[ \text{azim} = 0.5 \tan^{-1} \left( \frac{2\text{Re}\{S_{xy}\}}{(S_{xx} - S_{yy})} \right) \] (equation 3.39).

5. **Degree of Polarization** is the ratio of the polarized power to the trace power,
   \[ \text{deg} = \frac{\text{pol}}{\text{trace}} \] given by deg = pol/trace. While not a Stokes parameter this is still a useful quantity.

Latitude and longitude profiles for all these parameters for the example event are shown in figure 4.8.
Figure 4.8: Polarization parameter profiles with latitude and longitude for the \( \text{Pc3-4} \) event on January 10. Plots are given for the 4 Stokes parameters (trace and polarized power, ellipticity and azimuth) as well as the degree of polarization for each IMAGE station.
118

CHAPTER 4. ANALYSIS OF OBSERVATIONAL DATA

Things to note from the plot given in figure 4.8 are:

1. The trace power and polarized power show a similar profile. This results in a relatively high degree of polarization (\( \sim 80\% \)), which remains fairly constant throughout the array.

2. The change in sign of ellipticity at \( \sim 70^\circ \) latitude indicates a polarization reversal around this region. This was later interpreted as a signature of a possible FLR harmonic for the example event (§6.2.1).

3. The relatively low ellipticity and azimuth values indicates the signal is largely linearly polarized.

4.6 Determination of Wave Properties From the Profiles

4.6.1 Coherence Length

Two techniques were used to determine the coherence length. The first was based on the profiles of coherence with latitude and longitude. As shown in figures 4.6b and 4.7a, the value of coherence drops away in an approximately symmetric fashion from the reference point (where by definition coherence = 1). We should therefore be able to fit a 2nd-order polynomial curve to the coherence profiles, also shown in figures 4.6b and 4.7a. The chosen cutoff point for coherence was 0.65, so by measuring the latitudes/longitudes corresponding to this cutoff value (best obtained using the equation of the curve of best fit) we can achieve a geographic angular distance for the coherence length. To convert to km we use the equation for the arc length of a circle, given by

\[
l = r\theta, \tag{4.2}
\]

where \( l \) is the length of the arc (across the Earth’s surface), \( r \) is the radius and \( \theta \) is the angular difference in radians. The latitude circle (circle of constant longitude) was taken at the mean latitude for the IMAGE stations, i.e. \( \lambda = 69.04^\circ \). The radius of this circle, defined by

\[
R_e = R_E \cos \lambda, \tag{4.3}
\]

where \( R_E = 6328 \) km, the radius of the Earth, was thus \( R_e = 2263 \) km.

For the example event, the equation of the 2nd-order curve of best fit for the H-component coherence vs latitude profile was

\[
\gamma = (-2.10 \times 10^{-3})\lambda^2 + (2.68 \times 10^{-1})\lambda - 7.59. \tag{4.4}
\]

By employing the quadratic equation formula for the function \( a\lambda^2 + b\lambda + C = 0; \)

\[
\lambda = \frac{-b \pm \sqrt{b^2 - 4aC}}{2a}, \tag{4.4}
\]
where $C = c - \gamma$ and $c$ is the constant term of the quadratic equation $\gamma = a\lambda^2 + b\lambda + c$. Setting $\gamma = 0.65$, we have two solutions, $\lambda = 76.5$ and $\lambda = 51.3$. The difference in latitude is $25.2^\circ$, which is equivalent to a distance of $2.78 \times 10^3$ km. Polynomial fits can also be made for the upper and lower limits of the error, thus determining the coherence length limits. In this fashion the upper and lower limits were $5.06 \times 10^3$ and $0.94 \times 10^3$ km. An identical procedure was then made for the D-component coherence with latitude profile, with upper, middle and lower values of $3.93 \times 10^3$, $2.90 \times 10^3$ and $1.29 \times 10^3$ km.

The above technique is adequate when there is a high signal-to-noise ratio but problems arise when there is a change in the background noise between events. When the above procedure was applied during this project there was a large number of events with a coherence length of approximately $200$ km, which corresponds to the field of view of the magnetometer instrument due to ionospheric spatial integration (§2.7.2). This was an indication that the coherence of the noise contribution was dominant over the signal, making any derivations from the profile unreliable. One method to overcome this is to remove a noise-power curve from the data set before determining the coherence value. This curve can be determined using a statistical analysis of data over a long period of time (Ponomarenko et al., 2000). Another technique is to determine the coherence length using a mathematical fit for the coherence profile modelled from the coherence values of separate station pairs. This was the technique used in this thesis. One of the variables in this model is station distance, determined using geographic coordinates. Coherence length calculation in this manner was achieved using the following technique:

1. Assume the form of the coherence profile is a Gaussian, given by $\gamma = e^{-x^2/X^2}$, where $x$ is the distance from the reference station and $X$ is half the coherence length. The half factor is because the Gaussian is symmetric about its central position (here, at $x = 0$).

2. Take a station pair, say SOD:KIL which are separated in latitude by $182$ km and determine the coherence between the two, thus producing the values of $x$ and $\gamma$. For the example event, the H-component coherence for this pair was 0.97.

3. Determine the value of $X$ by rearranging the above equation and solving for $X$. Here we have $X = \sqrt{-x^2/\ln \gamma} = \sqrt{-182^2/\ln 0.97} = 1.03 \times 10^3$ km.

4. Multiply the value of $X$ by 2 to produce the coherence length. This becomes $2.07 \times 10^3$ km.
5. Repeat the process for the upper and lower limits of coherence. In this case the limits of \( \gamma \) are 0.98 and 0.89 (table 4.3), producing coherence length values of \( 2.56 \times 10^3 \) and \( 1.07 \times 10^3 \) km respectively.

When applied to each value of coherence a new profile of coherence length with latitude is formed, shown in figure 4.9. From this graph there is a peak in coherence length at around 66\(^\circ\) latitude. It is expected that coherence should decrease with increasing latitude as the noise level is higher for higher latitudes. The maximum value for the H-component is \( 2.06 \times 10^3 \) (2.74 \( \times 10^3 \) for the D-component) km, which corresponds closely with the value obtained using the polynomial-fit technique. The maximum value was therefore used for the coherence length measurements. The upper and lower errors for both components of this maximum value were \( 2.56 \times 10^3 \) and \( 1.07 \times 10^3 \) km.

One interesting feature which arose from this technique was the varying values of coherence length with interstation distance. It was found that the distance between the station pair influenced the coherence length value in an as yet unpredictable fashion. It is not entirely known why this should occur but is possibly related to noise contribution from unidentified sites between the station pair. To overcome this problem a different selection of station pairs was used, chosen because of the similarity in distance between each pair. The profile with latitude/longitude now becomes a plot of coherence length with the central latitude between each pair of stations. Because of the low spatial extent in longitude only two pairs could be used for the longitude profiles. The chosen station pairs are given in table 4.6. In the case of the January 10 example event, data were missing from the MAS station, and so only the KEV:KIL pair could be used for coherence length determination. This value was 0.75 for the H-component and 0.98 for D, producing a coherence length of \( 712^{\pm 356} \) (H-comp) and 2688 (D-comp) km.

4.6.2 Azimuthal Wave Number, \( m \)

The azimuthal wave number is a measurement of the number of wave cycles which can occur around the Earth in the azimuthal plane, and is defined by

\[
m = \frac{\Delta \varphi}{\Delta \lambda},
\]

where \( \Delta \varphi \) and \( \Delta \lambda \) are the phase and CGM longitude differences between two points on the Earth. When a series of points is used the \( m \)-number can be obtained by simply taking the gradient of a linear fit of cross-phase with longitude. Error values are derived from the error flags in the cross-phase via the lines of maximum and
4.6. DETERMINATION OF WAVE PROPERTIES FROM THE PROFILES

Table 4.6: Station pairs used in the production of the coherence length profile with latitude and longitude.

<table>
<thead>
<tr>
<th>Station Pair</th>
<th>Distance km</th>
<th>Central GEO Latitude</th>
<th>Station Pair</th>
<th>Distance km</th>
<th>Central GEO Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOR:NAL</td>
<td>212</td>
<td>78.0°</td>
<td>HAN:OUJ</td>
<td>245</td>
<td>63.4°</td>
</tr>
<tr>
<td>HOP:LYR</td>
<td>187</td>
<td>77.4°</td>
<td>NUR:HAN</td>
<td>199</td>
<td>61.4°</td>
</tr>
<tr>
<td>BJN:HOP</td>
<td>222</td>
<td>75.5°</td>
<td>UPS:HAN</td>
<td>265</td>
<td>61.1°</td>
</tr>
<tr>
<td>SOD:KIL</td>
<td>182</td>
<td>68.2°</td>
<td>Longitude</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PEL:ABK</td>
<td>160</td>
<td>67.6°</td>
<td>MAS:AND</td>
<td>233</td>
<td>19.9°</td>
</tr>
<tr>
<td>OUJ:PEL</td>
<td>263</td>
<td>65.7°</td>
<td>KEV:KIL</td>
<td>191</td>
<td>23.9°</td>
</tr>
</tbody>
</table>

minimum slope. In the case of the example event, for the H-component the slope of the phase profile with longitude is +7.7 and slopes of the lines of maximum and minimum errors were 12.1 and 3.4 respectively. The m-number thus written \( +7.7 \pm 4.4 \). Similarly for the D-component the m-number is \( +7.8 \pm 4.4 \). By convention the positive sign indicates westward propagation.

4.6.3 Ground Phase Velocity (Speed and Direction)

If we assume that the wave is propagating then we can describe the phase differences as time delays in the signal’s arrival at each station. Furthermore with information on interstation distance and signal frequency we can determine the north-south and east-west components of the velocity on the ground. This was achieved by calculating the wavelength \( \lambda_L \) of the signal, first in degrees of geographic latitude/longitude, then in km. The equations of the lines of the trendlines are given in table 4.7. The phase velocity \( v_p \) was then determined using the equation

\[
v_p = \frac{\lambda_L f}{L},
\]

where \( f \) is the frequency of the signal, \( \lambda_L \) is in km, and \( v_p \) is in km/s. From the equation of the line of best fit through the phase profile we can determine the difference in latitude/longitude between \( 0° \) and \( 360° \). Using the same technique as for coherence length we can then convert these to km. Table 4.7 shows the derivation of the H and D north-south and east-west components of velocity.

Once the components had been obtained a technique known as restoring the wavefront was used. Wave energy is contained within the wavefront and so any measurements of wave activity must take wavefront propagation into account. This
means components of the wave measured are actually components of the wavefront. As an example consider a wave propagating across three arbitrary stations as shown in figure 4.10a.

Assuming a plane wavefront, station A would measure the velocity of the wavefront in the direction of OA and similarly for station B. If the ray moves with angle $\alpha$ from the OB direction then the velocity of the ray $V$ can be found (figure 4.10b). Here, $\cos\alpha = V/V_{OB}$. Similarly, $\sin\alpha = V/V_{OA}$, and thus

$$\tan\alpha = V_{OB}/V_{OA}. \quad (4.7)$$

This same theory applies when dealing with two components of an array of stations as we have in this project, where $V_{OA}$ represents the north-south component and $V_{OB}$ the azimuthal. For the H-component in the example event, $\tan\alpha = 47.5/348 = 0.14 \Rightarrow \alpha = 7.77^\circ$ and $V = V_{OB}\cos\alpha = 47.5\cos(7.77) = 47.1 \text{ km/s}$. Care must be taken here as with our convention $\alpha$ represents the angle from the azimuthal, rather than the radial. To correct this, a subtraction of $\alpha$ from $90^\circ$ was taken.

The propagation velocity of the January 10 example event across the ground was therefore 47.1 km/s N82°W. The program Velocity.pro was used to develop the velocity values from the equations of the lines of best fit for the cross-phase with latitude/longitude profiles.
4.6. DETERMINATION OF WAVE PROPERTIES FROM THE PROFILES

Table 4.7: North-south and east-west components of velocity for both H- and D-components of the Pc 3–4 event on January 10. By convention a positive phase represents northward (latitude) and eastward (longitude) direction propagation.

<table>
<thead>
<tr>
<th>Component</th>
<th>Phase Equation</th>
<th>Lat./long. Diff.</th>
<th>Wavelength</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-Component</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North-south</td>
<td>$\varphi = -2.38\lambda + 160$</td>
<td>151°</td>
<td>$1.67 \times 10^4$</td>
<td>348</td>
</tr>
<tr>
<td>East-west</td>
<td>$\varphi = +6.22\lambda - 670$</td>
<td>57.8°</td>
<td>$2.28 \times 10^3$</td>
<td>-47.5</td>
</tr>
<tr>
<td>D-Component</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North-south</td>
<td>$\varphi = +1.54\lambda + 101$</td>
<td>234°</td>
<td>$2.58 \times 10^4$</td>
<td>-537</td>
</tr>
<tr>
<td>East-west</td>
<td>$\varphi = +7.61\lambda - 767$</td>
<td>47.3°</td>
<td>$1.87 \times 10^3$</td>
<td>-38.9</td>
</tr>
</tbody>
</table>

4.6.4 Relationship with IMF

Solar wind data were obtained from the WIND satellite and the plots developed as described in sections 3.1.4 and 3.2.4. The Magnetic Fields Investigation (MFI) and Solar Wind Experiment (SWE) plots for the example event are shown in figures 4.11 and 4.12.
Figure 4.10: a) Diagram of a series of wavefronts moving across three spatially-separated points OAB. b) Geometric reconstruction of the wavefront and the east-west (OB) component.
4.6. DETERMINATION OF WAVE PROPERTIES FROM THE PROFILES

Figure 4.11: Data plot from the WIND Magnetic Field Investigation (MFI) for the period corresponding to the January 10 example event. The top four panels represent the total interplanetary magnetic field (IMF, in nT) and each GSM component. The bottom three panels represent the spacecraft position in units of $R_E$ in GSM coordinates. Time resolution was 1 minute. The vertical and horizontal lines at 0449 UT represent the lines from which the measurements of $B_{\text{IMF}}$ (total and $x$-component) and spacecraft position ($x$-component) were obtained.
Figure 4.12: Data plot from the WIND Solar Wind Experiment (SWE) for the period corresponding to the January 10 example event. The top three panels represent the spacecraft position and panels 4–6 show the solar wind velocity in km/s, both in GSM coordinates. The bottom panel shows ion number density in parts per cubic centimetre. The vertical and horizontal lines at 0449 UT represents the lines from which the measurements of solar wind velocity (x-component) and ion density were obtained.
4.6. DETERMINATION OF WAVE PROPERTIES FROM THE PROFILES

The first step was to determine the time delay between the signal detected at the spacecraft (s/c) and on the ground. To this end we use the radial distance of the s/c from the Earth and the solar wind speed at the s/c at this time. For both we begin with an estimate of time delay, say around 60 minutes, and measure the \( x \)-component of each from the plots. Spacecraft distance was obtained using the 5th plot of figure 4.11. For the example event, at 0445 UT the position of the spacecraft was 228.311 \( R_E \), or \( 1.445 \times 10^6 \) km. At the same time the solar wind speed, obtained from the 4th plot of figure 4.12, was \(-429 \) km/s, with the negative sign indicating a direction antiparallel to the Earth-Sun line. When examining this plot it is clear that there are deviations in the solar wind velocity on either side of this time. To account for differences between our estimated and the actual time we include these as possible limits in our velocity determination. We took the largest and smallest values in a 15 minute time period on either side of 0445 UT. In this case these values were \(-432 \) and \(-425 \) km/s. Using this time window also produces limits on the s/c position, here as 228.307 and 228.314 \( R_E \). Given a distance and an assumed constant velocity, the time can now be determined. Here the delay is 56 minutes, with upper and lower limits of 57 and 55 minutes. This places the time of arrival of the signal at the satellite between 0448 and 0450 UT. As a double-check we take the mean time of 0449 UT and measure the solar wind velocity and s/c position, here as \(-430 \) km/s and 228.311 \( R_E \). This translates to a time delay of 56 minutes and 0 seconds from the arrival of the signal on the ground at 0545 UT, or exactly 0449 UT.

The measurements of the required parameters were obtained by drawing a vertical line through each plot corresponding to the required time (0449 UT) and horizontal lines through the intersection of the time line and the graph to the ordinate, as shown in figures 4.11 and 4.12. Measurements relevant to this study were the total interplanetary magnetic field (\( B_{IMF} \)) and the radial (\( x \)) component of the interplanetary magnetic field (\( B_x \)), required for ion-cyclotron resonance properties, the \( y \)- and \( z \)-components of the interplanetary magnetic field (\( B_y, B_z \)) for later modelling, and the ion number density (Ion N), for magnetopause modelling. For the example event, \( B_{IMF} = 5.62 \) nT, \( |B_x| = 4.42 \) nT, \( B_y = +3.05 \) nT, \( B_z = -1.7 \) nT, and Ion N = 5.17 \#/cc. Errors for these values were derived in a similar fashion to the s/c position, by taking upper and lower limits in the proximity of the chosen time. In this case the upper and lower limits for \( B_{IMF} \) were 5.67 and 5.56, 4.57 and 4.26 for \( |B_x| \), and 5.30 and 4.81 for Ion N.

For comparison of frequency with solar wind parameters we apply \( B_{IMF} \) and \( |B_x| \) to the upstream ion-cyclotron resonance equations (2.27) and (2.32). For each
we require the cone angle $\theta_{xB}$, which is defined by $|\theta_{xB}| = \cos^{-1} |B_x/B|$. For errors in cone angle we must obtain the maximum and minimum values of $|B_x/B|$. For example,

$$
\max\{|\theta_{xB}|\} = \cos^{-1} \left[ \frac{\max\{|B_x|\}}{\min\{|B|\}} \right],
$$

and vice versa for $\min\{|\theta_{xB}|\}$. Using these we have $B_{IMF} = 5.62 \pm 0.06$ and $|\theta_{xB}| = 38.1 \pm 3.4^\circ$. Upon application to the equation dictated by the theoretical model of Takahashi et al. (1984) (equation 2.27) we arrive at $f = 26.4 \pm 2.7$ mHz.

Using the empirical model of Le and Russell (1996) (equation 2.32) we have $f = 24.7 \pm 1.2$ mHz. When compared with our signal of frequency $20.9 \pm 1.7$ mHz we find that the values predicted by the two models fall just outside (within 1 mHz of) its upper limit. Given the resolution of the WIND data (60 seconds) and the precision with which we make our measurements it is reasonable to suggest that these models match the signal frequency. In Table B.3, such events are labeled with a “y”, indicating that their errors do not overlap exactly but are within a small frequency region (<2 mHz).

### 4.7 Magnetospheric Boundary Determination

#### 4.7.1 The Magnetopause

Previous work (e.g. Morris and Cole, 1987; Engebretson et al., 1990) has suggested that several properties of the wave (e.g. the amplitude profile) changed at the magnetopause. The amplitude profile shown in figure 4.6a indicates a peak at the point corresponding to HOP, at $74.04^\circ$ CGM latitude. This was a common feature in almost all of the daytime events, although the location of the peak varied. In this project we have made an attempt to compare the location of the Pc 3–4 amplitude peak and that of the projection of the magnetopause on the ground (the cusp). The latter was estimated using four models, which are described below.

The first was the Tsyganenko model (§1.3), which required the epoch, geographic coordinates, Dst and solar wind parameters, and produced 3-dimensional coordinates for intervals along a modelled geomagnetic field line. A more detailed discussion of this model can be found in section 6.2.5. The original program written for this was in FORTRAN code, called Geopack.f and was available courtesy of the National Space Science Data Center (NSSDC) at the Goddard Space Flight Center (GSFC). For magnetopause determination, the maximum distance of the field line can be determined using the on-line form available at http://nssdc.gsfc.nasa.gov/space/cgm/t96.html. The table produced by this web-
4.7. MAGNETOSPHERIC BOUNDARY DETERMINATION

Page showed information on the internal geomagnetic field, apex (maximum magnetic field line distance) and the minimum value of the magnetic field. Other parameters such as the geographic projection of the minimum strength magnetic field line, equatorial mapping of the starting point to the equatorial plane, and the location of the magnetic conjugate point were also produced. By entering the location of each IMAGE station, along with the time of the event and associated solar wind parameters we estimated the maximum distance of the field line corresponding with each station. Open field lines using this model were indicated by the ‘null’ comment “Mapping point is out of the modelling region”, which appeared in the output from the on-line form. The last IMAGE station without this comment was therefore the last station on a closed field line, and the error becomes the distance midway between this station and its nearest poleward neighbour.

In the January 10 example event the time and Dst were 0545 and −16, while the solar wind parameters, produced in section 4.6.4 were $B_y = 3.05$ nT and $B_z = −1.7$ nT. We applied the coordinates of each IMAGE station, starting at NAL, and assigned an altitude of 100 km. For the example event the first station not to bear the null comment was HOR (CGM latitude 74.04°), which had a modelled field line mapping to a distance of 19.5 $R_E$. Its nearest poleward neighbour was LYR (75.13° CGM latitude) and so the location of the magnetopause using this model was 74.04 ± 0.55°.

The next two models are those of Farrugia et al. (1989) and Rodger (1998), discussed in section 2.6. Both are empirical models which define the magnetopause as the balance between the magnetospheric and solar wind pressure. The former model is also given in equation (2.42),

$$L_{MP} = 107.4 \left( N_{SW} \times V_{SW}^2 \right)^{-1/6},$$

with the latter (equation 2.43),

$$L_{MP} = (129.4 \pm 2.6) \left( N_{SW} \times V_{SW}^2 \right)^{-1/6}.$$ 

The solar wind ion density $N_{SW}$ and speed $V_{SW}$ again were obtained from the WIND data shown in section 4.6.4 and the relationship between $L$ value and latitude was obtained using the NSSDC GEO-CGM webpage (Papitashvili, 2001). $L$ values were obtained using CGM coordinates varying from 55° – 75° CGM latitude, stepping in 1° increments and a constant 106° CGM longitude. A 4th-order polynomial fit to a plot of latitude with $L$-value (figure 4.13) produces a curve of equation

$$\lambda = -2.3 \times 10^{-3} L^4 + 1.01 \times 10^{-1} L^3 - 1.65 L^2 + 1.32 \times 10^1 L + 2.78 \times 10^1,$$  (4.11)
which was used to convert \( L \) value to CGM latitude.

Applying these parameters to the above equations produces the following for the location of the magnetopause: Equation (4.9): \( L = 13.05^{+0.46}_{-0.34}; \) CGM \( \lambda = 74.7^\circ \pm 0.4^\circ; \) Equation (4.10): \( L = 10.83^{+0.16}_{-0.07}; \) CGM \( \lambda = 72.5^\circ \pm 0.2^\circ. \)

Magnetopause location was also estimated using data from the DMSP satellite. Described in section 3.2.5, a neural network was applied to the data and the location of the cusp approximated. For the example event the cusp was identified on two separate instances with the DMSP satellite F13, first at 03:03:10 then 03:03:42 UT (average 03:03:26) with a corresponding geographic latitude of 81.1\( ^\circ \) and longitude of 131.0\( ^\circ \). Using the GEO-CGM webpage this corresponds to a \( L \) value of 15.02, translating to CGM \( \lambda = 76.0^\circ \) at IMAGE.

Now we compare the results from these four models with the location of the peak in amplitude. In the case of our example event the peak occurs at HOP and so the latitude is 72.94\( ^\circ ^{+0.55}_{-0.80} \) degrees. The value produced by the Tsyganenko model lies at 74.0 \( ^\circ \pm 0.6^\circ \), which is within the error margins of the amplitude peak. The Farrugia et al. estimate falls just outside upper bound of the limit but with a margin \(< 1^\circ \). We can therefore label this value with a “y”. The Rodger estimate satisfies the location of the peak within the error margins and the DMSP result lies several degrees poleward of the peak, corresponding more closely with the location of NAL.

For this event we can say that the location of the peak in amplitude may correspond to that of the magnetopause, but there is still some uncertainty. Two of the
four models predicted a location poleward of the amplitude peak. Further comparisons with other events reveal a similar relationship between the amplitude peak and the cusp region, casting doubt on this suggestion. This is discussed further in the following chapters.

4.7.2 The Plasmapause

While unclear in the latitude profiles, previous work (e.g. Menk et al., 1999; Milling et al., 2001) has shown that some ULF wave activity (e.g. Pc 5) is affected by the plasmapause. To determine if Pc 3–4 waves are also affected the location of the plasmapause was determined and compared with the ground profiles. This was done using three mathematical models described in section 2.6. Rycroft and Burnell (1970) gave the following (equation 2.44):

$$\lambda_{pp} = (62.0 - 1.0K_p - 0.4t \pm 1.8)^\circ, \quad (4.12)$$

where $\lambda_{pp}$ is the geographic latitude of the plasmapause, $t$ is the time in hours from local midnight (and is directional) and $K_p$ is the current $K_p$ index for the event. In the case of the example event $K_p = 2$ and $t = +7.25$ s, giving a value of $\lambda_{pp} = 57.1^\circ \pm 1.8^\circ$.

Equation (2.46) describes the plasmapause location according to Orr and Webb (1975):

$$L_{pp} = 6.52 - 1.44K_p + 0.18K_p^2, \quad (4.13)$$

where $L_{pp}$ is the $L$ value of the plasmapause and $K_p$ is the average value for the previous evening. As indicated in section 2.6 this equation is only valid at 0200 LT and a correction factor is required for other times, extracted from figure 2.22. The following procedure was used.

1. The $L$-value of the plasmapause at $K_p = 2$ was determined. For any time other than 0200 LT this is derived from the curve in figure 2.22. For this project this was reproduced by reading numerous values from the graph and reproducing them in a spreadsheet (values given in appendix C.3). The local time for the example event is 0715 which produces an $L$ value for the plasmapause location for $K_p = 2$ of 4.04.

2. The overnight $K_p$ value was found using the average value of the $K_p$ indices from the time periods 21–24, 00–03, 03–06 and 06–09 UT. If the event occurs before 0730 UT (is in the case of the example event) then the average values up to the time of the event were taken. This was 1+, or 1.33 for the January 10 event.
3. The $L$ value of the plasmapause at 0200 LT was found for the overnight $K_p$, by substitution into equation (4.13). This produces the value $X$ which, in this case was 4.92.

4. The $L$ value of the plasmapause was then calculated, using equation (2.47)

$$L_{pp} = \left[ \frac{L_1}{L_{Kp=2}} \right] \times X,$$

where $L_1$ is the $L$ value derived in part 1., $L_{Kp=2}$ is the $L$ value at $K_p = 2$, which on substitution into equation (4.13) is 4.36, and $X$ is the location of the plasmapause at 0200 LT, derived in part 3. For the example event the value of $L_{pp}$ is therefore $[4.04/4.36] \times 4.92 = 4.56$, or 62\(^\circ\) CGM latitude.

The third plasmapause model (equation 2.48) is from Carpenter and Anderson (1992), which is related to the maximum $K_p$ value over the previous 24 hours and applies to 0000–1500 MLT:

$$L_{pp} = 5.6 - 0.46K_{p_{max}},$$

which is equation (2.48) in this thesis. For the example event the maximum $K_p$ value over the time period 0600–0600 UT was 2, and so $L_{pp} = 4.68$, or 62.4\(^\circ\) CGM latitude.

We now compare the location of the plasmapause from all three models with the profiles shown in figure 4.6. The location derived in the Rycroft and Burnell (1970) model lies outside the lowest latitude for IMAGE and so could not be included with this study. This can be considered the most inaccurate model for our study of daytime events as it was intended for the time region 2100–0500 LT. The projections from both the Orr and Webb (1975) and Carpenter and Anderson (1992) models place the plasmapause between OUJ and PEL, which are the 4th and 5th points from the left on the profiles. Looking at these profiles one is tempted to account the decrease in amplitude for an indication of the plasmapause, as it occurs almost immediately poleward of its estimated location. However, this deviation in amplitude around this latitude range is common in all events, including many for which the location of the plasmapause is a large distance away (see figures 5.7a and b). Also, there is no deviation in the phase values around these latitudes.

Based on this evidence we conclude that the plasmapause probably has no influence on the ground characteristics for this event.
4.8 Comparison With Other Ground Arrays

Two additional ground magnetometer arrays were used for this study. The Magnetometer Array for Cusp and Cleft Studies (MACCS) array in arctic Canada (§3.1.3) was used to investigate the azimuthal spatial extent of the signals while the Antarctic station at Davis (§3.1.2) for conjugate point study. The example event on January 10 event was not identified at MACCS and, like the majority of the other events, can not be used as an example for this section. Instead we will use another event which was identified at both MACCS and the Antarctic. This event occurred on March 9, 1998 at 1030–1100 UT (local noon at IMAGE), had a frequency of 22.7 mHz and was number 32 in the March events of Appendix B.1.

4.8.1 Event Selection

As with the events at IMAGE, the signals detected at MACCS and the Antarctic stations required a sinusoidal time series, a peak in the activity in the power spectra, and a peak in cross-power and coherence for the declaration of an event to be made. For this event to correspond with those observed at IMAGE the event must meet the selection criteria for the same time and frequency as that at IMAGE. Cross-array station pairs used for later comparison were determined by the similarity in geographic latitude and are shown in table 4.8.

Longitudinal separation between IMAGE and MACCS implies a time delay exists between the arrival of the signal at the two station arrays. So the first step in the event selection process is to determine this delay. Given the azimuthal component of velocity obtained from the IMAGE longitude profiles (figure 4.7) and assuming the velocity remains constant during propagation between arrays this delay can be easily determined. The distance between IMAGE and MACCS was obtained from their longitudinal difference and equation (4.3) with the average latitude for the two arrays as the value of \( \lambda \). To determine this latitude the following equation was employed:

\[
\bar{\lambda} = \frac{1}{2} (\lambda_{\text{max}} + \lambda_{\text{min}})_{\text{IMAGE}} + \frac{1}{2} (\lambda_{\text{max}} + \lambda_{\text{min}})_{\text{MACCS}}.
\]  

(4.16)

The average latitude was 69.3° and the difference in longitude was determined for each pair of interarray stations. Azimuthal distances were obtained for the both westward and eastward, as waves propagating from IMAGE in the latter direction would propagate around the Earth before arriving at MACCS. These values are shown in table 4.8.
For Davis our interest lies in conjugate point properties and so it was compared with Longyearbyen (LYR) in Svalbard.

Table 4.8: Azimuthal distance determination for selected cross-array stations. Values for both the MACCS array and Davis station are given, compared with stations from the IMAGE array.

<table>
<thead>
<tr>
<th>Station</th>
<th>Lat.</th>
<th>Long.</th>
<th>Station</th>
<th>Lat.</th>
<th>Long.</th>
<th>Dist. W</th>
<th>Dist. E</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMAGE</td>
<td>GEO</td>
<td>GEO</td>
<td>MACCS</td>
<td>GEO</td>
<td>GEO</td>
<td>×10³ km</td>
<td>×10³ km</td>
</tr>
<tr>
<td>BHN</td>
<td>74.5°</td>
<td>19.2°</td>
<td>RE</td>
<td>74.7°</td>
<td>265.0°</td>
<td>3.35</td>
<td>7.21</td>
</tr>
<tr>
<td>SOR</td>
<td>70.5°</td>
<td>22.2°</td>
<td>CY</td>
<td>70.5°</td>
<td>291.4°</td>
<td>3.34</td>
<td>9.91</td>
</tr>
<tr>
<td>KIL</td>
<td>69.0°</td>
<td>20.8°</td>
<td>CB</td>
<td>69.1°</td>
<td>255.0°</td>
<td>4.97</td>
<td>9.24</td>
</tr>
<tr>
<td>ABK</td>
<td>68.4°</td>
<td>18.8°</td>
<td>PB</td>
<td>68.5°</td>
<td>270.3°</td>
<td>4.42</td>
<td>10.2</td>
</tr>
<tr>
<td>PEL</td>
<td>66.9°</td>
<td>24.1°</td>
<td>RB</td>
<td>66.5°</td>
<td>273.8°</td>
<td>4.82</td>
<td>10.9</td>
</tr>
<tr>
<td>OUJ</td>
<td>64.5°</td>
<td>27.2°</td>
<td>BL</td>
<td>64.3°</td>
<td>264.0°</td>
<td>5.88</td>
<td>11.3</td>
</tr>
</tbody>
</table>

For the January 10 example event the east-west component of H velocity was 47.5 km/s westward, which translates to an average time of 94 ± 30 seconds between the arrival at IMAGE and MACCS. For the March 9 event (of east-west velocity 38 km/s eastward) the value is 372 ± 98 s (or ~ 6 minutes). All of these times are insignificant when compared with the 30 minute window used for analysis and hence no alteration of the FFT time window at MACCS was required.

Next, we consider the condition of the data for these stations. Here there are two major problems. The first is data availability, as several stations were missing from the MACCS array for various times during the two month period. For example, data from CY was missing from March 12 until the end of the month. As with the IMAGE array these stations were omitted where necessary. The second problem was the difference in local times between arrays. MACCS is 7 hours behind IMAGE, meaning that many of the events in the local morning at IMAGE appear during the night at MACCS, and are thus more likely to be obscured by broadband disturbances from substorm activity. For this reason events which occur during the local night at MACCS were not considered. This excludes the January 10 event, which occurred at 0015 LT at MACCS, along with almost all of the other daytime events. The March 9 event, however, occurs at 1330 at IMAGE and 0630 at MACCS.

For the Antarctic data the same problem with data availability is regarded but
the time delay posed less of a problem, with Davis ahead of IMAGE by approximately 5 hours, allowing almost all the daytime events to be considered.

Now we can proceed with the event selection. Figure 4.14 shows the time series for the MACCS and Antarctic stations along with power spectra from two stations of each for the March 9 example event. The dotted lines between 1110 and 1120 UT in panel a) show the event. The circle in the dynamic power spectrum in figure 4.14b indicates the event at both MACCS and the Antarctic stations. We also have the static cross-power and coherence plots for the GH:CB and MAW:DAV pairs. The peaks in cross-power and coherence occur at 22.8 and 21.6 mHz for the former and 26.6 and 26.3 mHz for the latter. Also note that the time window for the latter begins at 1115 rather than 1030 in the previous case. It was decided that some flexibility would be allowed in the event selection process for the other arrays, but we note that the times and frequencies still fall within one time window and frequency resolution of each. This event therefore meets the selection criteria for an event in all three cases and is therefore an excellent candidate for comparison work.
Figure 4.14: a) Stacked time series of the Pc 3 event on March 9, 1998 from 1030–1130 UT bandpass filtered at 15–30 mHz. Plots of all the relevant MACCS stations and Davis are given. b) Dynamic power spectrum, with overlapping windows stepping every 15 minutes and a 2nd-order polynomial fit removed from the whole-day plot, for CB (left) and DAV (right). c) Amplitude against time (top); and cross-power (middle) and coherence (bottom) with frequency for two station pairs from MACCS (GH:CB).
4.8. COMPARISON WITH OTHER GROUND ARRAYS

4.8.2 Profiles and Values

As with the IMAGE events, information about amplitude a profile for the MACCS points were produced. These plots with latitude are shown in figure 4.16. For the longitude profiles the spatial separation make a similar plot difficult and so the longitude values for the MACCS array only are shown in figure 4.17.

Using the profiles the coherence length, azimuthal wavenumber and ground velocity were determined. These were obtained using the same procedure described in section 4.6, assigning the mathematical fit for the coherence and lines of best fit through the cross-phase profiles for the latter. For coherence length only two pairs of stations could be used from MACCS as in order to be consistent with IMAGE, and interstation distance of ~ 200 km was required. These were RB:GH (232 km) and CD:RB (254 km). The values from the MACCS and their comparison with IMAGE are shown in table 4.9.

Cross-array phase and polarization values were performed on the DAV-LYR pair but not for MACCS. To check for conjugate point properties the cross-phase between LYR and DAV was compared with those obtained from the Pc 5 resonance. We would expect any phase difference in the latter case to be a measurement of the distance of Davis from the true location of the conjugate of Longyearbyen. The resonance frequency was estimated by using the HOP:LYR dynamic cross-phase spectrum and the location of a large peak in cross-power in the static spectrum for both H- and D-components. Further confirmation was made by examining the time series for both stations of the IMAGE pair and Davis with a narrow filter bandwidth of 5 mHz. The estimated FLR frequency for the HOP:LYR pair was 5.6 mHz and the time series are shown in figure 4.15. The phase for this frequency for the LYR:DAV conjugate pair was $-70^\circ \pm 5$ for the H- and $-67^\circ \pm 5$ for the D-components. The phase difference for the same pair at the frequency of the March 9 event was $-24 (-37)\pm20^\circ$ for the H- (D-) components.

Alternatively the location of the conjugate point of Davis can be estimated using the T96 model. Entering the epoch, location of Davis and solar wind parameters for the March 9 event into the on-line form described in §4.7.1 the estimated location of the Longyearbyen conjugate point in CGM coordinates is $80.47^\circ$S, $174.47^\circ$E. This point is $3.71^\circ$S and $49.32^\circ$E of Davis station (CGM coordinates $76.76^\circ$S, $125.15^\circ$E). The equations of the lines of best fit through the phase profiles are $\varphi = -21.74\lambda + 1399$ ($\varphi = +0.86\lambda - 59.92$) for latitude and $\varphi = +8.48\lambda - 869$ ($\varphi = +11.36\lambda - 1196$) for longitude of the H- (D-) components. Given that Davis is NW of the Longyearbyen conjugate point and that a positive phase difference
represents southward or eastward propagation we must in this case assign the opposite sign as that of the gradient of the phase profile in the north-south direction, but the same sign in the east west direction. Applying the latitude difference to the gradient gives the magnitude of this change. The changes are thus $+80.66^\circ (-3.19^\circ)$ for the latitudes and $+418.2^\circ (+560.3^\circ)$ for the longitudes. Now the phase difference due to the spatial separation between the station and the conjugate point would be a combination of the latitude and longitude phases, or $498.9^\circ = 138.9^\circ (557.1^\circ = 197.1^\circ)$ for the H- (D-) components. The actual phase values can now be obtained by subtracting these values from the measured values of the H and D phase for the DAV:LYR pair. These values become $-163 \pm 40^\circ (-234 \pm 40^\circ)$. Now, if there are FLR's present one would expect to see the H-component in phase ($0^\circ$) and D-components out of phase ($180^\circ$) for an odd mode, and vice versa for an even mode (Sugiura and Wilson, 1964). For either mode the two components should be out of phase with each other. Given the size of the error estimates for this example this is indeed possible, $(\Delta \varphi = 71 \pm 80^\circ)$ but it is more likely that the two components are in phase with each other at conjugate points.

If we apply the same to the Pc 5 resonance at 5.6 mHz we find the phase correction required for the spatial distance is $-426.47^\circ$ for H- and $-552.88^\circ$ for the D-components, giving phase values of $-496^\circ = -136^\circ \pm 10$ and $-620^\circ = 100^\circ \pm 10$. Here the two components are approximately out of phase $(\Delta \varphi = 124 \pm 20^\circ)$ but it is unclear as to whether an odd or even mode exists.

The polarization parameters for the DAV station are shown in table 4.10.

Table 4.9: Coherence Length, azimuthal wavenumber and ground velocity for the IMAGE and MACCS arrays. Coherence length and ground speed in the DAV:MAW line are given for the Antarctic stations.

<table>
<thead>
<tr>
<th></th>
<th>IMAGE</th>
<th>MACCS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>D</td>
</tr>
<tr>
<td>Coherence Length</td>
<td></td>
<td></td>
</tr>
<tr>
<td>($\times 10^3$ km)</td>
<td>1.28</td>
<td>2.80</td>
</tr>
<tr>
<td>Azimuthal wavenumber</td>
<td>+7.0</td>
<td>+9.2</td>
</tr>
<tr>
<td>Ground Speed (km/s)</td>
<td>46.2</td>
<td>34.6</td>
</tr>
<tr>
<td>Direction</td>
<td>N85°W</td>
<td>S78°W</td>
</tr>
</tbody>
</table>
Figure 4.15: *Stacked time series for the H-component IMAGE station LYR and Antarctic station DAV for the March 9 event at 10:30 UT. Filter bandwidths are for the Pc 3 signal (top three), 15–30 mHz and for the Pc 5 signal (bottom three), 3–8 mHz.*

Table 4.10: Polarization parameters for Davis station for the event on March 9.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>$-76.7^\circ$</td>
</tr>
<tr>
<td>H Amplitude (nT)</td>
<td>$2.9 \times 10^{-1}$</td>
</tr>
<tr>
<td>D Amplitude (nT)</td>
<td>$3.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>Trace Power</td>
<td>$2.00 \times 10^{-3}$</td>
</tr>
<tr>
<td>Polarized Power</td>
<td>$1.02 \times 10^{-3}$</td>
</tr>
<tr>
<td>Ellipticity</td>
<td>$-0.20$</td>
</tr>
<tr>
<td>Azimuth</td>
<td>$+17^\circ$</td>
</tr>
<tr>
<td>Degree</td>
<td>0.51</td>
</tr>
</tbody>
</table>
Figure 4.16: a) Amplitude, b) coherence and c) cross-phase profiles with CGM latitude for the March 9 event for IMAGE and MACCS. Plots for the H-component only are shown with central stations of KIL and CB. The frequency resolution was 3.33 mHz and the measured frequency of the signal was at 23.3 mHz. The curve of fit through the coherence is a 2nd-order polynomial fit.
Figure 4.17: a) Coherence and c) cross-phase profiles with CGM Longitude for the Pc 3 event on March 9, 1998 at 10:30 UT for the MACCS array. The frequency resolution was 3.33 mHz and the measured frequency of the signal was at 23.3 mHz. The curves of fit through the coherence and cross-phase are second and first order polynomial fits.
CHAPTER 5

RESULTS

This chapter will present a summary of the results obtained in this study. A total of 170 events met the selection criteria specified in section 4.2, during January and March, 1998. The values determined for each event are all shown in appendix B.1.

5.1 Diurnal Occurrence

Figure 5.1 gives a histogram of event occurrence against time. From this figure there are two peaks in Pc 3–4 activity, at 0830 UT (1000 LT) and 2030 UT (2200 LT). This suggests that we have identified two classes of Pc 3–4 wave, during the day and night, hereafter referred to as “daytime” and “nighttime” events. The former constituted 125 of the 170 events and were the main focus of this thesis. Event occurrence was more frequent in March, accounting for 121 (69%) of the observed events.

Figure 5.1: Event occurrence with UT time for the 170 events which met the selection criteria for January and March, 1998.
CHAPTER 5. RESULTS

5.2 Spectral Appearance

As discussed in chapter 4 the events could be classified into three types, based on their appearance in the dynamic power spectrum. “Narrow” events appear as a power peak at a single frequency and time, with a low-power background. A “packet” event occurs within a larger group of power packets and “broad” events appear amidst a broadband power region. Figure 5.2a shows the dynamic power spectrum for March 22, 1998, during which an example of each type of event occurred. The equivalent static spectrum is given for the three types in figure 5.2b.

The narrow and packet events dominated the dayside, accounting for 95 (76%) of the 125 daytime events. There was also a dominance of broad events on the nightside, with 35 (78%) of the nighttime events falling into this category.

5.2.1 Relationship with IMF

For comparison with solar wind parameters we considered two relationships. The first was that of the frequency \( f \) of the signal compared with the interplanetary magnetic field strength \( B_{IMF} \) only. A plot of this relationship is given for the three types of Pc 3–4 signal in figure 5.3. The \( f/B_{IMF} \) ratio was different for the Pc 3 and Pc 4 events, with an average (and one \( \sigma \) error) of \( f = (2.74 \pm 1.19)B_{IMF} \) for Pc 4 (10–20 mHz), and \( f = (4.33 \pm 2.28)B_{IMF} \) for Pc 3 (20–50 mHz).

The second relationship was between the frequency of the signal, and the interplanetary magnetic field and cone angle as given by two mathematical models (section 4.6.4). The predicted and observed frequencies are given in table B.3. As shown in this table the upper and lower limits of the \( x \)-component of the interplanetary magnetic field \( B_x \) often vary greatly. This results in large deviations of the cone angle. For this reason any cone angle with an error margin greater than 30° were omitted. As an example consider the event which occurred on January 20 at 0900 UT (Event 23 of the January events in table B.3). The maximum and minimum values for \( B_x \) are 2.2 and 12.8 nT, with a resulting cone angle range of 0–81°. This event was thus neglected. After removal of the large-error cone angle events we were left with 38 narrow/packet and 9 broad events. The relationship between the predicted cone angle and frequency for these events are shown in figure 5.4. Note that none of the broad events had a cone angle less than 45°.
5.2. SPECTRAL APPEARANCE

Figure 5.2: a) Dynamic power spectrum for March 22, 1998 for the H-component of Kilpisjärvi (KIL). An example of a “narrow” (1200 UT at 24.5 mHz), “packet” (0545 UT at 23.6 mHz) and “broad” (2130 UT at 35.2 mHz) event are indicated by the circles. b) The equivalent static amplitude, cross-power and coherence spectra for each type of event (narrow = left, packet = centre, broad = right) measured for the H-components of the Kiruna and Kilpisjärvi (KIR:KIL) station pair. The vertical dashed lines represent each event.
Figure 5.3: Plot of the observed frequency $f$ and interplanetary field $B_{IMF}$ for the a) and c) narrow and packet, b) and d) broad daytime events. a) and b) represent all of the 125 daytime events while c) and d) are only for the 47 events with cone angle error less than 45°. The error bars shown in the top right corner of the plots represent the average variation in the magnetic field measured by the WIND satellite and limit of the observed frequency. The dashed lines represent the lines corresponding to $f = 6B_{IMF}$ and $f = 2B_{IMF}$.
Figure 5.4: Plot of the observed frequency $f$ and the cone angle $\theta_{xB}$ for the a) narrow and packet, b) broad events for the 47 events with low cone angle errors. The error bars shown in the top right corner of the plots represent the average variation in the cone angle and limit of the observed frequency.
Figure 5.5: Observed frequency against that predicted by the ion-cyclotron models of Takahashi et al. (1984) and Le and Russell (1996). Results are shown only for the 47 events which had low errors in cone angle. The dashed line represents the line for a perfect fit, i.e. $f_{\text{pred}} = f_{\text{obs}}$. Error bars on the top right hand side of each plot represent the average error with frequency.
The values for $B_x$ and $\theta_{xB}$ for the 38 narrow/packet and 9 broad daytime events were entered into the equations for frequency predicted by Takahashi et al. (1984) and Le and Russell (1996) (equations 2.27 and 2.32). A plot of observed against predicted frequency is shown in figure 5.5. The error bars were larger for the broad events than those of the narrow/packet class.

5.3 Profiles with Latitude and Longitude

5.3.1 Amplitude

The results show two types of amplitude with latitude profiles, for the narrow/packet (daytime) events and the broad (nighttime). The majority (88%) of the former events display a general increase in amplitude with increasing latitude up to a high-latitude peak, and then a decrease. Examples of such a profile for two events of different $K_p$ values are given in figures 5.7a and b. Note the location of the peak for the high $K_p$ is equatorward of that of the low. An example of a broad event is given in figure 5.7c. Of the broad nighttime events 64% showed a similar profile to this figure.

The general shape of the amplitude profile is shown in figure 5.8a (narrow/packet) and 5.8b (broad). The average peak was 1.8 (1.6) nT for the H- (D-) components of the narrow/packet events and 2.5 (2.3) nT for the broad events. Note the distortion in the profile between latitudes $63.0^\circ$ and $67^\circ$ for the narrow/packet events.

Daytime Events

The location of the amplitude peak was compared with both Dst and $K_p$ values, which are shown in figure 5.6. While the error margins are large, these show a general equatorward movement of the peak with increasing $K_p$ and decreasing Dst.

We then compared the location of the amplitude peak with that of the magnetopause as predicted by three models, described in section 2.5.1. The predicted location of the cusp from each model is included with the example event shown in figure 5.7a. Note that the peak does not occur in the magnetopause region predicted by either model. This was common in most events, with the results summarised in table 5.1. This table shows the percentage of the daytime narrow/packet events for which the amplitude peak fell within the error margins of the predicted location of the cusp. Those which were outside these margins by less than $2^\circ$ latitude are also shown. Note that the results predicted by Rodger (1998) were generally a closer fit but the errors estimates from this model were also higher.
Figure 5.6: a) Location of the average peak in amplitude for each $K_p$ value for the daytime events. b) Plot of Average Dst index with the location of the peak in amplitude. The errors represent the first standard deviation ($\sigma$) from the mean values of the latitude (a) and Dst (b).
5.3. PROFILES WITH LATITUDE AND LONGITUDE

Figure 5.7: Examples of amplitude vs latitude profiles for daytime (a and b) and nighttime (c) events. Panels a) and b) are for the events on 9 and 10 March, 1998 at 1115 and 0645 UT respectively (Events 35 and 74 of the March events in appendix B.1). Panel c) represents a typical broad profile for on 1 March, 1998 at 0030 UT (Event 1 of the March events in appendix B.1). The respective frequencies of each event are a) 29.4 mHz, b) 37.0 mHz and c) 20.7 mHz. Kp values for the two daytime events are a) 1+ and b) 4−. The location of the magnetopause estimated from the three mathematical models are included in panel a) and the estimated location of the auroral electrojet in panels b) and c). The predicted locations of the plasmapause from three models are included in all three events.
Figure 5.8: Diagram of the general shape of the amplitude profile for the (a) narrow/packet daytime and (b) broad nighttime for the 170 events in January and March, 1998.

Of the daytime events 54 occurred near a time when the DMSP spacecraft passed through the magnetopause. In all cases the measured location of the magnetopause was poleward of the amplitude peak. For example, for the event shown in figure 5.7b the DMSP satellite indicated the magnetopause at 80° latitude, beyond the region spanned by the IMAGE array.

Another feature in the amplitude profile is the distortion in the mid-latitude region of the IMAGE array as shown in figures 5.7a, b and 5.8a. This disturbance appeared in every daytime event as either a peak or trough, of varying intensity. The location was independent of $K_p$ and Dst, always appearing within the cluster of points between Kiruna (CGM $\lambda = 64.6^\circ$, $L = 5.5$) and Masi (CGM $\lambda = 66.1^\circ$, $L = 6.2$). Also in several cases (e.g. the event which occurred on March 3 at 0745 UT, Event 20 in March in appendix B.1) the peak in amplitude in the H-component occurred at the same time as a trough in amplitude for the D.

We compared the location of this disturbance with that of the plasmapause and
Table 5.1: Summary for the location of the magnetopause as compared with that of the maximum peak in amplitude. $\Delta \lambda$ refers to the difference between the error limits of the predicted and those of the actual (CGM) latitude. Results are given for the 95 narrow/packet daytime events only.

<table>
<thead>
<tr>
<th>Model</th>
<th>Exact Fit</th>
<th>$\Delta \lambda \leq 2^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tsyganenko (T96)</td>
<td>39% (H) 44% (D)</td>
<td>61% (H) 67% (D)</td>
</tr>
<tr>
<td>Farrugia et al. (1989)</td>
<td>21% (H) 20% (D)</td>
<td>61% (H) 47% (D)</td>
</tr>
<tr>
<td>Rodger (1998)</td>
<td>64% (H) 58% (D)</td>
<td>82% (H) 82% (D)</td>
</tr>
<tr>
<td>All three Models</td>
<td>4% (H) 2% (D)</td>
<td>36% (H) 29% (D)</td>
</tr>
</tbody>
</table>

the auroral electrojet region. As discussed in section 4.7 we used three models to estimate the former. Figures 5.7a and b show the location of the plasmapause using these models. Note that while the plasmapause lies in a considerably different region for each example the location of the disturbance remains fixed.

The location of the auroral electrojets was obtained from the IMAGE webpage (Viljanen, 2001). The results from this page demonstrate that there is rarely a well defined electrojet system during the late local morning (0600-1200 UT). This is the same time period during which the majority of the observed events occur. For the example event in figure 5.7b the light blue region indicates the location of the electrojet determined from figure 5.9a. As shown it covers a large area, including the location of the mid-latitude distortion. For the example event given in figure 5.7a however, there was no defined electrojet region. Also, as shown in figure 5.9c the electrojet region moves as the day progresses while the mid-latitude distortion remains constant, independent of time.
Figure 5.9: The electrojet boundaries as obtained from the IMAGE webpage (Viljanen, 2001) for a) 10 March, 1998, b) 1 March, 1998, c) average for the entire year 1998. The red plot indicates the eastward electrojet (EEJ) while the blue is the westward (WEJ). The upper and lower plots represent the outer and inner boundaries of the electrojet region.
Nighttime Events

In the nighttime amplitude profiles the most apparent feature is the peak which occurs around 66° latitude (figures 5.7c and 5.8b). This was compared with the location of the plasmapause and is shown in figure 5.10. The absence of a linear fit suggests that there is no relationship between the two. This peak was also compared with the location of the auroral electrojet as obtained from the IMAGE webpage. For the event given in figure 5.7c the electrojet boundaries were at 69° and 78° as shown in figure 5.9b. This region is considerably poleward of the amplitude peak, as shown in the figure. The average location of the electrojet averaged across the entire year is given in figure 5.9c. From this figure the electrojet location on the nightside is always poleward of 67°, which is also poleward of the peak in amplitude of the broad events.

Figure 5.10: Plots of the location of the plasmapause with that of the peak in amplitude for the (a) H-component and (b) D-component nighttime events.
5.3.2 Coherence

As discussed in section 4.4.2 the coherence with latitude profiles were plotted with three reference stations. For simplicity here only the results for those centred at Kilpisjärvi (KIL) are shown. There were two types of coherence profiles observed. Figure 5.11a shows an example of a “wide” coherence profile which was dominant in the narrow/packet events observed in the daytime (74%). Coherence length for this type of profile was determined using the maximum value of the coherence length values. Dominating the broad (nightime) events (66%) were the “sharp” coherence profiles, an example of which is given in figure 5.11b. For this type of event the coherence length was calculated using the average of the coherence length values. A similar trend was observed using other reference stations. For all events in general, the D-component was more coherent over a larger distance and hence produced larger coherence lengths. Coherence was typically high for all stations across the longitudinal extent of the IMAGE array (e.g. refer to figure 4.7b) which demonstrated that the signal remained coherent across the entire azimuthal spatial extent of the array.

5.3.3 Coherence Length

All of the values determined in this section are shown in appendix B.1. Figure 5.12 shows the values of the coherence length for the H- (panel a) and D- (panel b) components as obtained with the mathematical technique described in section 4.6.1. We used a mathematical approximation of a gaussian for the coherence length profiles and selected station pairs which were roughly equally separated in latitude. An example of a coherence length with latitude profile is given in figure 4.9. For the narrow/packet events the maximum coherence length from the profile was obtained. The average of these was 1560 (2000) km with a standard deviation ($\sigma$) of 840 (1000) km for the H- (D-) components. For the broad events the average coherence length was used, with an average across all the events of 390 (110) km, with $\sigma = 490$ (200) km. There appears to be no trend in coherence length of either type with time, Kp or Dst.
5.3. PROFILES WITH LATITUDE AND LONGITUDE

Figure 5.11: Examples of coherence vs latitude profiles for a) wide and b) sharp coherence profile events. a) Profile for the event on 9 January 1998 at 1015 UT, frequency = 12.6 mHz (Event 5 of the January events in appendix B.1). b) Profile for the event on 30 March 1998 at 0845 UT, frequency = 27.3 mHz (Event 150 of the March events in appendix B.1).
Figure 5.12: Summary of the coherence lengths for (a) H- and (b) D-components as determined from the IMAGE array.
5.3. Cross-Phase

Cross-phase with latitude profiles were essentially the same for the narrow/packet and the broad events. They could be divided into two categories. The first was the direct linear relationship between cross-phase and latitude as in the case of the event which occurred on January 22, 1998 at 2015 UT (Event 28 of the January events in appendix B.1) (the H-component plot in figure 5.13d). The second were those which demonstrated a “skewed” appearance, with a phase change of \( \sim \pi \) typically between Bear \( (L = 9.9) \) and Hopen Island \( (L = 11.8) \). The location of this phase skip appeared to be independent of \( K_p, Dst, \) amplitude peak location, time of day or frequency. Examples of such plots are given in the D-component profiles of figures 5.13a and d. These figures also demonstrate that the cross-phase profiles could be different for the H- and D-components. This was a common feature for many events.

Figure 5.13b is a reproduction of the January 10 event from chapter 4 while figures 5.13c and d give examples of phase profile for broad nighttime events. Figure 5.13c is that of the same event in figure 5.7c.

Because of the large number of events with deviations at high latitudes a linear fit was applied only to the IMAGE stations on the Scandinavian mainland (i.e. equatorward of Tromsø) as discussed in section ???. The reliability of this fit was then determined for each event using the standard error \( (\Delta \phi) \) for the linear fit. These are shown in table B.1. Only those with a standard deviation \( \Delta \phi < 30^\circ \) were considered in the calculation of the ground phase velocity. This left us with 117 (144) events for the H- (D-) components. For example, the \( \sigma \) values for the 4 events given in figure 5.13 were 21\(^\circ\) and 11\(^\circ\) (for the H- and D-components of panel a), 10\(^\circ\) and 10\(^\circ\) (panel b), 30\(^\circ\) and 34\(^\circ\) (panel c), and 30\(^\circ\) and 11\(^\circ\) (panel d).

The longitude profiles showed few points with IMAGE, which is primarily a latitudinal array. The profiles were in a similar fashion to that shown in figure 4.7b and a linear fit was produced. In most cases the phase values were relatively low, resulting in a high phase speed of the azimuthal component of the wavefront. The relative errors were also higher. Those with an unreliable linear fit for the latitude profiles were also neglected in the longitude profiles.
Figure 5.13: Examples of cross-phase with latitude profiles for a) the Pc 4 event on March 1, 1998 at 0730 UT, frequency = 16.5 mHz (Number 6 of the March events in appendix B.1); b) for the event on January 10, 1998 at 0545 UT, frequency 20.9 mHz (event 8 of the January events in appendix B.1); c) and d) the events which occurred on March 1 at 0030 UT, and January 22 at 2015 UT (Numbers 1 of the March and 28 of the January events in appendix B.1) In each case a linear fit was applied to each, with the standard error $\Delta \varphi$ from this fit given in each case.
5.3.5 Azimuthal Wavenumber, m

The m-numbers are summarised in figures 5.14a and b, again separated into categories as they appear on dynamic power spectra. The average m-number was $+3.9, \sigma = 3.1$ ($+4.0, \sigma = 4.8$) for the H- (D-) component. There seems to be a deviation in the m-number trend at local noon (1030 UT), distinguishing the events into those in the local morning and at other times. The m-numbers for the former are a closely spaced ($\sigma$ for H = 2.5, $\sigma$ for D = 3.4) cluster of values of mean value $+4.4 (+4.8)$ for H (D). The trend line shown on figures 5.14a and b indicate a tendency for the m-number to decrease in magnitude as local noon is approached. As shown in the plots most of the values for the events (86% for both H and D) were positive in sign.

5.3.6 Ground Phase Velocity (Speed and Direction)

Figure 5.15 shows the ground phase speed values for the events for which $\Delta \phi \leq 30^\circ$. The average phase speed was 54.5 km/s, $\sigma = 27.4$ kms/s (118 km/s, $\sigma = 318$ km/s) for the H- (D-) component. As shown in figure 5.15a, the majority of the H-component waves (82%) were directed poleward, as indicated by the positive sign. There appears to be a peak in the H-component phase velocity of $\sim 120$ km/s around local noon. The D-component phase speeds (figure 5.15b) appear to display two, possibly three distributions of speed during the local daytime with no peak at noon. A larger proportion of the waves in the D-component (51%) were directed equatorward with an average speed of 66 km/s. Note that the distribution of the broad events is more randomised than those of the narrow and packet events. There is also a row of packet events at $\sim 140$ km/s which is almost constant. For both components there appears to be no dependence on the phase speed with time, $K_p$ or Dst.

Propagation angle displayed a similar feature as the m-numbers. These are shown in figure 5.16. The linear fit seems to suggest that the magnitude of the propagation angle decreases as time approaches local noon. In total 89% of the events propagated in a westward direction with an average of $+43.3^\circ, \sigma = 32.0^\circ (+53.6^\circ, \sigma = 45.4^\circ)$ for the H- (D-) components. The distributions appear similar for both components.

The average ground phase velocity was hence 55 km/s N45°W for the H-component and 120 km/s N55°W for the D-component. As most of the events occurred in the local morning this implied that the majority of the events were propagating away from the local noon equatorial plane.
Figure 5.14: Summary of the m numbers for the events with $\Delta \varphi < 30^\circ$ for a) H- and b) D-components.
Figure 5.15: Ground phase speed with UT Time for a) H-component and b) D-component for the events with $\Delta \varphi < 30^\circ$. Ground speed is in km/s and a positive value indicates poleward propagation.
Figure 5.16: Ground phase velocity angle with UT Time for a) H-component and b) D-component. Propagation angles are plotted as measured from the CGM north-south line. Positive values correspond to eastward propagation and are given for all events with $\Delta \varphi < 30^\circ$. 
5.4 Polarization Profiles

The Stokes parameters discussed in section 3.5.2 were produced and plotted as a function of latitude and longitude for all the events. Again there was a large difference in the latitude profiles for the narrow/packet (daytime) and broad (nighttime) events. An example of each case is given in figure 5.17. A further example appears in figure 4.8. The average values for each station are shown in figure 5.18.

Features which were apparent in these profiles are listed below.

1. The contribution to the power which is polarized (polarized power) resemble the amplitude profiles (or the trace power).

2. The ellipticity is generally low. For the narrow/packet (daytime) events there appears to be a change in sign across the gap between Sørøya (SOR) and Bear Island (BJN). It is unclear whether a similar change occurs for the broad (nighttime) events.

3. The azimuth indicates a fairly strong north-south orientation for the waves, but a reversal occurs in the same region as the ellipticity change.

4. The degree remains constant (~ 0.6) across the entire IMAGE array for all types of signals.
Figure 5.17: Two examples of the four Stokes parameters with latitude. Left: A typical narrow/packet event which occurred on March 13 at 0815 UT, frequency = 31.5 mHz (Event 59 of the March events of appendix B.1). Right: A typical broad event which occurred on March 1 at 0030 UT, frequency = 20.7 mHz (Event 1 of the March events in appendix B.1).
Figure 5.18: Summary of the polarization Stokes Parameter plots with latitude for (left) the narrow/packet (daytime) and (right) broad (nighttime) events. Error bars represent the first standard deviation ($\sigma$) from the mean values.
5.5 Comparison with Other Ground Arrays

5.5.1 Azimuthal Extent (MACCS)

Because of the time difference between the two magnetometer arrays, most of the
daytime events at IMAGE occurred in the local night at MACCS, and so they were
obscured by substorm activity. Of the 125 daytime events only 38 also occurred in
the daytime at MACCS. Of these 26 had power at the same frequency and time at
both arrays, i.e. \( \sim 2/3 \) had an azimuthal coherence length \( \geq 7 \) hours in local time
\((\geq 4 \times 10^3 \text{ km})\).

Of these 26, 12 had high coherence across the MACCS array and 7 bore a similar
resemblance in amplitude and phase to those from IMAGE. Examples of amplitude,
coherence and cross-phase profiles for MACCS are shown in figures 4.16 and 4.17.
The propagation characteristics of these 7 events as compared with the same events
from IMAGE are shown in table 5.2.

In all cases the waves at MACCS were slower and had a coherence length < 50% of
those observed at IMAGE. Azimuthal wavenumber direction was preserved in
half of the cases but magnitudes differed somewhat. Four of the events had a
velocity direction the same as at IMAGE. The low amplitude and coherence values
at MACCS may have influenced the phase values and consequently ground velocity
and m-number determinations. Note that MACCS is predominantly a longitudinal
array and phase estimates based on a trendline with latitude may be inaccurate.

5.5.2 Conjugate Points (Davis-Longyearbyen)

Although there was a time difference between Longyearbyen and Davis the majority
of the events (94) were still observed in the local daytime at each array. Here
the problem was that of data availability as the data from Davis was missing for
several days. Such limitations in the data effectively removed a further 30 events
from the list. Of the remaining 64 events we have selected 6 which displayed a
well-defined wave in the time series and a peak in power at a common frequency
at both Longyearbyen (LYR) and Davis (DAV). We then chose 6 events, selected
because of their appearance in both conjugate points and of the existance of a
Pc 5 FLR in the dynamic power and cross-phase profiles. In a similar process
that that described in section 4.8.2 we determined the conjugate point location
for each of the six events of the conjugate stations. The difference between the
stations and their true conjugate points are shown in tables 5.3. The phase measured
between DAV and LYR would be a combination of the actual phase of the event
and the phase deviation from the spatial separation from the station from the true
5.5. COMPARISON WITH OTHER GROUND ARRAYS

Table 5.2: The 7 events observed at both the IMAGE and MACCS arrays. For each event the m-number, phase velocity (speed and angle) and coherence length (C.Len) for the H- and D-components are shown for both arrays.

<table>
<thead>
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<th>IMAGE Array</th>
<th>MACCS Array</th>
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<td>(B.1)</td>
<td>m Speed</td>
<td>m Speed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>km/s Direction</td>
<td>km/s Direction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>×10³ km</td>
<td>×10³ km</td>
</tr>
<tr>
<td>H</td>
<td>16</td>
<td>+7.8 30.9 N57°E</td>
<td>+2.6 20.2 N8°E</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>+1.4 94.0 N16°W</td>
<td>+0.6 18.3 S4°E</td>
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<td></td>
<td>36</td>
<td>+2.5 107 N41°W</td>
<td>+0.5 5.33 N4°E</td>
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<tr>
<td></td>
<td>32</td>
<td>+7.0 46.2 N85°W</td>
<td>+0.3 2.68 S2°W</td>
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<tr>
<td></td>
<td>73</td>
<td>+8.5 27.9 N75°W</td>
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</tr>
<tr>
<td></td>
<td>114</td>
<td>+2.5 107 N50°W</td>
<td>+0.7 29.0 N6°W</td>
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<tr>
<td></td>
<td>122</td>
<td>+2.7 56.3 N36°E</td>
<td>+0.2 0.74 N1°E</td>
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| D   | 16    | +0.4 924 S84°W | +1.1 149 S42°W |
|     | 27    | +21 11.7 S68°W | +0.9 15.3 S15°E |
|     | 36    | +0.7 376 S74°W | +3.6 58.0 N58°W |
|     | 32    | +7.0 33.3 N87°W | +2.4 61.8 N62°W |
|     | 73    | +6.8 30.5 S63°E | +1.3 47.1 S47°E |
|     | 114   | +10 36.7 S88°E | +0.6 39.3 S39°W |
|     | 122   | +11 43.2 N71°E | +2.1 21.6 N22°W |

conjugate point from its counterpart in the opposite hemisphere. The measured and corrected phases are given in table 5.4 for the selected Pc 3–4 events and 5.5 for the accompanying Pc 5 resonance. Note that for the latter the frequency resolution restricted our frequency for phase measurement to 3.33 mHz in all but Event 58, the frequency for which was measured at 6.67 mHz as it was closer to the estimated resonance frequency of 5.63 mHz. The differences between the H- and D-components are also shown in these tables.

Figures 4.15 and 4.14b give examples of the time series and dynamic power of one of these events. Table 5.3 gives a list of the selected events and some of their traits. Polarization properties were compared between Davis and at a site close to the conjugate point of Davis, at Longyearbyen. Their results are shown in table
5.6.
Table 5.3: The 6 selected events and the location of their modeled (T96) conjugate points. The latitude (Lat.) and longitude (Lon.) values are all in CGM coordinates and the altitude is 100 km. The first three are modelled using a magnetic field line originating at LYR and the last three from DAV. In each case the difference between the conjugate point and DAV and LYR respectively are given. Note that the geographic location of Longyearbyen is 78.2°N, 15.82°E and that of Davis is 68.60°S, 78.00°E

<table>
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<th>Event (B.1)</th>
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<th>Time</th>
<th>Freq.</th>
<th>Conjugate Pt. Lat. Lon.</th>
<th>Diff. from LYR Lat. Lon.</th>
<th>Diff. from DAV</th>
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<td>Jan 9</td>
<td>1130</td>
<td>13.4 73.47°N 105.8°E</td>
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<td></td>
<td>11</td>
<td>Jan 12</td>
<td>1030</td>
<td>28.5 72.23°N 96.75°E</td>
<td>2.90°S 16.42°W</td>
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<td>7</td>
<td>Mar 1</td>
<td>0745</td>
<td>23.4 74.39°N 99.08°E</td>
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<td>LYR</td>
<td>31</td>
<td>Mar 9</td>
<td>1015</td>
<td>26.3 80.47°S 174.5°E</td>
<td>3.71°S 49.32°E</td>
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<td></td>
<td>58</td>
<td>Mar 13</td>
<td>0815</td>
<td>16.2 77.65°S 69.33°E</td>
<td>0.89°N 55.82°W</td>
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<tr>
<td></td>
<td>79</td>
<td>Mar 17</td>
<td>0715</td>
<td>27.4 74.12°S 100.4°E</td>
<td>2.64°N 24.77°W</td>
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Table 5.4: Measured phase for the DAV:LYR pair for the selected Pc 3–4 events along with the difference in phase estimated from the spatial separation between the station and the conjugate point from table 5.3. The difference between the H- and D-components for the corrected phase values are also shown.

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<th>Phase Separation</th>
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<th>∆ϕ</th>
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<td>D</td>
<td>Error from Conjugate Pt.</td>
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<td>D</td>
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<td>D</td>
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Table 5.5: Measured phase for the DAV:LYR pair for the Pc 5 FLRs corresponding to the Pc 3–4 events. The difference in phase estimated from the spatial separation between the station and the conjugate point from table 5.3 and the difference between the (corrected) phase for both components.

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<th>Event (B.1)</th>
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<th>Phase</th>
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<th>Phase Separation</th>
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<th>∆ϕ</th>
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<td>D</td>
<td>Error from Conjugate Pt.</td>
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<td>D</td>
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Table 5.6: Polarization properties for the 6 events observed at both the IMAGE and Antarctic magnetometers. Values are given for Davis (DAV) station and for Longyearbyen (LYR). Ell., Azim. and Deg. refer to the ellipticity, azimuth and degree of polarization respectively.

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<td>0.97</td>
<td>3 × 10^{-1}</td>
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<td>11 Jan 12</td>
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<td>0.31</td>
<td>4 × 10^{-1}</td>
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<td>7 Mar 1</td>
<td>4 × 10^{-3}</td>
<td>0.00</td>
<td>0°</td>
<td>0.00</td>
<td>1 × 10^{-2}</td>
<td>−0.30</td>
<td>−20°</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>31 Mar 9</td>
<td>2 × 10^{-3}</td>
<td>−0.20</td>
<td>+17°</td>
<td>0.51</td>
<td>4 × 10^{-2}</td>
<td>+0.11</td>
<td>+65°</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>58 Mar 13</td>
<td>3 × 10^{-3}</td>
<td>0.00</td>
<td>0°</td>
<td>0.00</td>
<td>1 × 10^{-1}</td>
<td>+0.01</td>
<td>−43°</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>79 Mar 17</td>
<td>3 × 10^{-3}</td>
<td>−0.11</td>
<td>−28°</td>
<td>0.60</td>
<td>2 × 10^{-2}</td>
<td>+0.15</td>
<td>+41°</td>
<td>0.65</td>
<td></td>
</tr>
</tbody>
</table>

5.6 Summary

From the results presented in this chapter, the following features appear to be evident for Pc 3–4 ULF waves at high-latitudes:

1. Event occurrence was higher in January than in March.

2. Maximum daytime event occurrence occurs in the late morning, at around 0830 UT (1000 LT). These events are narrowband packets of Pc 3–4 wave power.

3. A smaller class of broadband event also occurs in the local nighttime, peaking at 2030 UT (2000 LT).

4. The ratio of frequency observed on the ground and interplanetary magnetic field \( f/B_{IMF} \) is between 2 and 6, with an average of 4.33 ± 2.28 for Pc 3 and 2.74 ± 1.19 for Pc 4.

5. The relationship between the observed frequency on the ground and those predicted by the upstream ion-cyclotron resonance of Takahashi et al. (1984) and Le and Russell (1996) are unclear. The latter model appears to fit the results more closely.
6. Amplitude with latitude profiles are different for the daytime and nighttime events. Daytime events show a curve of increasing slope with latitude until it reaches a peak near (but not necessarily at) cusp latitudes. The amplitude then decreases again. Nighttime profiles give are almost symmetric about a peak at $\sim 66^\circ$ CGM latitude.

7. Plasmapause and auroral electrojet appears to have no influence on the waves.

8. Daytime amplitude peak does not appear to be related to the location of the magnetopause, but the results are inconclusive.

9. Average coherence length for the daytime events is $1560 \pm 840$ for the H- and $2000 \pm 1000$ km for the D-components. For the nighttime the coherence is $390 \pm 490$ (110$\pm$200) km for the H-(D-) components. D-component signals observed on the ground maintain a higher coherence across a larger distance than the H. Coherence length for the longitude profiles exceeded the longitudinal extent of the IMAGE array.

10. Cross-phase profiles show a roughly linear fit at low-latitudes with a “skip” in the Arctic Ocean gap between the Scandinavian mainland and Bear Island ($6.4 < L < 12$). There are often significant differences between the H- and D-component phase with latitude profiles.

11. Average azimuthal wavenumber was $+3.9 \pm 3.1$ ($+4.0 \pm 4.8$) for the H- (D-) component. There seems to be a deviation in the m-number trend at local noon (1030 UT).

12. Ground speed is relatively low, 55 km/s N45$^\circ$W for the H-component and 120 km/s N55$^\circ$W for the D-component. Most of the events appeared to propagate poleward and away from local noon.

13. Polarization ellipticity is generally low, azimuth indicates north-south polarization and degree is constant across the IMAGE array at 0.6. Some events show a change in ellipticity across the Arctic Ocean gap.

14. Most of the waves were not observed at the MACCS array, 7 hours LT away from IMAGE.

15. Conjugate point measurements failed to identify a mode for the waves, including the accompanying Pc 5.
CHAPTER 6
GENERATION AND PROPAGATION MECHANISMS

In the development of a model for ULF wave propagation one must consider both generation and propagation mechanisms. This chapter will attempt to explain the results described in the previous chapter in terms of such possible mechanisms.

6.1 Generation

6.1.1 Surface Waves (Kelvin-Helmholtz Instability)

The generation of ULF waves by the Kelvin-Helmholtz Instability has been discussed in section 2.2.1. This mechanism is the result of two dynamic fluids in contact with each other, in this case the solar wind and the magnetosphere. The result is the production of approximately elliptically polarized waves with the rotation in the opposite sense along the dawn and dusk meridians. This is currently believed to be the generation mechanism for Pc 5 field line resonances at high latitudes (Samson, 1972; Samson, 1991 and references therein; Olson and Fraser, 1995; Ables et al., 1998).

The energy from the KHI is produced with no particular frequency, exciting field line resonances in a similar fashion to the plucking of a guitar string. The frequency with which the field line oscillates is dependent on the plasma density and field line length rather than the energy source. Thus it is conceivable that a sufficient energy source may be able to excite standing geomagnetic field line resonances in their fundamental and higher harmonic modes. If such an excitation were true, then one would expect to observe the FLR and its harmonics simultaneously, a result not supported by previous work on high-latitude Pc 3–4 waves (Tonegawa et al., 1984; Ziesolleck et al., 1997). The production of the FLR harmonic propagation mechanism is discussed in section 6.2.1. Here we will compare the properties of our observed Pc 3–4 events on the ground with those predicted by a surface wave generation mechanism.

The results expected for a KHI include a polarization reversal about local noon (Samson and Rostoker, 1972; Samson, 1972; Olson and Rostoker, 1978; Olson and Fraser, 1995; Ables et al., 1998) and the ellipticity of the polarization should indicate a circularly polarized wave, such that by our definition (equation 3.38) \( e = 1 \).
When we consider the results shown in chapter 5 we find that the average ellipticity shown in figure 5.18 is close to zero, indicating a linearly polarized wave. Note that in some cases (e.g. figure 5.17a) the ellipticity was closer to unity. The event shown in this figure was believed to be a field line resonant harmonic (section 6.2.1) and it is unclear as to the connection between the fundamental Pc 5 resonance (which is believed to be driven by KHI) and the Pc 3 higher harmonic (which appears to be unrelated to KHI). We also note that the ellipticity plots in figure 6.7b show that $|e| > 0.5$ across most of the profile, implying the wave is circularly polarized in agreement with KHI theory. Figure 6.1 shows a plot of the ellipticity against UT time for the events from chapter 5 for high (LYR), mid (KIL) and low (OUJ) latitudes of the IMAGE array. While there is clearly no trend in the LYR plot there does appear to be a trend for a reversal around noon for the KIL and OUJ plots. This trend is certainly not conclusive however, and we note again that the majority of the values shown in these figures are still close to zero. This implies that there is a small contribution to the wave which is elliptically polarized and the direction of this polarization may reverse at local noon.

These results suggest that the Pc 3–4 ULF waves observed in this thesis are not related to the Kelvin-Helmholtz Instability. While we agree with the conclusions drawn by previous workers which support this model for Pc 5 we do not believe the same mechanism produces the Pc 3–4.
Figure 6.1: Plots of ellipticity against UT time for a) Longyearbyen (LYR), b) Kilpisjärvi (KIL) and c) Oulujärvi (OUJ) for the events discussed in chapter 5. For the IMAGE array, local noon occurs at 1030 UT as indicated.
6.1.2 Ion Cyclotron Resonance

A review of the previous work and theory governing the production of ULF waves via the ion-cyclotron resonance mechanism has been presented in section ???. Briefly, if the cone angle $\theta_{Bz}$ (angle between the interplanetary magnetic field and the Earth-Sun line) is low ($< 45^\circ$) then the Earth’s bow shock can reflect some of the particles from the solar wind back upstream. These ions gyrate in a cyclotron motion about the interplanetary magnetic field (IMF) lines with a frequency in the Pc 3–4 range. Incoming fast mode waves from the solar wind matching this cyclotron frequency will be enhanced via resonance, and then propagate into the magnetosphere.

There has been an abundance of work produced which has shown a relationship between the interplanetary magnetic field and the observed frequency on the ground. In early papers (Troitskaya et al., 1971; Verő and Holló, 1978; Odera and Stuart, 1985) this was $f = 6B_{IMF}$ but later the ratio of 6 had decreased slightly to 5.8 (Russell and Hoppe, 1981), 4.8 (Odera, 1982) and most recently, 4.4 (Ponomarenko et al., 2002). It has also been shown (Bol’shakova and Troitskaya, 1968; Plyasova-Bakounina, 1972; Engebretson et al., 2000) that the cone angle also plays a role in the frequency of ULF waves observed on the ground. Takahashi et al. (1984) and Le and Russell (1996) produced a theoretically and empirically derived relationship between the IMF strength, cone angle and the observed ULF wave frequency on the ground. These are given in equations (2.27) and (2.32).

In this thesis, we have produced a relationship between the signal frequency $f$ and both the interplanetary magnetic field strength $B_{IMF}$ (figure 5.3), and the cone angle $\theta_{Bz}$ (figure 5.4). We have also produced the relationship between $f$, $B_{IMF}$ and $\theta_{Bz}$ as predicted by the models of Takahashi et al. and Le and Russell (figure 5.5). If the upstream ion-cyclotron resonance mechanism is producing the Pc 3–4 ULF waves then we would expect the following to be observed:

1. The ratio between $f$ and $B_{IMF}$ would be somewhere between 4 (Ponomarenko et al., 2002) and 6 (e.g. Troitskaya, 1971).

2. Event occurrence would maximise at $\theta_{Bz} = 0$ and decrease with increasing $\theta_{Bz}$ (e.g. Plyasova-Bakounina, 1972), ideally with no occurrence observed for $\theta_{Bz} > 45^\circ$ (e.g. Hoppe et al., 1981).

3. A relationship should appear between the observed frequency of the events and the frequency predicted by the models of Takahashi et al. (1984) and Le and Russell (1996), i.e. a plot of observed against modeled frequencies should produce a linear fit of gradient 1 and intercept zero.
The results shown in section 5.2.1 are unclear when comparing with these predictions. The majority of the events lie between the $f/B_{IMF}$ ratios of 2 and 6 (figures 5.3a and b), with only 27% between the two intervals predicted. This value improved significantly (76%) when we removed the events with cone angle error < 45° (figures 5.3c and d). There was, however, a tendency for the higher frequency signals to produce a higher $f/B_{IMF}$ ratio, with an average of $4.33 \pm 2.28$ for the Pc 3 signals against $2.74 \pm 1.19$ for the Pc 4. The Pc 3 events agree well with the ratio of $4.41 \pm 0.25$ given by Ponomarenko et al. (2002), but there is still inconsistency in the Pc 4. This inconsistency also appeared in the results of Gul’elmi (1974) (reproduced in figure 6.2, although they neglected most of the low-frequency discrepancies in their determination of the $f/B_{IMF}$ ratio (which was ~ 5.5). The difference for the Pc 4 also appeared in the results of Green et al. (1983) who offered that “several authors” had suggested that the low frequency waves were uncorrelated. This result implies that there may be a separate generation mechanism producing the Pc 4 waves but none of our results produces any other evidence for a difference between the two.

Figure 6.2: *Ground pulsation frequencies plotted against IMF magnitude (Gul’elmi, 1974). The solid line represents the trend of slope ~ 5.5 from which their relationship was obtained. Data points marked with crosses were not included in this analysis.*
Another puzzling result is the large number of events (65%) which were observed with a cone angle greater than 45° (figure 5.4). We first note that large deviations in the measured value of $B_x$ produced large errors in the cone angle, reducing the number of events with an error low enough for measurements to be meaningful to 47. Next, we note that the error bars associated with each measurement allows the possibility for a larger number of events (45%) to lie within the 45° upper limit. We cannot, however, account for the remaining events.

The plots shown in figure 5.4 also show that the majority of the events (83%) with a small cone angle error were Pc 4 signals, and a possible difference in generation mechanism may explain the large cone angle. We believe this to be unlikely, however, as the majority of the Pc 3 events which were shown in this figure also had a large cone angle. In terms of the expected cone angle maximising near 0° we note that the results given in a recent paper by Engebretson et al. (2000) showed few events with $|\theta_{Bz}| < 15°$. They omitted the events with a cone angle greater than 45° and we have no way of knowing the percentage of events which occurred above this value.

The results also show that all of the “broad” events had a cone angle $> 45°$. This suggests that these events were produced by a separate mechanism, which is likely, considering the appearance of these signals within broadband noise and the abundance of this type of signal in the local nighttime.

Figure 5.5 shows the relationship between the observed frequency and those predicted by the two models. There is certainly no linear or any fit which can be assigned to this plot, although the observed and predicted frequencies for the majority of the events are within an order of magnitude of each other. If we consider the size of the error margins than we find that in fact 12 (32%) of the observed frequencies match those predicted by Takahashi et al., with a further 4 (11%) falling within 2 mHz of that predicted. For the Le and Russell model the number of events which fall within the error margins is 19 (50%) with an additional 9 (24%) falling within 2 mHz of the predicted frequency. The model of Le and Russell (1996) clearly shows a better fit to the events observed.

We believe that the ion-cyclotron resonance mechanism remains the best for describing the generation of Pc 3–4 ULF waves. While the results are not entirely convincing, they seem to support the theoretically accepted relationships between the solar wind and the signals on the ground.
6.2 Propagation

6.2.1 Harmonics of FLR’s

Traditionally, the most popular model for Pc 3–4 at high latitudes is that of higher harmonics of Pc 5 field line resonances. These have been observed by several workers (e.g. Takahashi and McPherron, 1982; Tonegawa and Fukunishi, 1984; Tonegawa et al., 1984; Ziesolleck et al., 1997; Howard and Menk, 2001) but were only attributed to a small percentage ($\approx 20\%$) of those observed. A review of the work supporting the field line resonance (FLR) harmonic model has been discussed in section 2.3.2. In this section we consider this possibility with our 125 daytime events discussed in chapter 5.

As shown by Waters et al. (1991b) the signature of a FLR can be shown in an amplitude and phase latitude profile. A FLR of a given frequency will be shown by a peak in amplitude and a $\pi$ phase change at a particular latitude as shown in figure 2.6. A polarization reversal is also observed across this resonance latitude (Samson et al., 1971). Higher harmonics of the same FLR would be expected to display a similar trend in phase and polarization (Kivelson and Southwood, 1986). Also, one would expect to observe the fundamental mode and its harmonics simultaneously.

There are two problems associated with harmonics observed in the past. The first is the fact that Pc 3–4 waves are often observed in the absence of the Pc 5 resonances, even in the cases where harmonics are believed to occur (Tonegawa et al., 1984; Ziesolleck et al., 1997). It has been suggested that this may be due to ionospheric screening of the fundamental mode (Ziesolleck et al., 1997).

The second problem is one of harmonic order, as harmonics in the Pc 3 range are often the 5th or 7th harmonic (or higher) of Pc 5 especially at high-latitudes. It seems unlikely that higher order harmonics would be excited spontaneously in the absence of the lower, unless some unknown property made such excitations favourable. A paper by Poulter et al. (1988) offers evidence that the introduction of heavy ions into the density affects the higher harmonic orders such that they do not oscillate with frequencies of integral multiples of the fundamental resonance. Their work further developed an older paper by Poulter et al. (1984) and adopted a plasma density profile of Baily (1983), which included H$^+$ and O$^+$ ions along the magnetic field between the 120 km altitudes at conjugate hemispheres. They produced harmonics of approximately equal-spacing (8–9 mHz) in frequency with the ratio of each harmonic frequency with the fundamental of 2.6, 4.2 and 5.7 at geographic latitudes 60° (their figure 8) and 50°. It was assumed that the H$^+$ ions were dominant at these latitudes.
The first step in identifying harmonics was to look through the events for FLR signatures, i.e. a peak in amplitude, $\sim \pi$ phase change and polarization reversal at the same latitude. Taking into account the fact that low-Q resonances show a phase change across large spatial extents (Waters et al., 1994), we identified 38 daytime events (30%) which show this signature. We now consider the Pc 5 fundamental component of two of these 38 events.

In a recent paper by Mathie et al. (1999a) the Pc 5 FLR “Alfvén Continuum” at high latitudes has been presented. Their shown results were for the FLR’s seen across the IMAGE array between 0630 and 0800 UT on March 22, 1996. The $K_p$ for this time interval was 2. Based on these results they produced a plot of FLR frequency against CGM latitude (their figure 4), reproduced in figure 6.3. An approximate second-order polynomial fit for this plot is

$$f_R = 4.87 \times 10^{-2} \lambda^2 - 7.6081 \lambda + 298.56,$$

where $f_R$ is the frequency of the fundamental FLR in mHz and $\lambda$ is the latitude in CGM coordinates, restricted to $64^\circ < \lambda < 76^\circ$. Their paper used a CGM epoch of 1996 while we used 1998, but the differences between the two are negligible for this first-order approach. For the IMAGE array ($56 \leq L \leq 76$) this limits the frequency of the fundamental mode to 1.0–12 mHz with frequencies becoming constant in the plasmapause region (Milling et al., 2001).

We now consider one of the 38 FLR harmonics which was similar in time and $K_p$ to those of the FLR’s observed by Mathie et al. This was Event 87 in appendix B.1 which occurred at 0445 UT on March 21, 1998. The frequency was 25.2 mHz ($\pm 1.67$ mHz) and the $K_p$ index was 2–. Its amplitude, cross-phase and ellipticity profiles, as compared with the Pc 5 counterpart, are given in figure 6.4. As shown in this figure, the FLR signature was detected between SOR and BJN. According to Mathie et al. (1999a) the fundamental resonance frequency for BJN (latitude = 71.4$^\circ$) is 2.6–3.5 mHz. The ratio between the Pc 3 and Pc 5 frequency ranges is hence 5.5–8.7. If we assume that the Poulter et al. (1988) model of harmonic order remains at constant frequencies for continuing harmonic order the frequency range indicated in our results could be the 4th (5.7), 5th (6.7) or 6th (8.2) order harmonic.
Figure 6.3: The Alfvén speed continuum, calculated for the interval 0630–0800 UT on March 22, 1996, centred on the time of maximum amplitude of the 1.8 mHz signal. The dotted lines represent three examples, at 74.0° (1.8 mHz), 72.3° (3.0 mHz) and 67.8° (6.3 mHz) (Mathie et al., 1999a).
Figure 6.4: Amplitude, cross-phase and ellipticity profiles with CGM latitude for the H- and D-components of the selected FLR harmonic event. This event occurred on March 21, 1998 at 0445 UT and had a frequency of 25.2 mHz. The profiles for the a) Pc 3 and b) Pc 5 counterpart (frequency 3.3 mHz) are shown.
Other signatures of a FLR include an azimuthal wavenumber (m number) of \( \sim 3 \) (Inhester, 1987; Ziesolleck et al., 1997; Mathie et al., 1999b), an odd mode observed at conjugate points (Chen and Hasegawa, 1974a; Tonegawa and Fukunishi, 1984; Tonegawa et al., 1984; Matthews et al., 1996; Howard et al., 2001) and a H-component amplitude peak which is greater than the D (e.g. Samson, 1972; Matthews et al., 1996). The phase change is also less-pronounced for the D-component than for the H (Samson, 1972; Tonegawa and Fukunishi, 1984). The m-number for the March 21 event was 5.2 (6.1) for the H- (D-) component. The average for all 38 events, however, was 4.0 ± 3.3 for the H- and 4.9 ± 4.7 for the D-components, where the errors represent the first standard deviation from the mean. Eight of the 38 events showed \( m \geq 8 \) casting doubt on their validity as FLR harmonics. Removing these from the set produced m-numbers of 3.0 ± 2.7 and 3.1 ± 3.1 for the H- and D-components, in good agreement with the predicted value of 3.

As shown in section 5.5.2 no odd mode was observed for the six examples shown. Only three of these events were suspected to be FLR harmonics (Events 11 in January, and 7 and 58 in March in appendix B.1) but none of them appeared to show any difference to the other three. They certainly do not show the classical signature for an odd mode harmonic for either the Pc 3 or Pc 5. One point which may be relevant is that for the harmonic events 7 and 58 in March the phase difference between the H- and D-components (the last column in tables 5.4 and 5.5) differ by \( \sim 180^\circ \). This does not appear in the third harmonic suspect, but this event has an m-number > 8 and so was removed from the set. Sources of error with the conjugate point phase measurements used in this thesis are listed below.

1. The stations are generally located near the cusp and would therefore exist on open field lines for several events, thus preventing the establishment of standing waves along the field lines. Also one station may sometimes lie on a closed field line while the other does not.

2. The proximity to the cusp means the T-96 model may sometimes be unreliable, as even small changes to the field here can produce dramatic changes to the location of the conjugate point.

3. The corrections to phase were made using the linear phase fit for the stations in the Scandinavian mainland rather than those in the Svalbard archipelago. As shown in many of the phase latitude profiles in this thesis the latter group of points often showed deviation from the main body of points. Thus the corrections made may be invalid.
Regarding the difference between the H- and D-components, the amplitude profiles with latitude given for the January 10 event in chapter 4 (figure 4.6a) as well as that shown in figure 5.7a were both suspect resonant harmonics. Note that in these two cases as well as the two given in figure 6.4 the D-component amplitude peak is lower than the H-component. Also note the phase profiles shown in figures 5.13a and b along with those in figure 6.4 are all harmonic suspects. Again in each the phase skip for the D-component is not as pronounced as that in the H. While it appears in figure 5.13a that the difference is larger in the D-component note that the bulk of the lower latitude points have a higher phase than the D. On closer inspection the difference between the two are approximately equal. Also note that the same skip in latitude is not apparent for the events shown in figures 5.13c and d as these were not FLR harmonic suspects. Both of the H- and D-component differences were observed in 23 of the 30 (77%) harmonic events. These results agree with the phase profile plots for a FLR.

Our results therefore suggest that at least 30 events were higher harmonics of FLR's, representing 24% of the observed events. This is in agreement with the results of Takahashi and McPherron (1982), Ziesolleck et al. (1997) and Howard and Menk (2001), who gave the value at 20–25%. This appears to be the correct representation of the abundance of resonance harmonics for Pc 3–4 ULF waves at high latitudes. On comparison of the properties of these events with the expected signatures for the Kelvin-Helmholtz Instability generation mechanism (§6.1.1) they do not appear to be related. While the results remain inconclusive, they seem to support the claim that the FLR harmonics are produced by the ion-cyclotron instability (§6.1.2) along with the non-FLR harmonic events.

We believe that for these events higher harmonic standing waves in geomagnetic field lines are excited by a source in the solar wind, possibly the ion-cyclotron resonance mechanism. The fast mode waves are converted to shear Alfvén modes which oscillate in a similar manner to their Pc 5 counterpart. Because the two are driven by separate mechanisms it is also possible that these may dominate the spectrum over the Pc 5 (if mode coupling is efficient enough), which may explain its absence in the results of Tonegawa and Fukunishi (1984), Tonegawa et al. (1984) and Ziesolleck et al. (1997).

It should be noted that the majority of the resonance signatures were detected across the large gap between Sørøya and Bear Island which are separated by the Arctic Ocean, thus preventing location of resonant signatures more accurately. It is now apparent that a great deal of Pc 3–4 activity occurs in this region and so future research should look into means of closing this gap.
6.2. PROPAGATION

6.2.2 Direct Fast Mode Propagation

As discussed in section 2.3.4 the major problem associated with direct fast mode wave propagation is the transmission of the signal to the ground at high latitudes. A fast mode wave propagating in from the equatorial plane would arrive at the Earth in this plane and would not be observed by stations in the polar region. Three possibilities remain. As the solar wind impacts with the entire magnetopause waves may be able to enter the magnetosphere from a region corresponding to the high latitude region on the Earth and propagate directly through. This theory has been rejected by Zhang et al. (1993; 1995) who claim that such an entry would not be possible as low frequency waves (< 60 mHz) are unable to penetrate the ion-ion cutoff boundary at high latitudes for the He\(^+\) and O\(^+\) gyroresonances (figure 2.13). This model restricts the entry of Pc 3–4 waves to a region in the proximity of the equatorial plane of latitudinal extent dictated by the wave frequency.

A second possibility involves waves propagating through the magnetosphere (virtually) unimpeded and then “dispersing” through the ionosphere from the equatorial plane poleward. It is unclear how large wavelength waves such as those with Pc 3 frequencies can propagate in the ionosphere as it is a thin layer, almost 1-dimensional in comparison with such a wavelength. If such a propagation were possible one would expect dispersion to be great, thus preventing the distinction of coherent signals at high-latitudes. Also, atmospheric features known to affect the ionosphere, such as the auroral electrojets, would be expected to affect certain features of the waves such as the amplitude. Our results suggest that no such affects take place in the auroral electrojet region (e.g. figures 5.7b and c).

The remaining possibility has been discussed in terms of waves “breaking” around the plasmapause boundary (Moore et al., 1987). They proposed that the refractive index and wave speed gradient at the plasmapause would act as a boundary for incoming fast mode waves, encouraging diffraction around this boundary to the ionosphere. Also, the contours in both these properties from Moore et al. (1987) and Zhang et al. (1993) (shown in figures 2.12 and 2.13) imply that wave refraction will also occur, possibly guiding an equatorial wave into the polar regions.

A model for fast mode hydromagnetic wave propagation from the equatorial plane to the ground was proposed in the late 50’s by Francis et al. (1959) to describe sudden commencement transmission. Their model constructed magnetohydrodynamic (mhd) path lengths in the same manner as optical path lengths are determined using Fermat’s Principle. The waves began in the equatorial plane and time of flight determinations were made using the plasma density models of John-
son et al. (1958) and Johnson (1959) (figure 6.5), and proceeded in the following manner:

The hydromagnetic wave (Alfvén) velocity $V$ in the equatorial plane is given by

$$V = \frac{B}{\sqrt{\mu_0 \rho}} = \frac{K}{R^3 \sqrt{mn}},$$

where $B$, $\mu_0$, and $\rho$ are defined according to equation (2.1), $R$ is the distance from the centre of the Earth in km, $m$ is the ionic molecular weight, $n$ is the ion number density (in m$^{-3}$), and $K = 1.789 \times 10^{32}$ is a constant. The value of $B_0$ was taken to be 0.315 gauss ($3.15 \times 10^4$ nT) at the Earth’s surface.

The transit times for the ray paths shown in figure 6.6 between the points $S_i$ and $S_f$ are given by

$$T = \int_{S_i}^{S_f} \frac{dS}{V} = \int_{\theta_i}^{\theta_f} \frac{\sqrt{R^2 + (R')^2} d\theta}{KR^{-3}(mn)^{-1/2}} = \int_{\theta_i}^{\theta_f} F(R, R') d\theta,$$

where $(R, \theta)$ are polar coordinates, $R' = dR/d\theta$, and $dS$ is the arc differential. In this publication, $S_i$ was taken to be a point in the equatorial plane at 6 and 10 $R_e$ and $S_f$ was three separate points on the Earth at latitudes ($\theta$) $0^\circ$, $60^\circ$ and $180^\circ$ (or local midnight). As with optical waves, mhd ray paths are governed by Fermat’s
principle, which results in the Euler equation,

\[
\frac{d}{d\theta} \left( \frac{\partial F}{\partial R'} \right) - \frac{\partial F}{\partial R} = 0.
\]  

(6.4)

Now, \( F \) does not contain the independent variable \( \theta \) and so \( R' \partial F/\partial R = (dR/d\theta)(\partial F/\partial R) = dF/d\theta \). Also,

\[
R' \left[ \frac{d}{d\theta} \left( \frac{\partial F}{\partial R'} \right) - \frac{\partial F}{\partial R} \right] = \frac{d}{d\theta} \left[ R' \frac{\partial F}{\partial R'} - F \right] = 0,
\]  

(6.5)

and so

\[
R' \frac{\partial F}{\partial R'} - F = \text{constant} = C,
\]  

(6.6)

allowing the determination of \( R' \) and \( d\theta \). Now we apply the function \( F(R, R') \) as defined in equation (6.3) to equation (6.6) (derivation is shown in appendix E.5):

\[
R' = \pm R[\pm n(R^4/KC)^2 - 1]^{1/2},
\]  

(6.7)

and

\[
d\theta = \pm [n(R^4/KC)^2 - 1]^{-1/2}(dR/R).
\]  

(6.8)

Upon application to equation (6.3) the transit time along the ray path becomes

\[
T = \left| \int_{R_i}^{R_f} \frac{1}{K} \left( \frac{1}{KC} \right) \frac{mnR^5}{\sqrt{mn(R^4/KC)^2 - 1}} \right| dr,
\]  

(6.9)

and the constant \( 1/(KC) \) can be determined from

\[
\frac{1}{KC} = -\frac{\sqrt{R^2 + (R')^2}}{R^3\sqrt{mn}}.
\]  

(6.10)

In the case where the wave travels along the equatorial plane in a vertical descent path (\( \theta = 0^\circ \)), \( mn(R^4/KC)^2 \gg 1 \), and so the travel time is simplified to

\[
T = \int_{R_i}^{R_f} \frac{dR}{KR^{-3}(mn)^{-1/2}}
\]  

(6.11)

and the transit times for the other rays require a proper choice for the constant \( 1/(KC) \). According to Francis et al. (1959), all rays will be refracted away from the altitude of maximum velocity (the dotted circle in figure 6.6), except for the path satisfying \( R' = 0 \). In order to converge the rays to the Earth, a small minimum value was assigned to the value of \( R' \), thus fixing the constant \( 1/KC \) by equation (6.10).

The delay times between the arrival in the equatorial plane and at 60° latitude was 2.3 and 2.2 seconds for starting distances of 6 and 10 \( R_E \) respectively. For a wave of frequency 25 mHz this translates to a phase delay of \( \sim 20^\circ \).
CHAPTER 6. GENERATION AND PROPAGATION MECHANISMS

Figure 6.6: Three paths in the equatorial plane of hydromagnetic waves generated at 6 Earth radii. The dashed line represents the altitude of maximum velocity (given as $3 \times 10^3$ km). For these paths, $T_0$ is 33.1 seconds, $T_{60}$ is 35.4 seconds and $T_{180}$ is 43.6 seconds (Francis et al., 1959).

This idea was taken further by Matsuoka et al. (1997), who used an Alfvén speed profile which included a plasmapause density gradient from Takahashi and Anderson (1992) (reproduced in figure 6.7a) on a single Pc 3 event observed in October 1992 on the ground, and identified as a tailward-propagating fast mode wave in the magnetosphere by data from the Geotail satellite (Takahashi et al., 1994). They began with the wave equation of Southwood (1974) in terms of $E_y(x)$,

$$\frac{d^2 E_y}{dx^2} - k_y^2 \frac{dK^2}{dx} \left[ \frac{1}{(K^2 - k_y^2)(K^2 - k_z^2 - k_y^2)} \right] \frac{dE_y}{dx} + (K^2 - k_z^2 - k_y^2)E_y = 0, \quad (6.12)$$

where $K^2 = \omega^2/V_A^2$ and $V_A = V_A(x)$ is the Alfvén speed. Assuming an entirely radially propagating wave, or $k_y = k_z = 0$, $k_x = k_x(x)$. the second term of this equation was eliminated, leaving us with

$$\left( \frac{d^2}{dx^2} + \frac{\omega^2}{V_A^2} \right) E_y(x) = 0. \quad (6.13)$$

An approximate solution to this equation can be obtained using the WKB approximation,

$$E_y(x) = E_y e^{ik_x x}, \quad (6.14)$$

where

$$k_x^2 = \frac{\omega^2}{V_A^2}. \quad (6.15)$$

Matsuoka et al. (1997) then used this solution to estimate the propagation phase delay $\phi$ between two points along the radial direction; at $x = x_1$ and $x = x_2$. This
was defined by

\[ \phi = 2\pi \frac{T}{T} \],

where \( T \) is the period of the wave and \( \tau \) is the time delay between \( x_1 \) and \( x_2 \), given by

\[ \tau = \int_{x_1}^{x_2} \frac{dx}{V_A}. \]  

The results showed (their figure 10, reproduced in figure 6.7b) that this model was close to the experimentally determined phase values for \( L > 2.8 \) prompting Matsuoka et al. (1997) to suggest that waves poleward of this \( L \) value may be produced by tailward-propagating radial waves. There were, however, only two data points for these \( L \) values, the largest of which was \( L = 5.5 \), and neither of them fit the model presented. It was also unclear how a wave restricted to the \( x \)-axis would propagate to polar latitudes and no attempt was made to describe the increasing power with latitude as shown in their figure 9b.

Ray paths originating in the equatorial plane were also produced for fast mode
waves by Zhang et al. (1993). They also used a WKB approximation and produced ray paths for waves beginning in the equatorward plane at a distance of 11 \( R_E \), and propagating away in increments from 0° to 180°, with 90° being the Earthward direction. The plasma density profile they used were based on a model by Gallagher et al. (submitted to JGR, 1993, possibly Gallagher et al., 1995) and included a composition of 75% \( H^+ \), 20% \( He^+ \) and 5% \( O^+ \) throughout the magnetosphere. These are shown for two frequencies in figure 6.8.

![Figure 6.8: Meridian ray paths for (a) 100 mHz and (b) 50 mHz for fast mode waves starting in the equatorial plane. Initial ray normals angles of rays vary from 0° to 180° where 90° is toward the Earth (Zhang et al., 1993).](image)

As shown in figure 6.8a waves which have a frequency higher than the cutoff of 60 mHz are entirely refracted away from the Earth. Waves of frequencies below the cutoff will pass through, but only for entry angles < 45° (figure 6.8b). Their model did not extend any closer than 5 \( R_E \) to the Earth, probably because of the fact that the WKB approximation breaks down at the plasmapause.

To compare our results with the fast mode model we compare the results presented in chapter 5 with those expected for an incoming fast mode wave. Evidence to date which supports this model at high latitudes includes the accumulation of evidence over the years that fast mode Pc 3–4 waves have been observed by satellites in the high latitude magnetosphere (e.g. Takahashi et al., 1994 and references therein; Matsuoka et al., 1997). On the ground our results show that 92 (74%) of our daytime events propagated both poleward and away from local noon for the H-component (49 or 39% for the D-component). The m-number which approaches
zero and propagation angles which decrease as we approach local noon, shown in figures 5.14 and 5.15, loosely suggest that propagation angle may decrease as time approaches local noon. These results fit with the signature expected for a fast mode wave propagating away from the equatorial plane sunward direction. Our results also show, however, that only 37 (30%) of the daytime events propagated away from local noon for both the H- and D-components simultaneously. Also, our results and those of several previous workers (e.g. Bol’shakova and Troitskaya, 1984; Plyasova-Bakounina et al., 1986; Howard and Menk, 2001; Matsuoka et al., 2002) show an amplitude peak at high latitudes near the cusp region. If the wave was indeed arriving from the low-latitude equatorial plane one might suspect the reverse to be true.

One possibility for the amplitude peak at high latitudes has been suggested by Moore et al. (1987) in terms of waves breaking around the plasmasphere. In this case the maximum amplitude at high latitudes may be related to the entry region of Pc 3–4 energy after plasmaspheric diffraction and not from the cusp at all. One must concede that while it has been generally accepted in literature that this peak has been related to the cusp (e.g. Morris and Cole, 1987; Engebretson et al., 1990) there has been to date no direct evidence presented to support this claim. The spacing and sensitivity of the IMAGE magnetometers have prevented such measurements to be made in this study, but developments are currently underway by the author and coworkers to clarify this issue.

If, however, the wave breaking around the plasmasphere is correct then one may expect a relationship between the maximum peak of Pc 3–4 activity and the location of the plasmapause. Figure 6.9 shows this relationship for our 125 daytime events. Although a loose fit, there appears to be a decreasing trend in the location of the plasmapause with increasing location of the amplitude peak. This implies that as the plasmasphere expands and the plasmapause moves poleward the amplitude peak for Pc 3–4 activity moves equatorward. This is the opposite to what might be expected for a wave which is diffracted around the plasmasphere and arrives at the ionosphere. We note that this fit is not entirely convincing and conclude that this does not disprove the possibility of inward propagating fast mode wave.

We now compare the mathematical models proposed by Francis et al. (1959) and Matsuoka et al., (1997) with the results we have produced in chapter 5. The former model predicts a phase change of $\approx 20^\circ$ across an entire 60$^\circ$ of latitude while our results (e.g. figure 5.13) show this same difference across only 5$^\circ$ of latitude. The absence of the plasmapause in the Alfvén velocity profile almost certainly affects the results here. Considering the latter model we converted the $L$ values in the top
CHAPTER 6. GENERATION AND PROPAGATION MECHANISMS

Figure 6.9: Plots of the predicted location of the plasmapause from the three models used in this study with that of the peak in amplitude for the daytime Pc 3–4 events. Only those events with a propagation poleward and away from local noon. Error bars represent the first standard deviation (σ) from the mean values.

Panel of figure 6.7 into CGM latitudes using the curve in figure 4.13 and equation (4.11). With this correction in place the equation of the 2nd order polynomial curve of best fit through the modelled values was according to the following:

\[
\varphi = 4.17 \times 10^{-1} \lambda^2 - 3.50 \times 10^1 \lambda + 7.54 \times 10^2, \quad 3.0 \leq L < 4.2
\]

\[
\varphi = 2.08 \times 10^{-1} \lambda^2 - 2.27 \times 10^2 \lambda + 6.31 \times 10^3, \quad 4.2 \leq L < 6.5
\]

(6.18)

where \( \varphi \) is the cross-phase and \( \lambda \) is the CGM latitude. We note here that the Matsuoka model did not exceed \( L = 6 \) and we have extended the linear fit to \( L = 6.5 \) because the highest latitude station of the mainland Scandinavian body of points, Masi, is located at \( L = 6.2 \). The modelled phase values, centred at Kilpisjärvii (\( L = 6.0 \)) and phase-reversed to match the convention of the results in chapter 5, are shown in figure 6.10 along with the locations of each IMAGE station. For comparison, we have included phase profiles from the suspect FLR harmonic from figure 5.13b in figure 6.10a and a non-FLR harmonic which occurred at 0745 UT on March 3, of frequency 11.4 mHz in figure 6.10b. Note that the non-FLR harmonic fits the predicted values better than the FLR suspect, although the trend in phase was sharper for the model than the measured values. Note also that the D-component was significantly different in each case. We found that 49 (52%) of the 95 non-FLR daytime events showed a similar relationship, although few (16) rose as sharply at low latitudes as the phase values predicted by Matsuoka et al.,
and only 3 showed a similar relationship for both H- and D-components.
Figure 6.10: Phase vs latitude profiles for the events on a) March 3 at 0745 UT, frequency 11.4 mHz (Event 49 of the March events of appendix B.1) and b) January 10, 1998 at 0545 UT, frequency 20.9 mHz (Event 8 of the January events in appendix B.1). In both cases the predicted phase values from Matsuoka et al. (1997) along with the H- and D-components are shown.
The lower gradient in the low-latitude trend in cross-phase could be due to an underestimation of the plasma density within the plasmasphere. However, we do not believe this to be the case as a recent paper by Gallagher et al. (2000) gives similar values for the plasma density (their figure 1). Further developments by Goldstein et al. (2001) have suggested that the plasma density in this region is virtually constant. By setting the value of \( \rho = \text{constant} \) this would produce an Alfvén velocity profile which is higher than those quoted by Takahashi and Anderson (equation 2.1). This would in fact increase the values of the phase values within the plasmasphere as in the model these were interpreted as time delay between the signal’s arrival at each station, casting further doubt on the validity of this propagation mechanism.

Our results therefore suggest that approximately half of the events which were not suspected FLR harmonics could be incoming fast mode waves propagating directly from the solar wind. The wavefronts propagate along the shortest path dictated by Fermat’s principle which is dependent on the orientation of the refractive index contours of the magnetosphere. In this manner waves generated in the equatorial plane can be refracted to high latitudes as suggested by Moore et al. (1987). There are, however, several features in our results which cannot currently be explained by this model. These include the difference between the H- and D-components and the near-cusp peak in the amplitude profile. Papers which advocate the fast mode wave propagation model include Yumoto and Saito (1983), Odera et al. (1991) and Takahashi et al. (1994).

6.2.3 Waveguide/Cavity Modes

The idea of the waveguide/cavity mode was as a propagation mechanism for high-latitude ULF wave propagation originally suggested by Kivelson et al. (1984) and has been quietly achieving popularity over the years (Kivelson and Southwood, 1985; 1986; Allan et al., 1985; Zhu and Kivelson, 1989; Samson et al., 1992a; Wright, 1994; Pilipenko et al., 1999; Lee and Kim, 1999). Experimental evidence for this model, however is lacking, and only in recent years has there been a serious attempt to compare experimental results with the theoretical models (Kivelson et al., 1997; Mann and Wright, 1999; Waters et al., 2002).

An introduction to this type of wave has been provided in section 2.3.4. The general idea is that a cavity is established between an outer and inner boundary within the magnetosphere. Several modes within this cavity are in frequencies within the Pc 3 frequency range. Signals from these cavity modes are conveyed to the ground via geomagnetic field lines by field line resonances, (e.g. Waters et al.,
as mode conversion is most effective when the frequency of the compressional mode approaches that of the resonant wave (Pilipenko et al., 1999). Hereafter waveguide/cavity modes will be referred to as simply cavity modes, as a cavity mode is essentially a waveguide mode with a small azimuthal speed (Lee, 1996). The boundaries for the cavity are an outer magnetospheric boundary (typically the magnetopause) and an inner turning point (Mann and Wright, 1999), corresponding to a fast mode wave cutoff location which is frequency dependent (Harrold and Samson, 1992) and governed (Waters et al., 2000) by

\[ \omega^2 - V_A^2(x_t)[k_y^2(x_t) + k_z^2(x_t)] = 0, \]  

(6.19)

where \( \omega \) is the fast mode wave frequency, \( x_t \) is the location of the turning point and \( V_A \) is the Alfvén speed. Recent work by Mann et al. (1999) and Mills et al. (1999) has focussed on the KHI and it’s influence on the reflection properties of the dayside boundary for Pc 5 resonances. The results of Mann and Wright (1999) seem to support this claim and suggest a dawn-side excitation of FLR’s due to cavity modes. Here we will compare our results on the ground with those expected from the cavity mode resonance models produced by Pilipenko et al. (1999) and Waters et al. (2002). The latter presents a review of the work on cavity mode resonances.

Pilipenko et al. (1999) produced a 2-D model based on a cold plasma enclosed within a magnetic box model. They assumed a smooth variation in the Alfvén velocity across the \( x \)-axis of the cavity (figure 6.11) which allowed the application of the standard WKB approach. They showed that mode conversion is most efficient in the resonant case, and at frequencies \( f \sim V_1/2a \sim 30 \text{ mHz} \), where \( V_1 = 4 \times 10^2 \text{ km/s} \) and \( a = 2R_e \) are defined in terms of the step-like Alfvén velocity profile for the \( z \)-axis (figure 6.11), namely

\[ V_A = \begin{cases} V_1, & |z| < a, \\ V_2, & |z| > a, \end{cases} \]  

(6.20)

and \( V_2 = 1 \times 10^3 \text{ km/s} \). They then suggested that the means of penetration of Pc 3 waves to the ionosphere was different for the high- and mid-latitude Pc 3–4 waves, a finding supported by an earlier paper by Pilipenko et al. (1996).

The model of Pilipenko et al. (1999) made several predictions as to the appearance of ULF waves observed on the ground, including the absence of a coherent signal at conjugate points and a maximum occurrence at \( \sim 30 \text{ mHz} \). These, along with the other predictions for cavity modes from previous workers are listed later.

Waters et al. (2002) used a cavity mode model in both the 1-D and 3-D case and attempted to identify them with AMPTE/CCE and ISEE 1 and 2 satellite data.
They were unable to locate any evidence of cavity modes using the 1-D model and those identified from the 3-D model were evident even when the coherence length of the $B_z$ (field aligned) data was less than 0.4 $R_E$, casting doubt on the reliability of such identification.

For the 1-D model, Waters et al. (2002) assumed the boundaries are highly reflective and introduced a magnetized cold plasma and energy source. The relevant parameters of the modeled magnetosphere were the radial variation of the Alfvén speed and the fundamental FLR’s, which determines $k_z$ (Waters et al., 2000). Spacing between cavity mode harmonics were 3–4 mHz for frequencies between 20–80 mHz. Using this model they produced phase and polarization characteristics as a function of radial distance ($L$ value) for the cavity modes, reproduced for a cavity mode frequency of 36.8 mHz in figure 6.12.

The 3-D model was based on the dipole model of Lee and Lysak (1999) and removed the condition of the perfectly reflecting outer boundaries. Instead the boundary conditions imposed were a partial wave reflection on the dayside diminishing to zero reflection on the nightside, thus allowing energy to propagate out of the model. A perfectly reflecting boundary was placed at the ionosphere, assumed to be at a height of 0.5 $R_E$. Waves were introduced into the model using a radially-
directed pulse in the magnetotail. The results for a cavity mode of frequency 39 mHz are shown in figure 6.13. Note that the models described in figures 6.12 and 6.13 give a range from 1–10 R_E. This accounts for the majority of the IMAGE array, from UPS (L = 3.3) to BJN (L = 9.9).

In consideration of the models proposed and recent results, the following observations would be expected for a Pc 3–4 high-latitude cavity mode:

1. Event occurrence would be frequency dependent with a peak occurring at around 30 mHz (Pilipenko et al., 1999).

2. Amplitude profile with latitude would show multiple peaks corresponding to the latitude of each cavity mode harmonic for the event frequency. The location of the peaks would be frequency dependent, with peak spacing reducing with increasing frequency (REFERENCES; Waters et al., 2002).

3. Cross-phase profiles with latitude would reveal multiple $\pi$ phase changes indicating the FLR excited by the cavity mode harmonics. The location of these would also depend on signal frequency (REFERENCES; Waters et al., 2002).

4. Polarization characteristics would show a complex display of peaks in azimuth and ellipticity changes similar to those shown in figure 6.12 (Waters et al., 2002).

5. Wave occurrence would increase with increasing solar wind speed, maximising for $V_{SW} > 500$ km/s (Mann et al., 1999).

6. Waves would be related to the Kelvin-Helmholtz Instability generation mechanism (Mann et al., 1999).

7. Azimuthal wavenumber $m \sim 0$ (Allan and Wright, 1997; Pilipenko et al., 1999; Mann and Wright, 1999) and wave occurrence would not depend on local time.

8. No coherent waves would be observed at the conjugate points (Pilipenko et al., 1999).

To compare our results with those predicted by the cavity mode model we will use as an example one of our events with the closest frequency to the 36.8 (39) mHz used by Waters et al. (2002). This is Event 118 of the March events in appendix B.1 and occurred at 0545 UT on March 24, 1998. It had a frequency of 36 ± 1.67 mHz. Figure 6.14 shows the amplitude, cross-phase and polarization profiles with L value for this event.
Figure 6.12: Radial variation of the 36.8 mHz cavity resonance from the 1-D model. The top three panels show the absolute value (solid), real part (dotted) and phase (dashed) of the magnetic field and the bottom three panels show the azimuth, ellipticity and D value (Waters et al., 2002).
Figure 6.13: Dawn side magnetic field data from the 3-D MHD model. The absolute value (solid), real part (dotted) and phase (dashed) of the radial, azimuthal and field aligned of the cavity mode of 39 mHz are shown. (Waters et al., 2002).
Figure 6.14: (a) Amplitude and (b) cross-phase for the H- and D-components, (c) azimith and (d) ellipticity plots with L value for the event on March 24, 1998 (event 118 in appendix B.1).
We now address each point described above when compared with our results.

1. A plot of event occurrence against frequency is given in figure 6.15. The peak does not appear at 30 mHz, but almost represents a gaussian distribution centred at 20 mHz. We note that the sample rate of IMAGE of 10 seconds may provide difficulty in the determination of the high-frequency events. A relatively clear signal observed in the time series was part of the selection criteria for events in this study (§4.2).

2. There appears to be no relationship between the amplitude profile in figure 6.14a and those of figures 6.12 and 6.13. Furthermore the structure of the amplitude-latitude profile does not appear to be frequency-dependent.

3. No significant $\pi$ phase changes appear in figure 6.14b, and no features appear in the cross-phase which are similar to those in figures 6.12 and 6.13.

4. No similarity exists between the polarization properties of the observed event and the predicted model.

5. A plot of wave occurrence against solar wind speed for the 125 daytime events is shown in figure 6.16a. As is evident in this figure there was no relationship between signal activity and solar wind speed with only 29 (23%) appearing with a solar wind speed greater than 500 km/s, the peak occurring at around 400 km/s. Further analysis of these 29 events reveals no difference in the amplitude, phase or polarization properties than for the remainder of the daytime events.

6. As discussed in section 6.1.1, the waves we have investigated here do not appear to be related to the KHI generation mechanism. Closer analysis of the events where the solar wind speed was high revealed the same conclusion. For example, figure 6.10a shows an event which occurred during a period of high solar wind speed (550 km/s).

7. As shown in section 5.3.5 the average azimuthal wavenumber for all the events was $\sim +4$. Again, when we consider the subset of events with high solar wind speed the mean $m$ number becomes $5.1$, $\sigma = 3.3$ ($6.2$, $\sigma = 2.9$) for the H-(D-) component where $\sigma$ is the first standard deviation from the mean. These values are in fact higher than the mean of all the events, opposite to that expected for a cavity mode. Also, the plot of event occurrence against UT time for the 29 events of solar wind speed $> 500$ km/s, shown in figure 6.16b,
shows a resemblance in shape to the occurrence plot for all the events given in figure 5.1. This implies that there is nothing different for these events than the other daytime events.

8. Measurements of coherence at conjugate locations in this study should be regarded with some suspicion due to reasons discussed in section 6.2.1. However we note that in several of our daytime events power was also detected at both conjugate points.

Figure 6.15: A plot of event occurrence against frequency for the 125 daytime events, in 5 mHz (10–15, 15–20 etc) intervals.

These results lead us to conclude that the waves identified in our analysis were probably not produced by cavity mode resonances, at least not the events of high spatial extent identified in this study. While it is conceivable that a spatially-limited class of Pc 3–4 activity may exist at cusp latitudes and that these may indeed be generated by cavity mode resonances we do not believe that these events are the same as those identified here. Another possible mechanism for the cusp-latitude class of events is discussed in the following section.

6.2.4 The Transistor Model

The transistor model was proposed by Engebretson et al. (1991) and followed studies of cusp/cleft latitude Pc 3–4 waves with ground and satellite data (Engebretson
et al., 1986; 1989; 1990). Their model required no wave mode coupling or wave propagation across any field lines, rather the precipitation of electrons caused by pressure fluctuations in the magnetosheath. Magnetosheath pressure fluctuations were attributed to the upstream ion-cyclotron resonance mechanism and modulated electron beams were released from outer magnetosphere regions where the electrons were otherwise trapped, and conveyed wave information to the near-cusp ionosphere. Periodic precipitation of particles would modulate the ionization and conductivity of this region and thus modulate ionospheric current flow equatorward of the cusp. Overhead field lines would then be excited with the same frequency as the modulated electrons (i.e. the upstream waves) as these cusp-generated ac
currents flow equatorward. Engebretson et al. (1991) likened this behaviour to that of a transistor, as in both cases a small base current modulates a larger flow from collector to emitter. Figure 6.17 shows this model in further detail.

The paper offered no mathematical model to predict wave propagation and subsequent field line oscillation. Also, the data presented in this paper and in papers leading to its development (e.g. Engebretson 1990) described the waves observed as broadband relative to the signals in this thesis. The term ‘broadband’ is of course highly subjective as demonstrated here, where the high Nyquist frequency used in the studies by Engebretson et al. produced a domain in which our entire frequency range (10–50 mHz) appeared narrowband, and the waves were termed by the workers as such.
Figure 6.17: The transistor model as proposed by Engebretson et al. (1991). Particles are precipitated into the cusp ionosphere as electron beams modulated by the upstream waves: (a) Dusk meridian view; (b) Magnetic north view. (c) As the cusp-generated ac current moves equatorward it drives overhead field line oscillations with the same frequency as the upstream waves. The dashed line between regions 1 and 2 indicates the point of entry of the precipitating electron beam.
This confusion between broadband and narrowband signals was also noted by Olson and Szuberla (1997), who developed a simple model of the precipitation mechanism based on ground data between two cusp-latitude stations to estimate an upper limit for the coherence length. This work followed a previous paper by Olson and Fraser (1994) who showed that waves in the cusp region showed high correlation, even at the conjugate hemispheres, but low spatial coherence. Olson and Szuberla (1997) began with the assumption that a modulated electron beam is cylindrical in cross-section and nearly vertically incident. This would project a circular footprint on the ionosphere and magnetometers within this circle would observe roughly the same Pc 3 variations, and thus produce high coherence.

They considered a rectilinear region with a longitudinal and latitudinal extent of $15^\circ$ (740 km) by $5^\circ$ (560 km) into which two stations are placed, separated by 200 km. Precipitating electron beam footprint circles were produced from uniformly random distributions with the single restriction that no part of these circles could lie outside the $5^\circ$ latitudinal extent of the model box. This limitation on circle centre and radius was to maintain consistency with satellite observations of near-cusp particle populations and their ionospheric projection (e.g. Newell and Meng, 1988; 1992). For coherence length the problem becomes a determination of the probability that the circle (of radius $r$) will cover both stations. Their results were determined geometrically. Choosing a coherence cutoff of 0.65 based on previous statistical work (their section 2.2) they used the same probability ($P(r) = 0.65$) that the circle will be observed by both stations. Applying this value to their results gave a radius of 220 km. From the graph (their figure 5) it was shown that radii corresponding to lower probabilities will be less than this radius and so 220 km was the maximum circle radius (and thus coherence length) of an ion beam footprint.

Szuberla et al. (1998) took this idea one step further by taking a series of cusp-latitude stations. Based on a similar coherence analysis to those produced by Olson and Fraser (1994) and Olson and Szuberla (1997) they predicted a coherence which diminishes with interstation distance via

$$C_L \sim 1.4e^{-S/250}, \quad (6.21)$$

where $C_L$ is the coherence and $S$ is the inter-station distance in km. They predicted a coherence length of 140–180 km. To date, these are the only papers which attempt a mathematical model for waves detected on the ground due to the precipitating electron beams. They suggest that such waves are broadband and strongly localised within the cusp region. However, one should always be suspicious of any results on the ground which give a coherence length of $\sim 200$ km, as this value is the
upper limit of the range in which any signal is guaranteed to be coherent, due to ionospheric spatial integration (section 2.7.2). It seems no coincidence that the coherence length of 220 km predicted by Olson and Szuberla (1997) is identical to the diameter of the circle across which a point source at the ionosphere is spread (Hughes and Southwood, 1976b). We believe that the coherence technique adopted for these models was inadequate, by taking an average across all of the frequencies for the spectra they showed the coherence of the broadband noise contribution only.

While a theoretical justification of the transistor model remains outstanding the shear weight of experimental evidence accumulated in recent years (Engebretson et al., 1986; 1990; 1994; 1995; Olson and Fraser, 1994; Anderson and Engebretson, 1995; Baker et al., 1998; Engebretson et al., 2000; Matsuoka et al., 2002) demands a consideration in any model predicting high latitude Pc 3–4 wave propagation. Here we will attempt to predict the results on the ground obtained for comparison with those in this thesis.

The ion-precipitation model of Engebretson et al. (1991) gives the cusp/cleft as the source region for these waves. Propagation occurs in the ionosphere, presumably at the Hall layer, and further coupling with overhead magnetic field lines is encouraged via current variations in the ionosphere due to the wave propagation. Signals registered on the ground would therefore be expected to be a direct translation of the wave as it propagates in the ionosphere away from the point of entry. An amplitude with latitude profile would hence show a decrease with decreasing latitude, possibly in an exponential manner, and dissipation would be expected to occur quickly, as wave energy is lost due to both dispersion in the ionosphere (assuming waves of such high wavelength can propagate in the ionosphere at all) and overhead magnetic field line coupling. Phase values would show propagation away from the cusp, i.e. propagation would be equatorward with an angle related to the position of the cusp relative to the ground array at which the wave is observed. Also any environmental features known to influence features in the ionosphere, such as the increase in ionospheric conductivity produced by the auroral electrojets would be expected to influence the signals in some way.

The results of Newell et al. (1989) show that the cusp location is dependent on the east-west component of the interplanetary magnetic field, or $B_{y, IMF}$. They showed that when the IMF is southward ($B_{z, IMF} < 0$) and $|B_{y, IMF}| > 3$ nT the location of the cusp (defined as the peak probability of a spacecraft observing the cusp) shifts by 0.5 hour (1.5 hours) toward dawn (dusk) from local magnetic noon. Thus if wave propagation is related to the cusp location one would expect a relationship between its propagation angle and $B_{y, IMF}$. 
6.2. PROPAGATION

Amplitude profiles for the daytime events appear to show an exponential decay (figure 5.8a) but do not appear as sharply as might be expected. Our results showed that only 20 (16%) of our daytime events moved in an equatorward direction for the H-component but 69 (55%) moved equatorward in the D-component.

The former is a more reliable direction as phase values were generally lower for the D-component, and errors become relatively large. To check propagation angles we obtained values of $B_{yIMF}$ from WIND satellite data for each of the 125 daytime events and compared them with the propagation angles given in table B.1. These are shown in figure 6.18. It is clear from this figure that no such relationship exists between the IMF $y$-component and propagation angle, including those which were propagating equatorward.

Figure 6.18: *Plot of Propagation angle with $B_{yIMF}$ as obtained from the WIND satellite for (a) H- and (b) D-components. Points are shown only for those events which suit the conditions dictated by Newell et al. (1989), i.e. $B_{zIMF} < 0$. The two classes of events represent those with poleward and equatorward propagation. The convention adopted here is a positive angle indicates westward propagation.*
As mentioned earlier in this chapter, our results suggest that the location of the auroral electrojets are not related to any notable distortion to the latitude profiles. The results do not seem to support the theory for the ionospheric transistor model, at least not for the events observed in this study. It should be noted here that our selection criteria was based on observation of coherent signals across the entire extent of the IMAGE array. Due to the limited spatial extent of signals generated by this model both on the ground and in space the majority of them would not be expected to meet our selection criteria. We offer the suggestion that waves produced by the ionospheric transistor model, while probably abundant in the near-cusp region, are incapable of moving great distances, and thus do not represent the majority of Pc 3–4 waves observed equatorward of the cusp ($L < 10$). The suggestion of a different propagation mechanism for the high- and mid-latitude Pc 3–4 waves has been made before. Pilipenko et al. (1996; 1999) concluded that a possible cavity mode mechanism could be responsible for the high-latitude energy. While we do not believe that cavity modes generate the waves of large spatial extent observed in this study they may be responsible for those produced by the transistor model.

### 6.2.5 Field-Guided Propagation

An adequate model of wave propagation must be able to explain all of the 15 features shown in the results (section 5.6). In this thesis, we propose one further possibility for Pc 3–4 ULF wave movement, a model of field-guided propagation. Developments have been made on this model with respect to Pc 1–2 waves and VLF “whistlers” on the dayside (Jacobs and Watanabe, 1964; Obayashi, 1965) and involves a shear Alfvén mode wave propagating along the geomagnetic field lines. Previous workers have also shown (Kivelson et al., 1984; Waters et al., 1994; Menk et al., 2000) that Pc 3 fast mode waves produced in the upstream solar wind must propagate to the inner magnetosphere in order to establish low-latitude field line resonances there. We consider the possibility that a similar fast-Alfén mode conversion occurs at high latitudes. Field line lengths larger than the wavelength of Pc 3–4 prevent the establishment of standing oscillations (without FLR harmonics) and so field-guided travelling waves at high latitudes have been considered.

Our proposal begins with the assumption that the ULF waves enter the magnetosphere from the upstream solar wind as fast mode waves. Partial mode conversion occurs upon interaction with the geomagnetic field lines where the converted Alfvén mode waves are field-guided to the ionosphere. Remaining fast mode energy continues to propagate inward, coupling with further geomagnetic field lines as it does...
With this model the information conveyed to the ground would be a measurement of the converted Alfvén waves and not the fast mode source. Amplitude measurements would hence be a function of the coupling efficiency, which is known to maximise at a field line resonance location (Kivelson and Southwood, 1986; Inhester, 1987). If we assume that the coupling efficiency is constant away from the resonance region then an amplitude with latitude profile would show an exponential decay with decreasing latitude, with peaks occurring at each field line able to support a resonance harmonic.

Cross-phase profiles would be a measurement of the time delay in the arrival of the signal along each consecutive field line rather than a measurement of that of the fast mode speed in the magnetosphere. Previous work by the author and coworker (Howard and Menk, 2001) show that for an Earthward-propagating wave, the time of propagation along a high-latitude field line is actually larger than that of the transit time for the lower, including the time between field lines in the equatorial plane. This would create the appearance of a wave which is propagating poleward although the source wave propagates in the opposite direction. When the time delay of the signal arrival on the ground for the IMAGE stations was compared with the phase profile given in the results, there appeared to be correlation (refer to figure 2.18). We have further developed this model with the inclusion of a possible physical favourability for mode conversion, along with a more accurate production of the geomagnetic field lines. These were compared with the results of some of the daytime events.

To simulate the geomagnetic field we use the Tsyganenko T96 field model and assume that waves enter the geomagnetic field in the equatorial plane. As discussed in sections 1.3 and 4.7.1 the inputs for this model were date and UT time, location on the ground of the starting point (in geographic coordinates), solar wind dynamic pressure (in nPa), Dst index and the interplanetary magnetic field components $y$ and $z$ (in nT). The result was an ASCII file containing $x$, $y$ and $z$ GSM coordinates (in $R_E$) and total magnitude (in nT) of the geomagnetic field line, divided into a series of points beginning at the Earth’s surface ($\sqrt{x^2 + y^2 + z^2} = 1 R_E$). A plot of a few field lines in a 3-dimensional field is shown in figure 6.19.
Figure 6.19: Three-dimensional plot of 4 geomagnetic field lines produced by the T96 model for the Pc 3–4 event on January 10, 1998 at 0745 UT. Field lines are given for IMAGE stations LYR, HOP, BJN and SOR from the ground to the equatorial plane.

6.2.6 Geomagnetic Field Model Production

Solar wind parameters were obtained using data from the WIND satellite and the dynamic pressure calculated using

\[ p = \rho \times v_{SW}^2, \]  

where \( p \) is the pressure in Pa (kg/m\(^2\)), \( \rho \) is the mass density in kg/m\(^3\) and \( v_{SW} \) is the solar wind speed in m/s. Dividing \( \rho \) by the mass of a proton \((1.6726 \times 10^{-27} \text{ kg})\) and then by \(10^4\), \( v_{SW}^2 \) by \(10^6\) and \( p \) by \(10^9\) we have

\[ P = 1.6726 \times 10^{-6}(N \times V_{SW}^2), \]  

where \( P \) is in nPa, \( N \) is in #/cc and \( V_{SW} \) in km/s.
The first step was to establish which of the field lines were on the dayside. A first approximation of this was to examine the maximum distance of the field line corresponding to each station. Any field line which achieved a maximum distance >15RE was most likely to be on the nightside. This has been achieved in section 4.7.1 in the determination of the location of the magnetopause. Another method was to examine the direction of the x component for the field line of maximum length. Due to convention, a negative value indicates a field line antisunward of the origin (the centre of the Earth) and was thus on the nightside. Field line length was calculated using the sum of the distance between adjacent points, obtained via the 3-dimensional pythagorus theorem:

\[ L_j = \sum_{i=1}^{N-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2}, \]  

(6.24)

where \( L_j \) is the field line length in RE for the \( j \)th IMAGE station, \((x, y, z)_i\) are the coordinates for the \( i \)th point and \( N \) is the number of intervals produced by the T96 model. The distance between field lines in the equatorial plane was obtained from the difference between the two points where \( z = 0 \). If our assumption was correct the wave would need to travel this extra distance as well as along the field line to reach the ground. The difference in the distance travelled between field lines 1 (outer) and 2 (inner) is therefore

\[ \Delta L = L_1 - (D + L_2), \]  

(6.25)

where \( L_i \) is the field line length in equation (6.24) and \( D \) is the distance in the equatorial plane between field lines 1 and 2.

To determine the time delay on the ground a wave velocity was required. In this case we have chosen the Alfvén speed at each point, determined (equation 2.1) from

\[ V_A = \frac{B_0}{\sqrt{\mu_0 \rho}}, \]  

(6.26)

The magnitude of \( B \) was derived in the T96 model while \( \rho \) was obtained from a model of plasma density according to Chappell et al. (1971), Warner and Orr (1979) and Waters et al. (1996):

\[ \rho = \rho_0 \left( \frac{r_0}{r} \right)^\alpha, \]  

(6.27)

where \( \rho_0 \) and \( \rho \) are the proton number densities at \( r_0 \) and \( r \) and \( \alpha \) is an experimentally derived coefficient, typically 4 (Harris, 1974). This model is useful for regions in the plasmasheath but becomes unreliable Earthward of the plasmapause. A recent model of by Goldstein et al. (2001) using data from the Polar Spacecraft shows a flat (\( \alpha \sim 0 \)) plasma density within the plasmasphere. The region according
to the plasmapause was assumed to increase the plasma density to $\sim 400 \text{#/cc}$ (Carpenter and Anderson, 1992; Gallagher et al., 2000) where it was then held constant within the plasmasphere, as shown in figure 6.20. This figure only shows values of $L \geq 3$ as the measurements of Goldstein et al. (2001) were made only for $L \geq 2$. Equation (6.27) requires an initial value for $\rho_0$ and $r_0$ which was determined using the Magnetospheric Plasma Analyser on board the Los Alamos National Laboratory LANL-097A satellite (§3.1.6). For the March 20 event this satellite was at geosynchronous orbit with a geographic longitude of 69.9° which is similar to that of the IMAGE array ($\sim 20^\circ$). Table 6.1 gives the values obtained from this satellite.

![Figure 6.20](image)

Figure 6.20: a) The model of plasma density in the equatorial plane used in this thesis for the regions $3 \leq L \leq 12$ for the event on March 20, 1998 at 1045 UT. The location of the plasmasphere was determined using the models of Orr and Webb (1975) and Carpenter and Anderson (1992) and is variable, depending on the local time and $K_p$ of each event. The location of the initial value of $\rho_0$, determined from the LANL-097A satellite is shown. b) The corresponding Alfvén velocity profile using the magnetic field values obtained from the T96 model in the equatorial plane.

As an example of the development of this model we will take an event which occurred near local noon near equinox, a Pc 4 (frequency = 17.1 mHz) event which occurred at 1045 UT (1215 LT) on March 20, 1998 (Event 83 of the March events of appendix B.1). The information required for the T96 model and plasma density...
models for this event is shown in table 6.1.

Table 6.1: Values required for the T96 model for the event on March 20, 1998 at 1045 UT.

<table>
<thead>
<tr>
<th>Title</th>
<th>Value</th>
<th>Title</th>
<th>Value</th>
<th>Title</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>1998</td>
<td>Pressure (nPa)</td>
<td>0.93</td>
<td>$\rho_0$ (#/cc)</td>
<td>1.7</td>
</tr>
<tr>
<td>Day</td>
<td>079</td>
<td>Dst Index</td>
<td>-12</td>
<td>$R_0$ (Re)</td>
<td>6.618</td>
</tr>
<tr>
<td>Hour</td>
<td>10</td>
<td>$K_p$ Index</td>
<td>2</td>
<td>Plasmapause Location: CGM Lat.</td>
<td></td>
</tr>
<tr>
<td>Minute</td>
<td>45</td>
<td>$B_{IMF_y}$ (nT)</td>
<td>-7.1</td>
<td>Orr &amp; Webb</td>
<td>61.3°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B_{IMF_z}$ (nT)</td>
<td>-5.0</td>
<td>Carpenter &amp; Anderson</td>
<td>61.3°</td>
</tr>
</tbody>
</table>

With these data included in the T96 along with the geographic coordinates for each IMAGE station a file containing points along the field line were obtained for the station. For example in the case of BJN for the March 20 event the file contained 63 points, beginning with (0.27, 0.17, 0.95). The field line of maximum length (11.4 Re) occurred when $x = +10.4$ Re, i.e. on the dayside. The closest points to the equatorial plane were (10.42, 4.40, 0.62) and (10.47, 4.49, -0.01). Using linear interpolation we arrive at $E_{BJN} = (10.47, 4.49, 0.00)$ for the point crossing the equatorial plane. Applying the points to equation (6.24) with $N$ corresponding to the point in the equatorial plane (28) this produces a field line length of $L_{BJN} = 15.02$ Re. Applying a similar procedure for the station KIL we have a field line length of $L_{KIL} = 8.28$ Re. Here the interpolated point in the equatorial plane was $E_{KIL} = (6.71, 1.60, 0.00)$ and the distance between KIL and BJN in this plane $\Delta E = |E_{KIL} - E_{BJN}| = 4.50$Re. A wave propagating from the equatorial plane to the ground on the BJN field line would therefore have 2.24 Re further to travel than the same wave travelling from BJN to the KIL field line in the equatorial plane and then along its field line to the ground. Thus the wave would arrive at KIL before BJN. This is illustrated in figure 6.21.

6.2.7 Simulation Production

We can now use this model to predict the amplitude and cross-phase profiles with latitude based on the above magnetospheric model.

Amplitude

As mentioned before the amplitude profile on the ground would be expected to be a function of the coupling efficiency of the fast/Alfvén modes. For our model we
Figure 6.21: Example of wave propagation along geomagnetic field lines determined by the T96 model. The wave arrives at point A (the field line at BJN in the equatorial plane) and propagates along to field line to point B, travelling a distance of \( \sim 15.02 \) \( R_E \). The same wave propagates from A to C (the field line at KIL in the equatorial plane) and then along its field line to D, covering a total distance of \( \sim 12.78 \) \( R_E \). The wave will therefore arrive at point D before point B.

We have assumed the following:

1. Mode conversion is most efficient at resonant latitudes (Kivelson and Southwood, 1986; Inhestor, 1987).

2. Mode conversion is low and relatively constant away from the resonance region.

If this was the case then the peak in amplitude would occur at the field line corresponding to the lowest resonance at which the Pc 3–4 event was a harmonic. This is typically equatorward of the cusp, but is definitely in the neighbourhood, and the peak would also be expected to move equatorward with increasing \( K_p \) as the closed field lines (and hence the field lines able to support resonance harmonics) would begin at lower latitudes. Given these features it is conceivable how the amplitude peak could be mistaken for a signature of the cusp (e.g. Bol’shakova and Troitskaya, 1984).

We first derive the amplitude profile for the poorly coupled contribution. Assuming the coupling coefficient is equal with each field line coupling we can assign a decay constant \( k > 0 \) which is related to the coupling constant described in Kivelson
and Southwood (1986), i.e

\[
\frac{dA}{d\lambda} = kA
\]
or,

\[
A = A_0e^{k(\lambda - 56)},
\] (6.28)

where \(A = A(\lambda)\) is the amplitude, \(\lambda\) is the CGM latitude confined to the domain \(56^\circ \leq \lambda \leq 76^\circ\) and \(A_0\) is the amplitude at latitude \(\lambda_0 = 56\). Based on the results from this thesis we also assume the boundary conditions \(A(56) = 0.1 = A_0, A(71.35) = 1.6\). This produces a value of \(k = \ln 16/(71.35 - 56) \sim (1.8 \times 10^{-1})^\circ\). Figure 6.22a shows this amplitude profile.

We now determine the location of the regions where resonance harmonics are likely to occur. As discussed in section 6.2.1 this is determined by the conditions such as magnetic activity and local time and most importantly the frequency of the signal. In the March 20 example this is 17.1 mHz. Using the same ratios as those derived in Poulter et al. (1984) this represents the second harmonic of a 6.5 mHz, the third harmonic of a 4.1 mHz and the fourth harmonic of a 3.0 mHz Pc 5 FLR. When applied to the resonance frequency with latitude relationship from Mathie et al. (1999a) (equation 6.1) these correspond to latitudes \(67.9^\circ, 70.7^\circ\) and \(72.4^\circ\) respectively. We note that the conditions for the March 20 event were similar to those used by Mathie et al. (although our event is 3 hours later) which validates our usage of this model once again. The geomagnetic field lines at these latitudes satisfy the condition for favourable mode coupling (Kivelson and Southwood, 1986) and we would hence expect to observe a peak in amplitude at these latitudes. We do not believe that ULF wave is of long enough duration to sustain a harmonics, but rather the converted shear Alfén mode waves travel along the field lines. Thus, the energy converted by mode coupling would not be as great as for the fundamental resonance. The ratio of the respective efficiencies and the width of each resonance peak are at present unknown, so we shall assign a value for each based on the measurements obtained in this study. Let the peak in amplitude for the resonance be a factor of 1.5 greater than the non-resonance and let the full-width at half-maximum (FWHM) for these peaks be governed by the limitations on latitude given the frequency resolution of the signal. Thus the FWHM for each harmonic is \(3.5^\circ, 5.0^\circ\) and \(5.0^\circ\) respectively. Figure 6.22b shows the amplitude profiles for these modelled resonances. Note that the regions covered by these resonance tend to overlap.

The equation for the coupling parameter \(q\) is given (Inhestor, 1987) by

\[
q \sim \frac{k_y^2}{[k^2V_A^2(\partial V_A/\partial x)]^{2/3}}.
\] (6.29)
where $k_{\parallel}$ and $k_y$ are the components parallel to the magnetic field and in the $y$ direction of the wave vector and $V_A$ is the Alfvén speed. The Alfvén velocity profile for the March 20 event shown in figure 6.20b shows that in the plasmapause region the value of $V_A^2$ drops considerably. However, there is also a steep gradient and so $\partial V_A^2/\partial x$ would increase. These would act to cancel each other out, resulting in no noticeable change in the coupling parameter. Thus no change in amplitude would be expected to occur at the plasmapause region. This agrees with the results shown in section 5.3.1.

Finally, we combine the two effects to produce the total amplitude profile. This is shown in figure 6.22c along with the measured values of amplitude for this event. There is generally a good fit, although some discrepancy at the high latitudes.
Figure 6.22: Amplitude profiles with latitude of the field-guided propagation model for a) the poorly coupled fast mode component, b) the partially coupled shear Alfvén mode component. c) The combined amplitude model of the two contributions along with the measured values of the $H$- and $D$-components for the event on March 20, 1998 at 1045 UT.
Cross-Phase

Once the fast mode has converted to a field-guided Alfvén mode the next step is to determine the time of flight along each field line, and thus the delay in the arrival at each point on the ground. This calculation requires the Alfvén speeds for each point along the field line as well as the distance travelled between points. The former were obtained using equation (6.26)

\[ t_j = \sum_{i=1}^{N} \frac{L_{i,j} + \Delta E_j}{V_{i,j}}, \]

where \( L_{i,j} \) and \( V_{i,j} \) are defined in equations (6.24) and (6.26) respectively for the \( i \)th point in the T96 model and \( \Delta E_j \) is the distance in the equatorial plane, defined by \( \Delta E_j = E_{j-1} - E_j \) (\( \Delta E_1 = 0 \)). Time delays can then be converted to a phase in degrees by multiplying \( t_j \) by the wave frequency and then by 360°. In the March 20 example the time of flight across the BJN field line was 49.0 s and 28.3 s along the KIL field line. The average Alfvén speed in the equatorial plane for the two field lines was 1690 km/s which added a further 16.9 seconds for the transit from BJN to KIL in this plane. The time delay between the signal’s arrival at KIL and BJN on the ground was thus 3.9 s, which translates to a phase of 24.0°, with KIL leading BJN.

By applying this technique to all the IMAGE stations and measuring the delay with that at KIL, a profile of phase against latitude can be produced. For the selected March 10 event this is shown in figure 6.23a.

Next we add the partially-coupled FLR harmonic component to this profile. These occur at the same latitudes as those for the amplitude profile and we will assume a signature width equal to the FHM of the amplitude profiles. For consistency with the assumption made for the amplitude profile we will assume the signature is a factor of 3 less than that of a typical FLR, resulting in a phase change of \( \pi/3 \) at each harmonic location. This is shown in figure 6.23b.

The combined model of the two processes is given in figure 6.23c, as compared with the results for H and D from the March 20 event. There are some inconsistencies between the model and the measured values of H-component phase but the general shape of the profile seems to resemble those produced by the measured phase. There remains a large inconsistency between the H and D phase which could be an indicator of the low coupling parameter of the D-component. This could be due to the polarization of the wave.
Figure 6.23: Plot of the modelled profile of cross-phase with latitude for a) the poorly-coupled fast mode component and b) The partially coupled FLR harmonic. c) The phase profile for the combined model against those determined from the data for the event on March 20, 1998 at 1045 UT.
Comparison With Results

Here we compare the results from section 5.6 with those predicted from the field guided propagation model. We believe that the nighttime events are unrelated to this propagation mechanism, and these will be discussed in the next section. Thus we will neglect point 3 from this list. We believe the remainder of points 1–5 are related to the generation, rather than propagation mechanism.

1. The ion-cyclotron resonance mechanism may be inefficient away from equinox as antisymmetry in the geomagnetic field may alter the number of ions in the solar wind flowing upstream.

2. We can offer no suggestion for why occurrence should peak in the late morning but maintain it is probably related to the generation mechanism.

4. As discussed in section 6.1.2 the features represent those predicted by the upstream ion-cyclotron resonance mechanism.

5. Same as for 4.

6. The predicted amplitude profiles match fairly close those given in the results. We note that the coupling efficiency was chosen based on the results given, which probably introduced bias to the model. We are optimistic that further research will produce a more accurate model of wave mode coupling for “partial” FLR harmonics.

7. As the mode coupling of the waves occur in the high-altitude magnetosphere and are measured at the footprints of geomagnetic field lines we would not expect changes in the ionospheric conductivity, such as those produced by the auroral electrojets, to produce a noticable effect.

8. The plasmapause was included in our model for the cross-phase profile and no notable difference in the cross-phase profile was produced. Also, the plasmapause was considered in the wave-coupling efficiency as discussed by Inhestor (1987). Our model suggested that its inclusion produced any notable changes to the amplitude profile.

9. The effect on wave is largely unknown but we speculate that the wave source maintains its coherency on passage through the magnetosphere and thus the signal structure from the converted Alfvén wave observed on the ground would be virtually unaltered. Thus one would expect the coherence to be fairly high across a large spatial region. If these are the same waves which produce the
low latitude FLR’s (§2.3.1) then they would be expected to maintain their coherence throughout the magnetosphere. Perhaps at distances > 1600 km from the central station the overhead local plasma conditions are sufficiently different to change the signal structure between the two and thus reduce the coherence on the ground.

10. As discussed in the cross-phase model above the skip in phase can be explained in terms of “partial” FLR harmonic signatures at regions of high mode coupling efficiency. The differences in H- and D- could be due to the linear polarization of the wave.

11. While not discussed in the model above, we believe the azimuthal wavenumber is related to the propagation angle of the field-guided waves. As suggested by the coherence length with longitude profiles the signals extend in the longitudinal direction also and would be expected to be observed propagating away from local noon. Thus the further the signal is from noon, the larger its propagation angle, which would be shown by a high m-number. This feature appears in the results but is by no means conclusive.

12. As discussed in the model the speed on the ground could be measurements of the delay in the signal’s arrival on the ground for consecutive field lines. As the total distance between these is relatively low this would register as a low ground speed, which is shown in the results. The modelled cross-phase profiles compared well with those of the results which seems to support this claim.

13. The results show that ellipticity is generally low, indicating linear polarized waves. This suggests that when the fast mode waves are converted poorly they maintain their linear polarization with only a fraction converted to elliptical. It seems reasonable that waves which couple poorly with the geomagnetic field would retain the polarization of the source.

14. Given the coherence length of 1600 km one would not expect to observe coherent signals at MACCS which is located roughly 4500 km from IMAGE. Some of the more coherent signals may make the distance, as shown in the results, but these would be the exception rather than the rule.

15. The reliability in the results produced for the conjugate points is questionable, as discussed in section 6.2.1. However the results suggest that while the signal is observed at conjugate points there are no odd or even modes apparent.
This supports the claim that the waves are not generated by standing field line oscillations at all. If the field guided propagation model is correct then the phase between the two points would be a delay in the arrival of the signals at each conjugate point (assuming the waves propagate in both hemispheres in the same manner). This would hence be a function of field length and Alfvén velocity as in the cross-phase profiles in a single hemisphere. For example, around equinox the geomagnetic field symmetry about the equatorial plane would mean the arrival of the signal almost instantaneously at conjugate points. This has not been shown in this thesis, but is a prediction for future work on high-latitude Pc 3–4 wave conjugacy.

We believe that the field-guided model is the best for describing the results shown in chapter 5. While there remain discrepancies between the model and results we are confident that further research will help to resolve these.

6.3 Nighttime Events

Our results in chapter 5 have identified a new class of Pc 3–4 event, those which occur in the local nighttime. The nighttime amplitude profile reveals a peak which is unrelated to local time, frequency, $K_p$ or Dst index, and does not coincide with the location of the plasmapause. We remain sceptical that these waves are in fact Pc 3–4 but they do not display known features of Pi 2 signals either. The only suggestion we can offer is that these may be a new type of nighttime Pc 3 wave previously unseen or dismissed. Indeed the latter is tempting because the majority of the nighttime signals were identified within broadband substorm activity. Further research into these waves is clearly required.
CHAPTER 7

SOURCES OF ERROR AND CONCLUSIONS

The objectives of this project were twofold. The first was to improve the existing techniques for ULF signal extraction from a noise background and the second was to conduct a comprehensive ground study of the Pc 3–4 waves. The events were treated as a case study analysis but a large enough number of events were identified to perform a statistical analysis as well. The first limitation in this work is that it has been focused primarily on the ground. A complete study would involve both ground and satellite observations, with satellites placed at all of the major regions believed to influence ULF wave behaviour, i.e. the upstream solar wind, magnetosheath, plasmatrough, plasmasphere and ionosphere. The IMAGE magnetometer array maps to all of these regions, which is why the array was chosen, but only records the waves at the end of their journey through the magnetosphere.

7.1 Analytical and Result Limitations

Many features have appeared in the results which we believe are not physical properties of the signals studied. Here we will discuss the limits and caveats of the results presented in this project.

7.1.1 Timing Error Correction

There are two fundamental difficulties in the study of mid-frequency ULF waves using the IMAGE array. The first is the sample rate of 10 seconds. This rate represents a large percentage of the period of a Pc 3 signal and results in a small number of data points per wave cycle. In order to present a convincing sinusoidal signal it is therefore necessary for the wave to continue for several cycles, thus screening out the more impulsive signals. It also reduces the Nyquist frequency to 50 mHz, thus limiting our study to less than half of the Pc 3 frequency range (which continues until 100 mHz). Here we will note that the Pc scale was an arbitrary designation, based almost entirely on period ranges before any knowledge of signal generation and propagation was obtained (Jacobs et al., 1964). Indeed, studies conducted by the author and colleagues with magnetometers of higher sampling rates (not shown in this thesis) have suggested that no large amount of Pc 3 activity occurs at frequencies higher than 50 mHz. We do not believe that we are missing any vital information by not extending our frequency range beyond the Nyquist
frequency for a 10 second sample rate.

Timing errors have also been found in several of the stations. This is because a deviation of a few seconds was deemed insignificant for the IMAGE array, which was designed for the study of waves with much larger periods. For Pc 3–4 analysis, however, even a one-second deviation can significantly alter the phase of the wave between two stations (11° for a 30 mHz signal). Also, no test signal is included with the data which makes comparison of baseline effects such as timing errors very difficult. The only way we could examine this was to inspect the raw data and attempt to correct any baseline phase distortion across large periods of time. This assumes the baseline noise level has zero phase which may not be entirely accurate. As power spectra with large FFT lengths produce large noise contributions, the "fine-tuning" of this timing correction technique was not possible, and we believe that any timing error < 2s would not be detected. In all of the cases where a timing error was identified (table 4.2) the error was a fraction of the 10 second sample rate. This meant that a linear interpolation between data points was required, which makes an assumption of a linear fit between adjacent points which may be inaccurate. The error bars in our phase plots represent an assumption of the lower limit of our accuracy in timing error location (2.5 seconds) but should be regarded with some suspicion. Also, there were three stations which were excluded from the results as we were unable to completely correct any timing errors. These were Hornsund (HOR), Tromsø (TRO) and Andenes (AND) and we are uncertain as to whether these errors were due to internal or external interference.

7.1.2 Event Selection

Event selection is based on a signal’s distinction from the designated noise. The definition of noise is a largely subjective issue and there has been a great deal of confusion with regards to noise especially for signals in the Pc 3–4 frequency band. **** INCLUDE REVIEW ON EVENT SELECTION **** Even in the cases where coherence has been used an adequate analysis of statistical validity of this parameter has been lacking. Our study attempts to produce a standard technique of event distinction based on coherence measurements, including a rigorous analysis of the statistical implications of our analytical techniques. The size of the error bars for our coherence calculations is another indication of the inadequacy of the sample rate of the magnetometers. Our assigning of a coherence of zero for noise was also probably overstated. By comparing the peak in coherence with a peak in cross-power we have corrected this assumption to a small degree but a thorough analysis of background noise coherence should be investigated before more ade-
quate coherence designation can be made. This is a recommended topic for future work.

### 7.1.3 Ground Profiles

#### Amplitude

As discussed in section 5.3.1 a common feature in the daytime plots is the small peak (or dip) which occurs in the region of stations from 63.0° and 67° CGM latitude. We do not believe that this is a feature of the Pc 3–4 wave observed, but rather some external distortion of the signal, possibly by ground induction effects such as those near Masi (Viljanen et al., 1995). This assumption is supported by the IMAGE results of Mathie et al. (1999a; 1999b), Mathie and Mann (2000) and Howard and Menk (2001) which also reveal this distortion despite the fact that the former three were conducted at Pc 5 frequencies.

There are two limitations to the values of amplitude produced here. The first is the limit of reading of the signal, which at 0.1 nT is represented in the error bars given in the amplitude profile (figure 4.6a). We suspect that the amplitude readings may be more accurate than this as features occasionally appeared at amplitudes lower than this limit of reading. The second is the technique we have used to convert the power reading back into nT. As discussed in section ?? this included the addition of contributions of power from the neighbouring side bands in order to minimise leakage due to the FFT procedure. This includes the risk that external signals at frequencies in these side bands will be introduced into the amplitude value. Here we have set an absolute value of 2.5 mHz for each signal when this should in fact be frequency dependent so as to maintain a constant $f/\Delta f$ ratio. Testing of this variation by the author (not shown in this thesis) suggests that the affect is minimal, and we do not believe that the exclusion of this step would greatly influence the amplitude values produced.

#### Coherence

Coherence (and coherency) remains a controversial and often misused parameter in the determination of signal reliability. As demonstrated in chapter 3 a complete statistical analysis is required to achieve a meaningful value of coherence. This includes a step-by-step consideration of every analytical procedure performed on the data (e.g. filtering, windowing) as each of these make a contribution to the statistical validity. The key parameter here is the number of degrees of freedom, increasing this value increases the validity, but also the bias. As discussed in section
3.6.4 care must be taken to balance the two, and the large error bars shown in the coherence profiles demonstrate this conflict. There are several ways in which the number of degrees of freedom can be improved, all of which have a bias side-effect as demonstrated in table 7.1.

Table 7.1: Techniques to improve the degrees of freedom and resulting side-effects.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Side Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Increase the number of sub-windows used in the FFT.</td>
<td>Frequency resolution decreases.</td>
</tr>
<tr>
<td>2. Increase the range of the limit for signal/noise from 0–0.65.</td>
<td>Increase in the risk of introducing noise as a coherent signal.</td>
</tr>
<tr>
<td>3. Reduce confidence from 87%.</td>
<td>“Trust” in the validity of results decreases.</td>
</tr>
<tr>
<td>4. Change the window used.</td>
<td>Other windows may not be as effective at removing leakage artifacts.</td>
</tr>
<tr>
<td>5. Smooth the data instead of using averaged subwindows.</td>
<td>Low frequency bias - high frequency contributions removed.</td>
</tr>
</tbody>
</table>

As mentioned in chapter 5 the coherence profiles present two difficulties:

1. Coherence measurements are taken for both signal and noise contributions, meaning coherence profiles reflect whichever is dominant. Those with dominant noise will measure a coherent signal only across the 240 km region covered by spatial integration (§2.7.2).

2. Limited degrees of freedom produce large errors in the coherence values, preventing any meaningful values being extracted from the profiles.

It is for these reasons that the coherence length calculations were obtained using the mathematical estimate technique described in section 5.3.3. One further result was that the D-component coherence values were generally higher than the H-component. The reason for this is unknown.

Even with a thorough statistical analysis the validity of coherency is still limited. As demonstrated by Samson (1980) **CHECK **** coherency is a unidirectional parameter, i.e. it varies with rotation of the axes along which the measurements are made. The subsequent mathematical derivation showed that a more accurate means to determine signal coherency is with the degree of polarization between the two axes, because of its invariability with direction. Furthermore, Samson (1980) showed that in the $n = 2$ case the degree of polarization is the same as the...
maximum coherence for all directions. There are currently developments by the author and coworkers toward the development of a technique to measure the degree of polarization for a pair of stations, i.e. for the $n = 4$ case.

**Cross-Phase**

Problems and limitations in cross-phase determination have already been discussed. We will note here that the same statistical evaluation was conducted for cross-phase as for coherence as shown in section 3.6.6. We found that these values, while at times significant, were not as large as the errors we imposed from the timing error limitation. Future analyses in cross-phase using the averaged window technique should be aware of such a statistical approach, as any attempts to improve the quality of the phase values will inevitably be subject to statistical limitations.

There is a large gap in the IMAGE magnetometer array in the Arctic ocean between Sørsøya ($L = 6.8$) and Bear Island ($L = 9.9$), and many of the FLR’s and our suspect harmonics occurred in this gap. This $L$ region corresponds to that at which a great deal of FLR and related activity occurs. Nevertheless, we cannot exclude the possibility that some external property (such as coastal effects from the proximity of a deep, saltwater ocean) can be influencing the phase values in this region. More study is required, preferably with an array which can concentrate stations in this $L$ region without ocean restrictions.

One further problem in the cross-phase values is the $2\pi$ ambiguity resolution. We have applied every technique we could to account for this discrepancy, but in some cases the results were still unclear. We note this as a possible reason for why some of our events have a phase skip in the opposite direction to that expected.

**7.1.4 Spatial extent of signals, MACCS**

There are two problems which arise in the production of the MACCS results. The first is the fact that the majority of the stations lie in the polar cap region. This would obscure any data related to field line resonances as they cannot be sustained on open field lines. The second is the longitudinal extent of MACCS which exceeds that of the latitudinal. Phase values may be distorted here as the linear assumption we applied to the latitude phases (for consistency with the technique used for IMAGE) may be better served when applied to the longitude values at MACCS.
7.1.5 Conjugate Points, Davis:Longyearbyen

The sources of errors associated with the conjugate point study conducted in this thesis is given in section 6.2.1. We conclude that the results produced here for the conjugate points should be regarded with suspicion. More work is clearly required here, perhaps with a station pair which more closely approximate conjugate points and are further from the cusp region.

7.2 Conclusions

Based on our results produced in this study we have identified four classes of high-latitude Pc 3—4 ULF wave. With the exception of the nighttime events these are believed to be created by the same generation mechanism, i.e. ion-cyclotron resonance in the upstream solar wind. These fast mode waves enter the magnetosphere and propagate to the ground via the processes listed below.

1. Broadband, spatially-limited wave activity localised in the near-cusp regions. While these waves were not the focus of this study we believe these may dominate this region of the magnetosphere. They are produced either by modulated precipitating electrons from the magnetosheath or by cavity modes near the cusp.

2. Broadband nighttime signals associated with substorm activity. These appear to be a new discovery and we have conducted a preliminary analysis of their properties, but cannot account for their production. We are considering a future investigation into the properties of these waves.

3. Higher harmonics of the fundamental Pc 5 field line resonance. The incoming fast mode waves mode convert to a standing shear Alfvén wave and excites the 4th, 5th or 6th order harmonic. If coupling is efficient or not, these could appear in the absence of the fundamental Pc 5. This mechanism accounts for approximately 25% of the narrowband, spatially coherent Pc 3—4 waves observed on the ground in the local daytime.

4. Earthward propagating waves account for the remaining 75% of the spatially coherent signals. There are two possibilities for the propagation mechanism of these waves.

   (a) The fast mode waves propagate through the magnetosphere along optical path lengths dictated by Fermat’s Principle and are refracted and
7.2. CONCLUSIONS

diffracted to the high latitude ionosphere. This process is shown in figure 7.1. Our results suggest that approximately half of the propagating waves may be explained by this process.

(b) When compared with 5 non-FLR harmonic events we found that **** of the events showed characteristics similar to those predicted by this model. We believe this model is the more correct to describe this type of Pc 3–4 wave.

Figure 7.1: An illustration of the fast mode “direct” propagation model. Incoming fast mode waves are refracted along refractive index contours to the high-latitude ionosphere. Waves of too large and entry angle are refracted away.
Figure 7.2: An illustration of the Alfvén mode model proposed in this thesis. Incoming fast mode waves are converted to an Alfvén mode upon interaction with geomagnetic field lines favourable to support resonance harmonics. Remaining fast mode energy continues to propagate inward and couples with further field lines during its journey.
APPENDIX A

PUBLICATIONS RESULTING FROM THE WORK PRESENTED IN THIS THESIS

A.1 Conference Presentations


### A.2 Publications


APPENDIX B

SUMMARY OF RESULTS

B.1 Results for Each Event

NOTE: In all cases the events cited in text are given in boldface while those used in the mathematical model are given as italics.

Table B.1: Information and results from the ground IMAGE study. Results for both H- and D- components are shown.
Table B.1 (continued).
Table B.1 (continued).
Table B.1 (continued).
Table B.2: The location of the plasmapause in $L$-value (where appropriate) and CGM latitude and relating information for the three models discussed in section 2.6.
Table B.2 (continued).
Table B.2 (continued).
Table B.3: Solar wind parameters from the WIND satellite. Cone angle has been defined as described in section 2.2.4. “Middle”, “Upper” and “Lower” refer to the central value and the upper and lower error limits respectively. (y, “y”, “n”, n) indicate a fit within the error limits, a deviation of $\leq 2$ mHz and $\leq 5$ mHz, and outside the 5 mHz limits. Checks with the DMSP data are also given.
Table B.3 (continued).
Table B.3 (continued).
Table B.4: The locations (CGM latitude) of the maximum amplitude and the magnetopause as predicted in the three models discussed in section 2.5.1. “Mid”, “Up” and “Low” refer to the central value and the upper and lower error limits respectively.
Table B.4 (continued).
Table B.4 (continued).
Table B.5: Comparison of the maximum amplitude with the magnetopause calculations. (y, “y”, “n”, n) indicate a fit within the error limits, a deviation of $\leq 2^\circ$ and $\leq 5^\circ$, and outside the $5^\circ$ limits.
Table B.5 (continued).
Table B.5 (continued).
B.2 Events Used in the Mathematical Model

Table B.6: Events used in the mathematical model in section ???. Relevant parameters for determination of the Alfvén and T96 model and time of flight calculations are included. Values for the density (\(\rho\)) and spacecraft altitude (S/C Alt.) for obtained from the LANL-097A satellite. The \(y\) and \(z\) components of the interplanetary magnetic field (\(B_{IMFy}\), \(B_{IMFz}\)), and Dynamic pressure (Dyn.P) were taken from the WIND satellite.

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<th>Dst</th>
<th>Dyn.P</th>
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<th>B_{IMFz}</th>
<th>(\rho)</th>
<th>S/C Alt.</th>
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APPENDIX C

INDICES AND PARAMETERS

C.1 $K_p$ and Dst Indices

Table C.1: $K_p$ values for January, 1998 as obtained from the National Geophysical Data Center (NGDC) webpage (Morris, 2002).
Table C.2: $K_p$ values for March, 1998 as obtained from the National Geophysical Data Center (NGDC) webpage (Morris, 2002).

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<td>1+</td>
<td>1+</td>
<td>0+</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Table C.3: Dst values for January and March, 1998 as obtained from the National Geophysical Data Center (NGDC) webpage (Morris, 2002). Values for time 0000–1200 are shown.
Table C.4: Dst values for January and March, 1998 as obtained from the National Geophysical Data Center (NGDC) webpage (Morris, 2002). Values for time 1200–2400 are shown.
### C.2 Magnetic Declination (D) Angles

Table C.5: Magnetic declination angles as obtained from the Corrected Geomagnetic Coordinates Webpage (Papitashvili, 2001) for all the ground stations used in the analysis. All station which were in geographic coordinates required a rotation by this value to convert to geomagnetic coordinates. D values correspond to an epoch of 1998 and altitude of 100 km.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Station name</th>
<th>Declination Angle (D)</th>
<th>Abbreviation</th>
<th>Station name</th>
<th>Declination Angle (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMAGE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NAL</td>
<td>Ny Ålesund</td>
<td>−1.30°</td>
<td>HAN</td>
<td>Hanksalimi</td>
<td>+6.86°</td>
</tr>
<tr>
<td>Lyr</td>
<td>Longyearbyen</td>
<td>+1.82°</td>
<td>NUR</td>
<td>Nurmiyärvi</td>
<td>+5.66°</td>
</tr>
<tr>
<td>HOR</td>
<td>Hornsund</td>
<td>+1.82°</td>
<td>UPS</td>
<td>Uppsala</td>
<td>+2.56°</td>
</tr>
<tr>
<td>HOP</td>
<td>Hopen Island</td>
<td>+8.60°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BJu</td>
<td>Bear Island</td>
<td>+4.50°</td>
<td>RE</td>
<td>Resolute Bay</td>
<td>−36.56°</td>
</tr>
<tr>
<td>SOR</td>
<td>Sørøya</td>
<td>+6.13°</td>
<td>CY</td>
<td>Clyde River</td>
<td>−48.21°</td>
</tr>
<tr>
<td>KEV</td>
<td>Kevo</td>
<td>+8.81°</td>
<td>CB</td>
<td>Cambridge Bay</td>
<td>+18.56°</td>
</tr>
<tr>
<td>TRO</td>
<td>Tromsø</td>
<td>+4.06°</td>
<td>GH</td>
<td>Gjoa Haven</td>
<td>−6.24°</td>
</tr>
<tr>
<td>MAS</td>
<td>Masi</td>
<td>+6.85°</td>
<td>PB</td>
<td>Pelly Bay</td>
<td>−23.43°</td>
</tr>
<tr>
<td>AND</td>
<td>Andenes</td>
<td>+2.27°</td>
<td>RB</td>
<td>Repulse Bay</td>
<td>−25.37°</td>
</tr>
<tr>
<td>KIL</td>
<td>Kilpisjärvi</td>
<td>+5.09°</td>
<td>BK</td>
<td>Baker Lake</td>
<td>−0.55°</td>
</tr>
<tr>
<td>ABK</td>
<td>Abisko</td>
<td>+3.88°</td>
<td>CD</td>
<td>Cape Dorset</td>
<td>−32.40°</td>
</tr>
<tr>
<td>MUO</td>
<td>Muonio</td>
<td>+7.60°</td>
<td>IQ</td>
<td>Iqaluit</td>
<td>−35.75°</td>
</tr>
<tr>
<td>KIR</td>
<td>Kiruna</td>
<td>+4.74°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SOD</td>
<td>Sodankylä</td>
<td>+8.04°</td>
<td>MAW</td>
<td>Mawson</td>
<td>−65.63°</td>
</tr>
<tr>
<td>PEL</td>
<td>Pello</td>
<td>+6.60°</td>
<td>DAV</td>
<td>Davis</td>
<td>−78.06°</td>
</tr>
</tbody>
</table>
C.3 Values used For the Plasmapause Model

Table C.6: The values used to produce the plot in figure 2.22 and the values from which the L value of the plasmapause at $K_p = 2$ was determined (from Orr and Webb, 1975).

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Station name</th>
<th>Declination Angle (D)</th>
<th>Abbreviation</th>
<th>Station name</th>
<th>Declination Angle (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMAGE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>NAL</td>
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<td>+6.86°</td>
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<td>Nurmijärvi</td>
<td>+5.66°</td>
</tr>
<tr>
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<td>Hornsund</td>
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<td>UPS</td>
<td>Uppsala</td>
<td>+2.56°</td>
</tr>
<tr>
<td>HOP</td>
<td>Hopen Island</td>
<td>+8.60°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B Jain</td>
<td>Bear Island</td>
<td>+4.50°</td>
<td>RE</td>
<td>Resolute Bay</td>
<td>−36.56°</td>
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<td>CY</td>
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<tr>
<td>KEV</td>
<td>Kevo</td>
<td>+8.81°</td>
<td>CB</td>
<td>Cambridge Bay</td>
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</tr>
<tr>
<td>TRO</td>
<td>Tromsø</td>
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<td>+2.27°</td>
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<td>Repulse Bay</td>
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</tr>
<tr>
<td>KIL</td>
<td>Kilpisjärvi</td>
<td>+5.09°</td>
<td>BK</td>
<td>Baker Lake</td>
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<tr>
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<td>+3.88°</td>
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<tr>
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<td>Kiruna</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>SOD</td>
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<td>Mawson</td>
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<tr>
<td>PEL</td>
<td>Pello</td>
<td>+6.60°</td>
<td>DAV</td>
<td>Davis</td>
<td>−78.06°</td>
</tr>
</tbody>
</table>

Note: The Declination Angle is measured from the magnetic equator to the station's magnetic declination.
APPENDIX D

MAGNETOMETER DATA CONVERSION PROCESSES

D.1 The IMAGE Magnetometer Array Data Conversion

This section provides details on the conversion of the IAGA format from the IMAGE data to the ASCII format for the Newcastle data. An example of this is given in figure D.1.

Figure D.1: An example of the format used in data analysis. The header of this file includes the name of the station and coordinate (KIL = Kilpisjärvi; H = H-component), the year of the file, month (3 = March), day of the month, hour, minute and second of the first point in the file, the sample rate in seconds and the number of points in the file. The values in the four columns beneath are read from left to right and represent the amplitude (in nT) of each consecutive sample point).

<table>
<thead>
<tr>
<th>KILH 1998 3 4 0 0 0.0 10.0 8640</th>
</tr>
</thead>
<tbody>
<tr>
<td>10881.800 10881.700 10880.900 10880.800</td>
</tr>
<tr>
<td>10880.900 10881.400 10881.700 10882.100</td>
</tr>
<tr>
<td>10882.800 10883.400 10883.900 10884.300</td>
</tr>
<tr>
<td>10884.900 10885.400 10885.500 10885.600</td>
</tr>
<tr>
<td>10885.600 10885.600 10885.500 10885.300</td>
</tr>
<tr>
<td>10885.300 10885.400 10885.300 10885.100</td>
</tr>
<tr>
<td>10884.900 10885.000 10884.700 10884.300</td>
</tr>
<tr>
<td>10883.700 10883.400 10883.300 10883.000</td>
</tr>
</tbody>
</table>

The York format was developed by D.K. Milling and R.J. Bunting in 19** and the IAGA-York transformation software by R.J. Bunting. The York-Newcastle software was written by F.W. Menk in 19**. The complete transformation from IAGA to Newcastle was made at York by F.W. Menk in the following manner:

1. The data were split into a header containing the station information and time details, and a column (rather than a row) of numbers. This was done via a C program called recsplit.c and output a file in the form yyyymmddhh.iaga.split.

2. A FORTRAN program read_image.f read the IAGA format and converted it to a binary format for the University of York. The output file here was in the
APPENDIX D. MAGNETOMETER DATA CONVERSION PROCESSES

3. The geographic coordinates of each station in the file were rotated into CGM coordinates using the program coord_change.f. This program contained an array of declination (D) angles (Appendix C.2) for each IMAGE station, obtained from the CGM coordinates webpage (http://nssdc.gsfc.nasa.gov/space/cgm/cgm.html). For example, the angle of rotation for NAL was $-2.37^\circ$ while that of LYR was $+0.79^\circ$. The epoch data for this transformation was January 1998.

4. The program coord_change.f recorded the data with its new coordinate system to a file of type sss_ddmmyy.hd, where ‘sss’ represents the abbreviation of the station (e.g. KIL).

5. The data were then converted from York binary to Newcastle ASCII format using ****.

6. Finally, the new files in *.idl format were transferred to the computers at Newcastle by the File Transfer Process (FTP).

D.2 The Antarctic Magnetometer Array Data Conversion

Conversion of data from ADAS to Newcastle format was made according to the following process:

1. Header information containing the station name, date, time and sample rate were extracted from the ADAS file via a subroutine magre.pro. This program also extracted the data and checked that no data were missing or corrupted.

2. Once extracted a correction factor needed to be applied to the x and y channels to compensate for gains in the signal of one relative to the other. This was achieved with another IDL subroutine, induction_compensation.pro. The Davis x channel required a reduction of 1.07664 while no correction was needed for the Mawson system.

3. Using the main program, trueind.pro, the DC component was removed by removing a running mean from the data set. Conversion into nT was then made using calibration information obtained from the engineers stationed at each Antarctic base (REFERENCE) and the author (Howard, 2000). For 1998 the Davis data set required a rescaling by a factor of $2.294 \times 10^{-3}$ and
Mawson by $2.304 \times 10^{-3}$. No file was created with this program, rather the calibrated data remained in specified arrays created by the IDL program.

4. Conversion to Newcastle format was made with the program \textit{adas2idl.pro}. This wrote the data from the IDL arrays into ASCII files in the Newcastle format.

5. Finally, the data were rotated into geomagnetic coordinates using the declination (D) angles for MAW and DAV obtained from table C.2. The $X$ and $Y$ coordinates were first converted to $H$ and $D$ using the following equations:

\begin{equation}
H = \sqrt{X^2 + Y^2}, \quad \text{(D.1)}
\end{equation}

\begin{equation}
D = \tan^{-1}(Y/X). \quad \text{(D.2)}
\end{equation}

The declination angles were then added to the $D$ value in equation (D.2) and the $X$ and $Y$ components restored using

\begin{equation}
X = H \cos D, \quad \text{(D.3)}
\end{equation}

\begin{equation}
Y = H \sin D. \quad \text{(D.4)}
\end{equation}
APPENDIX E
DERIVATION OF MATHEMATICAL FORMULAE

E.1 Frequency and $B_{\text{IMF}}$ Relationship (§2.2.4)

We begin with equation (2.16), which is the general expression for the right-handed waveform in the upstream solar wind:

$$\omega_r - \mathbf{k} \cdot \mathbf{v}_{0j} + \Omega_j = 0$$

from the definition of the vector dot product. Also by definition, $|k| = 2\pi/\lambda$ and $v_{ph} = \omega/|k| \Rightarrow |k| = \omega/v_{ph}$. So,

$$\omega_r - (\omega_r/v_{ph})v_{0j} \cos \theta_{kB} + \Omega_j = 0$$

$$\Rightarrow \omega_r(1 - (v_{0j}/v_{ph}) \cos \theta_{xB}) + \Omega_j = 0$$

$$\Rightarrow \omega_r = \frac{\Omega_p}{(v_{0j}/v_{ph}) \cos \theta_{xB} - 1},$$

which is the expression shown in equation (2.17). Now we apply the doppler-shifted wave frequency expression given in equation (2.18):

$$-\omega_{s/c} = \omega_r - \mathbf{k} \cdot \mathbf{v}_{sw}.$$  

Now, $\mathbf{k} \cdot \mathbf{v}_{sw} = |k||v_{sw}| \cos \theta_{kx} = (\omega_r/v_{ph})v_{sw} \cos \theta_{kx}$. So equation (E.3) becomes

$$\omega_{s/c} = -\omega_r - \omega_r(v_{sw}/v_{ph}) \cos \theta_{kx}$$

$$= -\omega_r[1 - (v_{sw}/v_{ph}) \cos \theta_{kx}]$$

$$= \omega_r[(v_{sw}/v_{ph}) \cos \theta_{kx} - 1].$$

Now, substituting the expression for $\omega_r$ shown in in equation (E.2) into the above equation reveals

$$\omega_{s/c} = \Omega_p(v_{sw}/v_{ph}) \cos \theta_{kx} - 1$$

$$\Omega_p(v_{0j}/v_{ph}) \cos \theta_{kB} - 1,$$

which is the relationship described in equation (2.19).

We now move to the definition of the bulk velocity of the reflected wave, described by equation (2.22):

$$\mathbf{v}_{0j} = \mathbf{v}_r = (v_{||i} + v_{||r}) \hat{B}.$$  

(E.6)
Here we decompose this expression in terms of the vectors given in figures 2.5. From this figure,

\[ \hat{n} \cdot \hat{x} = \cos \varphi = \frac{v_n}{v_{sw}}, \quad (E.7) \]
\[ \hat{n} \cdot \hat{B} = \cos(\psi - \varphi) = \frac{v_n}{v_{||i}}, \quad (E.8) \]

where the hat above vector \( \mathbf{v} \) (\( \hat{\mathbf{v}} \)) represents the unit vector in the direction of \( \mathbf{v} \).

So,

\[ \hat{n} \cdot \hat{x} \hat{n} \cdot \hat{B} = \frac{v_n}{v_{sw}} \frac{v_n}{v_{||i}} = \frac{v_{||i}}{v_{sw}}, \quad (E.9) \]

Therefore we can apply the following definition:

\[ \mathbf{v}_{0j} = (1 + \delta) \left( \frac{\hat{n} \cdot \hat{x}}{\hat{n} \cdot \hat{B}} \right) v_{sw} \hat{B}, \quad (E.10) \]

which is the expression described by equation (2.23). Setting \( \delta = 1 \) and applying vector geometry we have

\[ v_{0j} = 2v_{sw} \frac{|\hat{n}||\hat{x}| \cos \theta_{nx}}{|\hat{n}||\hat{B}| \cos \theta_{nB}}, \quad (E.11) \]

This is the equation described in (2.24).

### E.2 The Polarization Equations (§3.5)

We begin with the \( x \) and \( y \) components of the waves \( \mathbf{E}_x \) and \( \mathbf{E}_y \), given by equation (3.12) and including them in trigonometric form:

\[ \mathbf{E}_x = a e^{i(\omega t - k_x z)} \hat{x} = a[\cos(\omega t - k_x z) + i \sin(\omega t - k_x z)] \hat{x} \]
\[ \mathbf{E}_y = b e^{i(\omega t - k_x + \phi) z} \hat{y} = b[\cos(\omega t - k_x + \phi) + i \sin(\omega t - k_x + \phi)] \hat{y}. \quad (E.12) \]

Now we equate the real and imaginary parts of the above equation and let \( \text{Re}\{\mathbf{E}_x\} = \mathbf{E}_x \) and \( \text{Re}\{\mathbf{E}_y\} = \mathbf{E}_y \),

\[ \frac{E_x}{a} = \cos \omega t, \]
\[ \frac{E_y}{b} = \cos(\omega t + \phi) = \cos \omega t \cos \phi - \sin \omega t \sin \phi, \quad (E.13) \]

as shown in in equation (3.14). From equation (E.13),

\[ \frac{E_x^2}{a^2} = \cos^2 \omega t, \]
which, on substitution into (E.13) reveals

\[ \frac{E_y}{b} = \frac{E_x}{a} \cos \phi - \sqrt{1 - \frac{E_x^2}{a^2}} \sin \phi \]

\[ = \frac{E_x}{a} \cos \phi - \sqrt{1 - \frac{E_x^2}{a^2}} \sin \phi, \]

as shown in equation (3.15). Now, rearranging,

\[ E_y - \frac{E_x}{a} \cos \phi = \sqrt{1 - \frac{E_x^2}{a^2}} \sin \phi \]

\[ \Rightarrow \frac{E_y^2}{b^2} - \frac{2E_xE_y}{ab} \cos \phi + \frac{E_x^2}{a^2} \cos^2 \phi = (1 - \frac{E_x^2}{a^2}) \sin^2 \phi \]

\[ \Rightarrow \frac{E_y^2}{b^2} - \frac{2E_xE_y}{ab} \cos \phi + \frac{E_x^2}{a^2} \cos^2 \phi = \sin^2 \phi - \frac{E_x^2}{a^2} \sin^2 \phi \]

\[ \Rightarrow \frac{E_y^2}{b^2} + \frac{E_x^2}{a^2} \cos \phi - \frac{2E_xE_y}{ab} \sin \phi = 0, \]

which is the equation of an ellipse given in equation (3.16).

Now we derive an expression relating \( \psi \) to \( a \) and \( b \). We begin with equation (3.17) where \( F = F(E_x, E_y) \).

\[ dF = \frac{\partial F}{\partial E_x} dE_x + \frac{\partial F}{\partial E_y} dE_y. \] (E.14)

Now let \( F = \sqrt{E_x^2 + E_y^2} \) and apply to equation (E.14),

\[ \frac{\partial F}{\partial E_x} = 2E_x(E_x^2 + E_y^2)^{-1/2} \] (E.15)

\[ \frac{\partial F}{\partial E_y} = 2E_y(E_x^2 + E_y^2)^{-1/2}. \] (E.16)

Also, at a maximum, \( dF = 0 \) and so application into (E.14) reveals

\[ E_x dE_x + E_y dE_y = 0, \] (E.17)

as given in equation (3.18). Now, let \( F \) be the ellipse described by equation (3.16). Then

\[ \frac{\partial F}{\partial E_x} = \frac{E_x}{a^2} - \frac{\cos \phi}{ab} E_y \] (E.18)

\[ \frac{\partial F}{\partial E_y} = \frac{E_y}{b^2} - \frac{\cos \phi}{ab} E_x, \] (E.19)

and applying these to equation (E.14), with \( dF = 0 \) reveals

\[ \left( \frac{E_x}{a^2} - \frac{\cos \phi}{ab} E_y \right) dE_x + \left( \frac{E_y}{b^2} - \frac{\cos \phi}{ab} E_x \right) dE_y = 0, \] (E.20)

which is the same as equation (3.19) in the text. Now we apply the definition of \( \psi \) such that the major axis of the ellipse is oriented at an angle given by \( \tan \psi = \frac{E_y}{E_x} \).

Equation (E.20) becomes

\[ \left( \frac{1}{a^2} - \frac{\cos \phi}{ab} \tan \psi \right) E_x dE_x + \left( \frac{1}{b^2} - \frac{\cos \phi}{ab} \cot \psi \right) E_y dE_y = 0. \]

\[ \Rightarrow \left( \frac{1}{a^2} - \frac{\cos \phi}{ab} \tan \psi \right) E_x dE_x + \left( \frac{1}{b^2} - \frac{\cos \phi}{ab} \cot \psi \right) E_y dE_y = 0. \]
So,

\[
\left(\frac{1}{a^2} - \frac{\cos \phi}{ab} \tan \psi\right) E_x dE_x = \left(\frac{1}{b^2} - \frac{\cos \phi}{ab} \cot \psi\right) E_y dE_y = 0.
\]

\[\Rightarrow \frac{1}{a^2} - \frac{1}{b^2} = \frac{\cos \phi}{ab} (\tan \psi - \cot \psi)\]

\[\Rightarrow \frac{b^2 - a^2}{ab \cos \phi} = \tan \psi - \cot \psi.
\]

Now,

\[
\tan 2\psi = \frac{\sin 2\psi}{\cos 2\psi} = \frac{2\sin \psi \cos \psi}{\cos^2 \psi - \sin^2 \psi} = \frac{2\tan \psi}{1 - \tan^2 \psi}
\]

\[\Rightarrow \cot 2\psi = \frac{1}{\tan 2\psi} = \frac{1}{2 \left(\frac{1}{\tan \psi} - \frac{1}{\tan \psi}\right)} = 1/2\{\cot \psi - \tan \psi\}
\]

\[\Rightarrow \tan \psi - \cot \psi = -2 \cot 2\psi.
\]

So,

\[-2 \cot 2\psi = \frac{b^2 - a^2}{ab \cos \phi}\]

\[\Rightarrow -1/2 \tan 2\psi = \frac{ab \cos \phi}{b^2 - a^2} \Rightarrow \tan 2\psi = \frac{2ab \cos \phi}{a^2 - b^2},
\]

which is equation (3.21) in the text.

We now derive the values of \(A\) and \(B\), the lengths of the major and minor axes. As indicated in the notes (section ??) to determine this value we must solve equations (3.16) and (??) simultaneously. We begin with the result of substituting the positive component of equation (??) into (3.16).

\[
\frac{B^2 E_y^2}{b^2} + \frac{E_x^2}{a^2} - \frac{2E_y E_x}{ab} \cos \phi - \sin^2 \phi = 0
\]

\[\Rightarrow \frac{E_y^2}{a^2} + \frac{E_x^2}{a^2} - \frac{2E_y E_x}{ab} \cos \phi - \sin^2 \phi = 0
\]

\[\Rightarrow \frac{2E_y^2}{a^2} - \frac{2E_x^2}{a^2} \cos \phi - \sin^2 \phi = 0
\]

\[\Rightarrow \frac{2E_x^2}{a^2} (1 - \cos \phi) = \sin^2 \phi
\]

\[\Rightarrow E_x = \pm a \sin^2 \phi
\]

\[\Rightarrow E_x = \pm \frac{a \sin \phi}{\sqrt{2 - 2 \cos \phi}}.
\]

And, on substitution back into (??),

\[E_y = \pm \frac{b}{a} \frac{a \sin \phi}{\sqrt{2 - 2 \cos \phi}} = \pm \frac{b \sin \phi}{\sqrt{2 - 2 \cos \phi}}.
\]

Similarly, using the negative part of equation (??) we have,

\[E_x = \pm \frac{a \sin \phi}{\sqrt{2 + 2 \cos \phi}},
\]

\[E_y = \pm \frac{b \sin \phi}{\sqrt{2 + 2 \cos \phi}}.
\]

Now, the magnitude of the vector to these points is defined as \(\sqrt{E_x^2 + E_y^2}\), which
for the major axis $A$ is

$$
A = \sqrt{\frac{(a^2 + b^2) \sin^2 \phi}{2 - 2 \cos \phi}}
$$

$$
\Rightarrow 2A = \sqrt{\frac{\sin \phi}{\sqrt{1 - \cos \phi}}} \sqrt{a^2 + b^2}
$$

$$
= \frac{\sin \phi}{\sqrt{1 - \cos \phi}} \sqrt{2a^2 + 2b^2}
$$

$$
= \sqrt{\frac{2(a^2 + b^2)}{1 - \cos \phi}} \sin \phi.
$$

Similarly for the minor axes,

$$
2A = \sqrt{\frac{2(a^2 + b^2)}{1 + \cos \phi}} \sin \phi,
$$
as shown in equation (??). Finally, apply the ellipticity equation described by (??),

$$
e^2 = 1 - \frac{B^2}{A^2}
$$

$$
= 1 - \left(\frac{2(a^2 + b^2)}{1 + \cos \phi}\right) \frac{\sin^2 \phi}{\sin^2 \phi}
$$

$$
= 1 - \left(\frac{1 - \cos \phi}{1 + \cos \phi}\right) \frac{\sin^2 \phi}{\sin^2 \phi}
$$

$$
= \frac{2 \cos \phi}{1 + \cos \phi},
$$
as described by equation (??).

### E.3 The Stokes Parameters (§3.5.2)

We begin with the expression for the coherency matrix, given (equation 3.23) as

$$
J = \begin{bmatrix}
< H_x H_x^* > & < H_x H_y^* > \\
< H_y H_x^* > & < H_y H_y^* >
\end{bmatrix}
$$

Using the definition of $H_x$ and $H_y$ described by equations (3.22) we have

$$
< H_x H_x^* > = [A_x e^{i(\varphi_x + \varphi)}] [A_x e^{-i(\varphi_x + \varphi)}] = A_x^2
$$

$$
< H_x H_y^* > = [A_x e^{i(\varphi_x + \varphi)}] [A_y e^{i(\varphi_y - \varphi)}] = A_x A_y e^{i(\varphi_x - \varphi_y)}
$$

$$
< H_y H_x^* > = [A_y e^{i(\varphi_y + \varphi)}] [A_x e^{-i(\varphi_x + \varphi)}] = A_y A_x e^{i(\varphi_y - \varphi_x)}
$$

$$
< H_y H_y^* > = [A_y e^{i(\varphi_y + \varphi)}] [A_y e^{-i(\varphi_y + \varphi)}] = A_y^2
$$

So the matrix $J$ becomes

$$
J = \begin{bmatrix}
A_x^2 & A_x A_y e^{i(\varphi_x - \varphi_y)} \\
A_x A_y e^{i(\varphi_y - \varphi_x)} & A_y^2
\end{bmatrix}
$$

(E.23)
By definition, det\(J\) for the \(2 \times 2\) matrix is thus
\[
J = A_x^2 A_y^2 [A_x A_y e^{(\varphi_y - \varphi_x)i}] [A_x A_y e^{(\varphi_y - \varphi_x)i}]
\]
\[
= A_x^2 A_y^2 - A_x^2 A_y^2 e^{(\varphi_y - \varphi_x)i}
\]
\[
= A_x^2 A_y^2 - A_x^2 A_y^2 e^{(\varphi_y - \varphi_x)i}
\]
\[
= 0,
\]
(E.24)
and the value of det\(J\) is thus 0.

Now we derive an expression for \(D\) which represents the elements of the unpolarized matrix \(U\). We begin with the four simultaneous equations derived from the matrix in equation (3.29),
\[
J_{xx} = A + D
\]
(E.25)
\[
J_{xy} = B
\]
(E.26)
\[
J_{yx} = B^*
\]
(E.27)
\[
J_{yy} = C + D.
\]
(E.28)
Adding equations (E.25) and (E.28) reveals
\[
J_{xx} + J_{yy} = A + C + 2D
\]
\[
\Rightarrow A + C = J_{xx} + J_{yy} - 2D.
\]
(E.29)
Now, from the matrix given in equation (3.29),
\[
\det J = |J| = (A + D)(C + D) - BB^*
\]
\[
= AC + AD + CD + D^2 - BB^*
\]
(E.30)
and \(AC - BB^* = 0\) (equation 3.25). Thus,
\[
D^2 + (A + C)D - |J| = 0
\]
(E.31)
Now, substituting equations (E.25) and (E.28) into (E.31),
\[
D^2 + (J_{xx} + J_{yy} - 2D)D - |J| = 0
\]
\[
\Rightarrow D^2 + (J_{xx} + J_{yy}) - 2D^2 - |J| = 0
\]
\[
\Rightarrow -D^2 + (J_{xx} + J_{yy}) - |J| = 0
\]
\[
\Rightarrow D^2 - (J_{xx} + J_{yy}) + |J| = 0,
\]
(E.32)
which describes a quadratic equation of independent variable \(D\). We now solve for \(D\) using the quadratic equation formula,
\[
D = \frac{J_{xx} + J_{yy} \pm \sqrt{(J_{xx} + J_{yy})^2 - 4|J|}}{2},
\]
(E.33)
or
\[
D = \frac{1}{2}(J_{xx} + J_{yy}) \pm \frac{1}{2}\sqrt{(J_{xx} + J_{yy})^2 - 4|J|},
\]
which is equation (3.30) in the text. If \( A > 0 \) and \( C > 0 \) then \( A + C > 0 \). So from equation (E.29),
\[
J_{xx} + J_{yy} - 2D > 0
\]
\[
\Rightarrow J_{xx} + J_{yy} > 2D
\]
\[
\Rightarrow D < \frac{1}{2}(J_{xx} + J_{yy}),
\]
which is required for the root with the negative sign in equation (3.30). Conversely, the root with the positive sign requires \( A \) and \( D \) to be negative, thus violating their initial definitions.

### E.4 Statistical Equations (§3.6)

We begin here with the “well known” trigonometric relations obtained from Chadwick (1989):
\[
\sum_{t=1}^{N} \cos \omega_p t = \sum_{t=1}^{N} \sin \omega_p t = 0
\]
\[
\sum_{t=1}^{N} \cos \omega_p t \cos \omega_q t = \begin{cases} 0 & p \neq q \\ N & p = q = N/2 \\ N/2 & p = q \neq N/2 \end{cases}
\]
\[
\sum_{t=1}^{N} \sin \omega_p t \sin \omega_q t = \begin{cases} 0 & p \neq q \\ 0 & p = q = N/2 \\ N/2 & p = q \neq N/2 \end{cases}
\]

And using the least squares estimate version of equation (3.40), given by equation (3.42):
\[
x_t = \hat{\mu} + \hat{\alpha} \cos(\omega_p t) + \hat{\beta} \sin(\omega_p t).
\]
Substituting the definition of \( \hat{\theta} = [\hat{\mu}, \hat{\alpha}, \hat{\beta}]^T \) from equation (3.41) into (E.40) we have:
\[
x_t = \hat{\mu} + \hat{\alpha} \cos(\omega_p t) + \hat{\beta} \sin(\omega_p t)
\]
\[
= \pi + 2 \sum_{t=1}^{N} x_t \cos(\omega_p t)/N + \pi + 2 \sum_{t=1}^{N} x_t \sin(\omega_p t)/N
\]
\[
= \pi + 2 \sum_{t=1}^{N} x_t \frac{\sum_{p=1}^{N/2-1} \cos(\omega_p t)/N + 2 \sum_{t=1}^{N} x_t \sum_{p=1}^{N/2-1} \sin(\omega_p t)/N + 
\sum_{t=1}^{N} x_t (-1)^t / N \cos \pi t}{\sum_{t=1}^{N} x_t (-1)^t / N \cos \pi t}
\]
\[
= \pi + 2 \sum_{t=1}^{N} x_t \frac{\cos(\omega_p t)/N + 2 \sum_{t=1}^{N} x_t \sum_{p=1}^{N/2-1} \sin(\omega_p t)/N }{\sum_{t=1}^{N} \sum_{p=1}^{N/2-1} \cos(\omega_p t)/N + 2 \sum_{t=1}^{N} \sum_{p=1}^{N/2-1} \sin(\omega_p t)/N + 
\sum_{t=1}^{N} (-1)^t x_t / N}.
\]
Now, \( \cos \omega_p t = \cos(2\pi pt/N) \) and \( \sin \omega_p t = \sin(2\pi pt/N) \) since \( \omega_p = 2\pi/N \). Therefore,

\[
x_t = a_0 + \sum_{p=1}^{N/2-1} [a_p \cos(2\pi pt/N) + b_p \sin(2\pi pt/N)] + a_{N/2} \cos \pi t \quad t = 1, \ldots, N,
\]

where

\[
\begin{align*}
a_0 &= \pi \\
a_p &= 2[\sum_{t=1}^{N} x_t \cos(2\pi pt/N)]/N \\
a_{N/2} &= \sum_{t=1}^{N} (-1)^t x_t/N \\
b_p &= 2[\sum_{t=1}^{N} x_t \sin(2\pi pt/N)]/N
\end{align*}
\]

as given by equations (3.43) and (3.44).

Now, we have the expression for variance \( \sigma^2 \) given by equation (3.47), and using equations (E.36)–(E.39),

\[
N\sigma^2 = \sum_{t=1}^{N} \left( \hat{\alpha} \cos \omega_p t + \hat{\beta} \sin \omega_p t \right)^2 \\
= \sum (\hat{\alpha}^2 \cos^2 \omega_p t) + \sum (\hat{\beta}^2 \sin^2 \omega_p t) + 2\sum (\hat{\alpha} \hat{\beta} \cos \omega_p t \sin \omega_p t) \\
= \hat{\alpha}^2 \sum \cos^2 \omega_p t + \hat{\beta}^2 \sum \sin^2 \omega_p t + 0 \\
= \hat{\alpha}^2 N/2 + \hat{\beta}^2 N/2 \\
= (\hat{\alpha}^2 + \hat{\beta}^2)N/2
\]

when \( p \neq N/2 \). When \( p = N/2 \), \( \sin^2 \omega_p t = 0 \) and \( \cos^2 \omega_p t = N \), thus leaving \( N\sigma^2 = \hat{\alpha}^2 N \).

Now,

\[
N\sigma^2 = (\hat{\alpha}^2 + \hat{\beta}^2)N/2 = N(a_p^2 + b_p^2)/2 = NR_p^2/2.
\]

So, from the definition of variance,

\[
\begin{align*}
\sum_{t=1}^{N} (x_t - \pi)^2 &= \sum_{p=1}^{N/2} NR_p^2/2 = N \sum_{p=1}^{N/2-1} R_p^2/2 + NR_{N/2}^2/2 \\
&= \sum_{p=1}^{N/2} NR_p^2/2 = N \sum_{p=1}^{N/2-1} R_p^2/2 + Na_{N/2}^2 \\
&= \frac{\sum_{t=1}^{N} (x_t - \pi)^2}{N} = \sum_{p=1}^{N/2-1} R_p^2/2 + a_{N/2}^2
\end{align*}
\]

as given as Parseval’s theorem in equation (3.49).

### E.5 Time of Flight For A Fast Mode Wave (§6.2.2)

Here we will derive the expression for \( R' \) as given in equation (6.7). We begin with the two equations for the ordinary differential equation (6.6) and the definition of \( F \) given in equation (6.3),

\[
R' \frac{\partial F}{\partial R'} - F = C \tag{E.44}
\]

\[
F(R, R') = \frac{\sqrt{R^2 + (R')^2}}{KR^{-3}(mn)^{-1/2}} \tag{E.45}
\]
From equation (E.45),

\[ F = \frac{R^3}{K} (mn)^{1/2} (R^2 + (R')^2)^{1/2} \]

\[ \Rightarrow \frac{\partial F}{\partial R} = \frac{R^3}{K} (mn)^{1/2} R' (R^2 + (R')^2)^{-1/2} \]

\[ \Rightarrow R' \frac{\partial F}{\partial R'} - F = \frac{R^3}{K} (mn)^{1/2} (R')^2 (R^2 + (R')^2)^{-1/2} - \frac{R^3}{K} (mn)^{1/2} (R^2 + (R')^2)^{1/2} \]

\[ = \frac{R^3}{K} (mn)^{1/2} \left[ \frac{(R')^2}{(R^2 + (R')^2)^{1/2}} - (R^2 + (R')^2)^{1/2} \right] = C. \]  

(E.46)

And so,

\[ \frac{(R')^2}{(R^2 + (R')^2)^{1/2}} - (R^2 + (R')^2)^{1/2} = \frac{KC}{(mn)^{1/2}} R^{-3} \]

\[ \Rightarrow \frac{(R')^2 - (R^2 + (R')^2)^{1/2}}{(R^2 + (R')^2)^{1/2}} = \frac{KC}{(mn)^{1/2}} R^{-3} \]

\[ \Rightarrow \frac{-(R^2)}{(R^2 + (R')^2)^{1/2}} = \frac{KC}{(mn)^{1/2}} R^{-3} \]

\[ \Rightarrow \frac{(R^2 + (R')^2)^{1/2}}{R^2 + (R')^2} = \frac{mn}{(KC)^2} R^{10} \]

\[ \Rightarrow \frac{(R')^2}{R^2 + (R')^2} = \frac{mn}{(KC)^2} R^{10} - R^2. \]

(Hence we have,

\[ (R')^2 = R^2 \left[ mn \left( \frac{R^4}{KC} \right)^2 - 1 \right] \]

\[ \Rightarrow R' = \pm R \left[ mn \left( \frac{R^4}{KC} \right)^2 - 1 \right]^{1/2}, \]  

(E.48)

which is given in equation (6.7).
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E.5. TIME OF FLIGHT FOR A FAST MODE WAVE (§6.2.2)


E.5. TIME OF FLIGHT FOR A FAST MODE WAVE (§6.2.2)


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http://nssdc.gsfc.nasa.gov/space/cgm/cgm.html


E.5. TIME OF FLIGHT FOR A FAST MODE WAVE (§6.2.2)


http://www.pitt.edu/~jrclass/stat/notes/ztable.html


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